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W. F. R. WELDON, FRANCIS GALTON AND KARL PEARSON

EDITED BY

KARL PEARSON

ASSISTED BY

EGON S. PEARSON

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VOLUME XVI DECEMBER, 1924 NOS 3 AND 4

ON AN INTERSPECIFIC HYBRID OF *DIGITALIS*.

BY ERNEST WARREN, D.Sc. (London).

Introduction.

INTERSPECIFIC hybrids are always interesting, and, if fertile in the least degree with themselves, or with either of the parent species, the results of breeding are likely to throw some light on obscure hereditary problems.

A preliminary account of this investigation was read at the Durban Session of the South African Association for the Advancement of Science* in July, 1921; and in the present place further results will be detailed, and a brief review will be given of the problem whether these experiments point to the existence of relatively rigid, unchangeable factors in the inheritance of ordinary measurable characters.

1. THE PARENT-SPECIES.

The present hybrid was obtained by crossing the common Garden Foxglove (♀) [*Digitalis gloxiniæflora* of the florists (= *D. gloxinioides* Clem., doubtless a garden variety of *D. purpurea* L.)] with a Yellow Foxglove (♂) which was supplied to me by a florist as *Digitalis lutea*. The so-called *lutea* was selfed for several generations, and in all obvious characters, at any rate, it was perfectly homozygous. There existed, however, a certain degree of sterility both with self and cross fertilization, and on this account it was thought advisable to submit specimens to the Kew Botanical Gardens for an expression of opinion as to the species to which the experimental plants belonged. Dr Arthur W. Hill, the Director, kindly replied to the effect that the plants were doubtless a fertile hybrid of *Digitalis lutea* L. × *Digitalis ambigua* Murr. There is no reason to question this determination, but the original definitions of these two species seem to be meagre and inadequate, and, since the existence of varieties and natural hybrids is admitted, it appears perhaps doubtful whether *lutea* L. and *ambigua* Murr. are really specifically distinct. In the following pages the Yellow Foxglove employed is called *lutea*, since the plants possessed a homozygous constitution, as far as could be tested, and agreed substantially with the definition of that species, except that they tended to be somewhat more hairy.

A considerable number of plant breeders have obtained hybrids between *Digitalis purpurea* (Pl. I, fig. 1) and *D. lutea* (Pl. I, fig. 2), but, as far as I am aware, these hybrids have been invariably reported as being quite sterile. In the case of the present experiments in Natal the hybrids have proved to be less completely sterile, since backcrosses have been raised with both parent species, and plants have also been obtained both by selfing the hybrids and by crossing them together.

* Warren, E., *South African Journal of Science*, Vol. XVIII. pp. 359—373, 1922.

2. Dominance of One of the Parents.

The hybrid between *gloxiniæflora* (= *purpurea* var.) and *lutea* differs from many other plant hybrids in that it is not markedly intermediate, at any rate in its general aspect or more obvious characters, between the two parent species. It is stated* that the hybrid is considerably nearer to the female parent (matroclinous) than to the male parent. Thus, the reciprocal crosses should tend to be conspicuously different from each other, the offspring of *purpurea* (♀) × *lutea* (♂) resembling *purpurea* more or less closely, and those of *lutea* (♀) × *purpurea* (♂) resembling *lutea*.

In the case of the Natal plants, I have only succeeded in obtaining the hybrid *gloxiniæflora* (♀) × *lutea* (♂) (Pl. I, fig. 5) and there is no question but that it resembles *gloxiniæflora* in general size and appearance much more closely than *lutea*. I am inclined to doubt, however, whether the reciprocal cross would be so distinctly matroclinous (i.e. whether the offspring of *lutea* (♀) × *gloxiniæflora* (♂) would markedly resemble *lutea*), for recently Professor J. Ben Hill of the Pennsylvania State College has kindly sent to me photographs of the hybrid plants and flowers which he has obtained by crossing *purpurea* (♀) × *ambigua* (♂) and *ambigua* (♀) × *purpurea* (♂) and it is quite clear that these reciprocal crosses are not so different from each other as might be expected, judging from certain published statements. For example, in the cross *ambigua* (♀) × *purpurea* (♂) the purple spots (totally absent in *lutea* and doubtless also in *ambigua*) of the flowers of the male parent (*purpurea*) are transmitted to the hybrid, while the general shape of the calyx and corolla appears to be intermediate rather than definitely like that of *ambigua*. Further, the general leaf-shape would seem to be not very dissimilar in the two reciprocal hybrids. The evidence therefore would seem to indicate that a matroclinous tendency is more evident when *purpurea* is the female parent than when *lutea* is the seed-plant.

In this connection the results of backcrossing are remarkable. The backcross plants hybrid (♀) × *glox.* (♂), (Pls. I, II, III, figs. 7, 9, 14, 15), are similar in shape of leaf and in floral characters to the reciprocal backcross, *glox.* (♀) × hybrid (♂) (Pl. II, fig. 12), and they differ but little from the hybrid itself; but, on the other hand, the plants obtained from the backcross, hybrid (♀) × *lutea* (♂), are strongly patroclinous, being in all conspicuous characters very similar to the pure *lutea* (Pl. III, figs. 17, 18, 19). I feel some confidence that a repetition of the whole experiment would only confirm the occurrence of these well-marked, definite differences in the nature of the dominance or prepotency of one or the other parent in the various crosses.

3. Fertility.

The accompanying Table I will give some notion of the degree of sterility, and the intensity of the germinating power of the seeds and of the vigour of the seedlings in the various crosses which were made.

* Babcock, E. B. and Clausen, R. E., *Genetics in Relation to Agriculture*, p. 229, 1918, New York.

(1) *Parents.* Selfed *glox.* exhibited a high degree of fertility, but selfed *lutea* showed considerable sterility, in that rather more than half of the pollinations failed to be effective.

(2) *Species crossings.* In the cross, *glox.* (♀) × *lutea* (♂), rather less than one-third of the pollinations were effective (see Table I). The capsules grew to a normal size, but later they became creased. Most of the ovules seemed to be fertilized and grew to a certain size, and then very many ceased to develop further*. Only about 4 seeds per capsule were capable of producing seedlings sufficiently large and vigorous to survive. The seeds were markedly dimorphic in size; on the average there were 4 large, sound seeds and 200 small seeds per fertile capsule, the latter were, however, capable of germinating, but the seedlings were too weak to survive.

TABLE I.

Fertility Table.

Pairings	Number of parents used	Number of pollinations	Number of capsules with good seed	Average number of seeds per fertile capsule	Number of seeds used	Number of seeds which germinated	Number of seedlings raised
Glox. (selfed)	50 at least	300	230	400	?	?	3000
Lutea (selfed)	6	60	25	200	?	?	50
Hybrids:							
Glox. (♀) × *lutea* (♂)	13 *glox.* × 4 *lutea*	70	21	4 (large)	84	?	33
				200 (small)	4200	many	0
Lutea (♀) × *glox.* (♂)	4 *lutea* × 3 *glox.*	30	5	1	5	1	0
F_2 generation:							
Hybrid *A* (selfed)	1	60	5	20	100	39	28
Hybrid *A* (♀) × hybrid *B* (♂) ...	*A* × *B*	25	2	23	46	18	12
Backcrosses:							
Hybrid *A* (♀) × *glox.* (♂) ...	1 hybrid × 5 *glox.*	40	17	4	68	?	11
Glox. (♀) × hybrid *A* (♂) ...	2 *glox.* × 1 hybrid	10	3	10	30	?	1
Hybrid *A* (♀) × *lutea* (♂) ...	1 hybrid × 4 *lutea*	25	7	14	98	?	30
Backcross offspring:							
[Hybrid *A* (♀) × *glox.* (♂)] selfed	4 *glox.*-backcross	25	0	0	0	0	0
[Hybrid *A* (♀) × *lutea* (♂)] selfed	4 *lutea*-backcross	40	12	125	395	310	180

In the reciprocal cross, *lutea* (♀) × *glox.* (♂), there was much greater sterility, since only about one-sixth of the pollinations were fruitful, and there was on the average only one reasonably good seed per fertile capsule, and no small seeds capable of germinating were observed. Out of 5 apparently sound seeds only one germinated, but the seedling lacked vigour and died.

* An account of the histology of these abortive ovules and seeds is given in my paper previously cited.

(3) *Hybrids.* Out of 84 large seeds 33 plants were raised and 32 of these flowered (see Table I). In most of the plants the pollen was entirely defective, the pollen grains were shrivelled and deformed and often very scanty; the anthers were very slow in opening, and when they did dehisce the pollen was sometimes caked into a mass and quite worthless. In the case of 4 plants, however, the pollen was not so hopelessly bad, and examination with the microscope revealed some grains apparently sound. These 4 hybrids were selfed, crossed with one another and backcrossed with both parent species. Only two (*A* and *C*) of the plants appeared to be capable of producing seeds which could germinate, while three (*A*, *B*, *C*) possessed effective pollen. When plant *B* was selfed, crossed with *A* pollen, or backcrossed with *glox.* pollen, the ovules were duly fertilized and exceptionally large capsules were formed. After a considerable period the capsular wall began to crease and further development of the young seeds stopped abruptly and no fully grown seeds were produced. In the case of Hybrid *A* crossed with *B* pollen a similar course of events occurred, only instead of all the young seeds ceasing development a few reached maturity.

It is interesting to note that the effectiveness, or otherwise, of the sexual elements of these plants was dependent on very delicate conditions, both innate and environmental. Plant *A* carried two flowering stalks at the same time; both produced capsules, but on one stalk none of the capsules bore a single mature seed, while on the other stalk over 8 per cent. of the capsules bore some good seeds. On very hot, dry days, when the anthers dehisced, the pollen was found to be caked and useless, while on cool days the pollen was normal in general aspect and had some fertilizing power. Such changes in the environment had no obvious influence on the parent species.

On selfing Hybrid *A* one-twelfth of the capsules contained mature seed; five of such capsules were obtained with an average of 20 seeds per capsule.

On crossing Hybrid *A* (♀) with Hybrid *B* (♂) a similar result was found.

Backcrossing Hybrid *A* with the pollen of *glox.* and *lutea* showed a somewhat increased fertility in the relative number of capsules producing good seed (see Table I).

The mature seeds formed by Hybrid *A* did not exhibit dimorphism in size, since the minute seeds capable of germinating, which were formed alongside the large seeds in the capsules of the first cross, did not occur; in other words, the great majority of the seeds ceased growing so early that they were incapable of becoming small mature seeds.

(4) *Second Generation of Hybrids* (F_2). The seeds obtained from Hybrid *A* were 146 in number and these were placed on damp blotting paper in order that the number capable of germinating might be ascertained with accuracy. Fifty-seven, or a little over one-third, of the seeds germinated (see Table I). From these, 40 healthy young plants with three leaves were raised. Unfortunately 36 of these plants were destroyed by a rat in one night. The remaining four plants grew fairly well, but with no excessive vigour, and they did not flower during that

season. In the subsequent season only one plant flowered, the other three plants gradually died. The pollen from this one plant was exceedingly scanty and many of the anthers failed to dehisce. I could not find a single good pollen grain under the microscope. Pollination with *glox.* pollen was tried and fair-sized capsules were formed, but not a single mature seed was obtained.

(5) *Backcrosses.* With Hybrid *A* fertilized with *glox.* pollen, 68 mature seeds were obtained from 40 pollinations and 11 plants were raised (see Table I). With a *glox.* plant crossed with pollen of Hybrid *A*, 30 mature seeds were obtained from 10 pollinations, but they lacked vigour and only one plant was raised. This plant flowered (Pl. II, fig. 12) and was completely sterile.

From Hybrid *A* fertilized with *lutea* pollen 98 mature seeds were obtained from 25 pollinations and 30 plants were raised. Thus, in the backcross with *lutea* there was a markedly greater fertility than in the backcross with *glox.* and there was obviously a greater germinating power in the seeds.

Of the 11 *glox.*-backcross plants only 5 flowered and these exhibited exceptional vigour in their leaf-development. The remaining plants were mostly healthy and grew to an ordinary size, but they did not flower. The five plants were entirely sterile both with their own pollen, which was exceedingly scanty and defective, and with *glox.* pollen.

The nature of the sterility of the *lutea*-backcross plants differed considerably from that of the *glox.*-backcrosses. Of the 30 *lutea*-backcross plants 11 flowered, and of these, 7 appeared to be sterile, although the pollen was fairly abundant and normal in appearance. The remaining four plants were very fairly fertile with their own pollen. Out of 40 pollinations 12 capsules were obtained containing on the average 125 seeds per capsule. There was no dimorphism in seed-size and the seeds germinated vigorously and many plants were raised.

Thus I arrived at the unexpected result that although the *lutea* plants used for the experiment were only moderately fertile when fertilized with themselves, while the *glox.* plants were very fertile, yet in backcrossing the hybrid of these two species the pollen of the *lutea* plants was more effective than that of the *glox.* and further, some of the *lutea*-backcross plants were quite fertile, but the *glox.*-backcross plants were apparently completely sterile.

(6) *Offspring of selfed lutea-backcross plants.* It was not possible to investigate in detail the fertility of these plants, but the pollen appeared to be good, and from casual fertilization by bees it was clear that a considerable number of the plants could produce good seed.

4. Comparison of the Different Generations with Respect to Various Characters.

We will now compare the means and variation distributions in the different generations of a number of ordinary, racial, measurable characters of the leaf and flower.

(1) *Leaf.*

(*a*) *Ratio.* A markedly contrasted character is given by the relatively wide leaves of *gloxiniæflora* and the narrow leaves of *lutea*. The ratio $\frac{\text{Breadth}}{\text{Length}} \times 1000$ gives a useful index of the general shape. The distribution of these ratios in the

TABLE II.

Leaf-Ratio $\frac{Breadth}{Length} \times 1000$.

Grades of Ratios × 1000	*Glox.*	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid × hybrid	Hybrid *Glox.* (♀) × *lutea* (♂)	Offspring of selfed *Lutea*-backcross	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	(*PP* × *YY*) × (*PP* × *YY*)	*PP* × *YY*	(*PY* × *YY*) × (*PY* × *YY*)	*PY* × *YY*	*YY* × *YY*
100—129	—	—	—	1	—	3	7
130—159	—	—	—	1	15	4	19
160—189	—	1	—	11	35	5	16
190—219	7	1	—	8	37	1	5
220—249	17	0	—	5	7	3	2
250—279	17	2	1	1	—	1	—
280—309	13	2	1	5	—	1	—
310—339	22	3	1	1	—	—	—
340—369	5	2	0	—	—	—	—
370—399	3	1	0	—	—	—	—
400—429	1	—	1	—	—	—	—
Totals	85	12	4	33	94	18	49
Means	285·0	297·0	324·5	213·6	186·0	181·2	159·8
Standard Deviation	48·75	59·40	56·13	49·83	25·18	51·54	29·77
Coefficient of Variation	17·1	20·0	17·3	23·4	13·5	28·4	18·6

various pairings is shown in the accompanying table. From a Mendelian aspect the gametic constitution of the different generations is represented by the usual symbols, where *P* means purple foxglove or *glox.* determinant and *Y* signifies yellow foxglove or *lutea* determinant.

Inspection of the table will show that in this character the mean of the hybrid is approximately intermediate between the means of *glox.* and *lutea*; also, the *lutea*-backcross and the offspring of selfed *lutea*-backcross plants are both nearly intermediate between *lutea* and the hybrid. The means of the *glox.*-back-

cross and of the offspring of the hybrid are seen to be less conformable, and since this is the case in a number of other characters it is doubtful if it is due simply to the smallness of the numbers, and to the fact that only one hybrid plant was available as seed-plant. Probably dominance or prepotency of one of the parents comes into play.

With regard to the absolute size of the leaves, the hybrids (Pl. I, fig. 5) tended to have larger leaves than even the pure *glox.*, and this was also the case with the *glox.*-backcross (Pls. I, II, III, figs. 7, 9, 14, 15).

(*b*) *Pilosity. Glox.* is pubescent and *lutea* is described as glabrous. As a measure of the hairiness of a leaf the number of hairs occurring on a definite length of leaf-margin was counted. The mean of the number of hairs on ten lengths of 2 mm. each near the middle of a typical leaf was taken as an index of the pilosity of the plant. The distribution of leaf-pilosity is given in the accompanying table.

TABLE III.

Pilosity.

Average number of hairs projecting over a length of 2 mm. of leaf-edge	*Glox.*	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid × hybrid	Hybrid *Glox.* (♀) × *lutea* (♂)	Offspring of selfed *Lutea*-backcross	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	(*PP* × *PY*) × (*PP* × *PY*)	*PP* × *PY*	(*PY* × *YY*) × (*PY* × *YY*)	*PY* × *YY*	*YY* × *YY*
0— 4	—	—	—	—	2	3	9
5— 9	—	—	3	3	10	8	12
10—14	—	2	0	4	23	6	4
15—19	1	3	1	10	16	3	1
20—24	3	3	—	5	9	1	—
25—29	3	1	—	3	2	1	—
30—34	3	—	—	1	0	1	—
35—39	5	—	—	2	1	—	—
40—44	2	—	—	0	—	—	—
45—49	2	—	—	0	—	—	—
50—54	4	—	—	0	—	—	—
55—59	1	—	—	1	—	—	—
Totals	24	9	4	29	63	23	26
Means	37·0	18·7	9·5	20·4	14·5	11·6	6·4
Standard Deviation	11·18	4·97	4·33	10·35	5·85	7·50	4·00
Coefficient of Variation	30·2	26·6	45·6	50·7	40·3	64·7	62·5

It will be seen that the mean of the pilosity of the hybrids is very nearly intermediate between the mean hairiness of *glox.* and of *lutea*; also, *lutea*-backcross and the offspring of selfed *lutea*-backcross are approximately intermediate between *lutea* and the *hybrid.* The *glox.*-backcross and the offspring of selfed or fraternally fertilized hybrid are less conformable, as in the case of the leaf-ratio.

(*c*) *Thickness of Leaf-lamina.* Here also we have a strongly contrasted character, the thickness of leaf being much greater in *lutea* than in *glox.* Sections of fully grown leaves were made, and the vertical distance between the upper and lower surfaces was measured under the microscope with an ocular micrometer. Several measurements were made on the same leaf and the mean was taken as the leaf-thickness of the plant. The distribution of leaf-thickness in the offspring of the various pairings is given in the accompanying table.

TABLE IV.

Thickness of Lamina.

Grades in divisions of ocular micrometer ×10 (1 division = 0·0027 mm.)	*Glox.*	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid × hybrid	Hybrid *Glox.* (♀) × *lutea* (♂)	Offspring of selfed *Lutea*-backcross	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	(*PP* × *YY*) × (*PP* × *YY*)	*PP* × *YY*	(*PY* × *YY*) × (*PY* × *YY*)	*PY* × *YY*	*YY* × *YY*
40— 47	2	—	1	1	—	—	—
48— 55	1	3	0	3	—	—	—
56— 63	7	3	2	8	—	1	—
64— 71	—	4	0	4	2	2	—
72— 79	—	—	1	1	4	3	—
80— 87	—	—	—	—	9	3	1
88— 95	—	—	—	—	5	5	2
96—103	—	—	—	—	2	1	0
104—111	—	—	—	—	1	1	1
112—119	—	—	—	—	—	0	1
120—127	—	—	—	—	—	1	2
128—135	—	—	—	—	—	—	3
Totals	10	10	4	17	23	17	10
Means	55·5	60·2	59·5	60·0	84·9	85·8	113·1
Standard Deviation	6·93	6·64	11·31	7·50	9·63	15·22	17·54
Coefficient of Variation	12·5	11·0	18·9	12·5	11·4	17·7	15·5

It will be seen that the mean leaf-thickness of *glox.* is only about one-half of that of *lutea*. This difference is due entirely to the great amount of spongy parenchyma in the latter species, since the palisade tissue is thinner and much less regular in *lutea* than in *glox.*

In this character the hybrid is by no means intermediate, in that the mean leaf-thickness is only very slightly greater than that of *glox.* Thus, the female *glox.* parent is dominant or prepotent in this character. On the other hand the *lutea*-backcross and the offspring of selfed *lutea*-backcross are strikingly intermediate between *lutea* and the hybrid.

(*d*) *Venation of the leaves.* In *glox.* the coarser veins of the leaf tend to run in two systems, a longitudinal series and a transverse series, the latter being more or less at right angles to the former. These veins project considerably on the under surface of the leaf. Thus, the lamina of the leaf is divided by the coarser veins

TABLE V.

Angle of Slope of Central Bundle of Indentations.

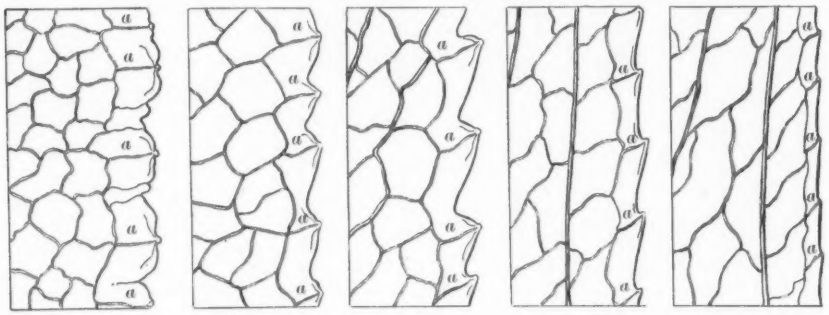
Angle made by central bundle of indentations with perpendicular from long-axis of leaf. Grades of Degrees.	*Glox.*	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid × hybrid	Hybrid *Glox.* (♀) × *lutea* (♂)	Offspring of selfed *Lutea*-backcross	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	(*PP* × *PY*) × (*PP* × *PY*)	*PP* × *PY*	(*PY* × *YY*) × (*PY* × *YY*)	*PY* × *YY*	*YY* × *YY*
0— 5	12	3	—	—	—	—	—
6—11	10	4	—	1	—	—	—
12—17	2	3	3	4	—	—	—
18—23	2	0	0	6	4	2	—
24—29	—	1	1	8	2	1	—
30—35	—	—	—	7	5	4	—
36—41	—	—	—	1	8	7	1
42—47	—	—	—	—	3	2	2
48—53	—	—	—	—	3	3	5
54—59	—	—	—	—	1	2	1
60—65	—	—	—	—	1	—	3
66—71	—	—	—	—	2	—	3
72—77	—	—	—	—	—	—	1
78—83	—	—	—	—	—	—	4
Totals	26	11	4	27	29	21	20
Means	7·1	10·1	17·5	23·7	39·5	39·1	61·3
Standard Deviation	5·34	6·81	5·20	7·27	13·92	9·95	13·23
Coefficient of Variation	75·4	67·5	29·7	30·7	35·2	25·5	21·5

into irregular squarish or polygonal compartments or islands. In *glox.* the margin of the leaf is crenate, and running out to the apex of each eminence there is typically a stout, short vein directed nearly at right-angles to the long-axis of the leaf. Towards the apex of the leaf these veins tend to have a slight, forward slope (Text-fig. 1, see below).

In *lutea* we also have two systems of coarser veins, the one corresponding to the transverse series of *glox.* being set at an angle of about 30° to the long-axis of the leaf. The coarser veins of both systems are fewer than in the case of *glox.* and thus the leaf is divided into larger compartments which tend to have the shape of obliquely placed, elongated parallelograms. The indentation of the margin is slight in *lutea*, and corresponding to the fewness of the coarser veins the indentations are sometimes wider apart than in *glox.* Running out to the apex of each eminence there is usually one clearly defined middle vein set at an angle of about 60° to the perpendicular from the long-axis of the leaf. In *glox.* this angle is only about 7°. It is clear that the nature of the margin of a leaf depends very largely on the character of the venation.

As a readily measurable venation character the slope of the middle bundle running to the apex of the leaf-indentations was taken. We have seen above that this angle is markedly different in the two parent species, and it serves as a kind of measure of the general venation differences in the offspring of the various pairings. The actual angle measured was that made by the bundle with the perpendicular to the long-axis of the leaf.

In *glox.* the angle is about 7°, in *lutea* it is as much as 61° and in the hybrid it is 24°. Thus, in this character the hybrid is not halfway between the two parent species, but it is nearer to *glox.* The *lutea*-backcross and the offspring of selfed *lutea*-backcross are nearly intermediate between *lutea* and the hybrid. The venation of the different generations may be studied in some detail in Pl. VI.



Text-fig. 1. Diagrams of the indented margin and of the coarser venation of the under surface of the leaf of the various generations. Note the differences in the angle of slope of the central bundles (*a*) of the indentations. The coarser venation produces small and deep squarish or polygonal compartments in *glox.* (*P*), while in *lutea* (*Y*) the compartments are large and very shallow, obliquely placed parallelograms. ×4·5.

(*e*) *Indentations of Leaf. Glox.* is obviously crenate, while *lutea* often has a margin which is practically entire, but indications of teeth can always be found and sometimes there is a weak serration. The indentations of the hybrid are usually much more developed than in either of the parent species, these indentations being of the nature of serration with teeth pointing forwards (Pl. VI). The height of an indentation was measured by the vertical depth of the out-cutting of the leaf-edge taken from the tangential plane of the margin of the leaf (Text-fig. 1). About 12 measurements were made on each leaf and the mean was taken. The distribution of indentation-depth is given in the accompanying table.

Here we have a peculiarity not previously met with in the characters that have been dealt with, namely that the mean of the hybrid does not lie somewhere between the means of the two parent species, but the character is intensified. The margin of the hybrid is not crenate, but is often deeply serrate with the

TABLE VI.

Indentations of Leaf.

Depth of indentations in divisions of ocular micrometer (1 division = 0·074 mm.)	*Glox.*	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid × hybrid	Hybrid *Glox.* (♀) × *lutea* (♂)	Offspring of selfed *Lutea*-backcross	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	(*PP* × *YY*) × (*PP* × *YY*)	*PP* × *YY*	(*PY* × *YY*) × (*PY* × *YY*)	*PY* × *YY*	*YY* × *YY*
0— 2	1	1	—	—	11	13	12
3— 5	4	2	—	1	22	5	7
6— 8	6	2	—	12	11	3	1
9—11	8	3	—	3	16	0	—
12—14	4	1	—	1	2	1	—
15—17	2	0	2	3	—	—	—
18—20	2	0	1	2	—	—	—
21—23	2	1	1	2	—	—	—
24—26	—	0	—	1	—	—	—
27—29	—	0	—	2	—	—	—
30—32	—	0	—	—	—	—	—
33—35	—	0	—	—	—	—	—
36—38	—	1	—	—	—	—	—
Totals	29	11	4	27	62	22	20
Means	10·5	11·4	18·2	12·7	5·8	3·0	2·3
Standard Deviation	5·35	9·69	2·49	7·11	3·42	3·05	1·77
Coefficient of Variation	50·9	85·0	13·7	55·9	58·9	101·7	77·0

teeth pointing forwards towards the apex of the leaf. In the *lutea*-backcross and in the offspring of selfed *lutea*-backcross the mean of the character is not half-way between the means of the parents, but is nearer to that of *lutea* (Pl. VI, figs. 1*a*—3*g* on the left).

(*f*) *Rate of Desiccation of Leaf.* It happened to be noticed that plucked *lutea* leaves withered much more quickly than *glox.* leaves. Such loss of water by leaves is, of course, of great physiological importance to the living plant, and two series of careful weighings with a chemical balance were undertaken. The larger series was weighed in April when the plants had finished flowering and the leaves

TABLE VII.

$$Ratio\ \frac{Weight\ of\ water\ lost\ in\ 4\ hours}{Original\ weight\ of\ leaves} \times 1000.$$

Grades of Ratios ×1000	April 1st—10th, 1922					November 2nd—7th, 1922		
	Glox.	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid *Glox.* (♀) × *lutea* (♂)	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*	*Glox.*	Offspring of selfed *Lutea*-backcross	*Lutea*
	PP × *PP*	*PY* × *PP*	*PP* × *YY*	*PY* × *YY*	*YY* × *YY*	*PP* × *PP*	(*PY* × *YY*) × (*PY* × *YY*)	*YY* × *YY*
50— 99	—	—	—	—	—	1	—	—
100—149	3	—	—	—	—	9	—	—
150—199	3	1	—	—	—	5	—	1
200—249	9	1	—	—	—	2	2	1
250—299	6	2	—	—	—	4	8	4
300—349	5	0	—	—	—	1	14	5
350—399	1	0	—	2	—	1	10	7
400—449	0	1	1	0	—	3	8	8
450—499	0	0	3	1	3	2	9	5
500—549	1	2	3	2	1	—	1	4
550—599	—	0	1	3	4	—	1	1
600—649	—	1	3	2	4	—	—	2
650—699	—	—	1	3	2	—	—	—
700—749	—	—	—	1	2	—	—	—
Totals	28	8	12	14	16	28	53	38
Means	253·1	380·8	545·3	570·9	596·4	233·4	371·6	406·1
Standard Deviation	82·85	154·99	73·19	104·30	79·00	121·05	79·15	102·85
Coefficient of Variation	32·7	40·7	13·4	18·2	13·3	51·9	21·3	25·3

were getting old. A smaller series was weighed in November when the plants were actually in flower and the leaves were younger and more vigorous. It will be seen that the younger leaves when plucked lose water less quickly than older leaves. Two or three mature leaves from each plant were picked and immediately weighed all together, and then laid on wire netting in order to allow free access of air. After a period of 4 hours the leaves were weighed again, and the loss of weight represented the weight of water which had been transpired by a given weight of fresh leaves.

The distribution of the ratios for the two different months is given in our Table VII.

We have already seen that the leaf-lamina of *lutea* is much thicker than that of *glox.* and we should therefore perhaps expect that desiccation would be slower in *lutea*, but this is very far from being the case. The structure of the leaves in the two species, however, gives us some insight into the causes which lead to this

TABLE VIII.

Number of Epidermal Cells in a given Length of Leaf.

Average number of epidermal cells of lower surface of leaf in diameter (=0·353 mm.) of microscope field	*Glox.*	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid × hybrid	Hybrid *Glox.* (♀) × *lutea* (♂)	Offspring of selfed *Lutea*-backcross	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	(*PP* × *YY*) × (*PP* × *YY*)	*PP* × *YY*	(*PY* × *YY*) × (*PY* × *YY*)	*PY* × *YY*	*YY* × *YY*
7·0— 7·9	—	—	—	—	—	—	3
8·0— 8·9	—	—	1	1	2	—	4
9·0— 9·9	—	2	1	3	9	8	11
10·0—10·9	—	2	1	4	8	7	4
11·0—11·9	—	1	1	1	19	6	3
12·0—12·9	—	3	—	4	9	1	—
13·0—13·9	1	0	—	3	2	—	—
14·0—14·9	2	1	—	5	2	—	—
15·0—15·9	6	1	—	2	1	—	—
16·0—16·9	6	—	—	1	—	—	—
17·0—17·9	3	—	—	1	—	—	—
18·0—18·9	2	—	—	—	—	—	—
Totals	20	10	4	24	52	22	25
Means	16·2	11·9	10·0	12·9	11·3	10·4	9·4
Standard Deviation	1·27	1·91	1·12	2·25	1·46	0·90	1·13
Coefficient of Variation	7·84	16·05	11·20	17·44	12·92	8·70	12·03

unexpected result. In *lutea* the palisade cells are very short and irregularly spaced and sometimes scarcely distinguishable, while the spongy parenchyma although abundant bears large intercellular spaces. In *glox.* the palisade cells are deeper, more regularly placed and are mostly in mutual contact. Further, the venation system is weaker in *lutea* than in *glox.*, and Professor J. W. Bews* of the Natal University College has suggested that a more rigid venation tends to retard evaporation by holding out the lamina and preventing collapse of the tissues.

In April picked mature *lutea* leaves lose more than half their weight in water in the course of 4 hours, while *glox.* leaves only lose about one-quarter. In this

TABLE IX.

Number of Stomates in a given Area of Leaf.

Number of stomates in microscope field with diameter of 0·353 mm.	*Glox.*	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid × hybrid	Hybrid *Glox.* (♀) × *lutea* (♂)	Offspring of selfed *Lutea*-backcross	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	(*PP* × *YY*) × (*PP* × *YY*)	*PP* × *YY*	(*PY* × *YY*) × (*PY* × *YY*)	*PY* × *YY*	*YY* × *YY*
5— 6	—	—	1	4	—	—	3
7— 8	—	1	1	7	4	9	6
9—10	—	3	1	2	12	8	8
11—12	—	3	1	4	20	3	6
13—14	—	2	—	3	12	2	2
15—16	1	0	—	1	3	—	—
17—18	1	1	—	0	1	—	—
19—20	1	—	—	0	—	—	—
21—22	0	—	—	3	—	—	—
23—24	6	—	—	—	—	—	—
25—26	5	—	—	—	—	—	—
27—28	2	—	—	—	—	—	—
29—30	2	—	—	—	—	—	—
31—32	0	—	—	—	—	—	—
33—34	2	—	—	—	—	—	—
35—36	1	—	—	—	—	—	—
Totals	21	10	4	24	52	22	25
Means	25·6	11·5	8·5	10·8	11·5	9·7	9·3
Standard Deviation	4·84	2·68	2·24	4·92	2·17	1·90	2·26
Coefficient of Variation	18·9	23·3	26·4	45·5	18·9	19·6	24·4

* Bews, J. W. and Aitken, R. W., "Researches on the Vegetation of Natal," Series I, Memoir 5, *Botanical Survey of South Africa*, Pretoria, 1923.

character the hybrid leaves are not half-way between *lutea* and *glox.* but are very much nearer to *lutea.* The *lutea*-backcross plants are nearly intermediate between *lutea* and the hybrid and similarly the *glox.*-backcross plants are nearly intermediate.

In November when the leaves are not so old and more vigorous the loss of water in *lutea* is about two-fifths instead of three-fifths.

(*g*) *Size of Cells of Lower Epidermis of Leaf.* The number of cells cut through by a diameter of the microscope field gives a measure of the cell-size. These cells are considerably larger in *lutea* than in *glox.* The number of cells extending over a diameter was determined for 10 different fields in each leaf and the mean taken. The distribution of this character in the various generations is given in Table VIII on p. 217.

The mean number of cells extending over the given diameter is about 9 in *lutea* and 16 in *glox.* while in the hybrid the number is almost exactly inter-

TABLE X.

Length of Guard-cells of Leaf.

Grades of number of divisions of ocular micrometer ×10 (1 division = 0·0041 mm.)	*Glox.*	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid × hybrid	Hybrid *Glox.* (♀) × *lutea* (♂)	Offspring of selfed *Lutea*-backcross	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	(*PP* × *YY*) × (*PP* × *YY*)	*PP* × *YY*	(*PY* × *YY*) × (*PY* × *YY*)	*PY* × *YY*	*YY* × *YY*
50— 53	—	—	—	1	—	—	—
54— 57	5	—	—	1	—	—	—
58— 61	13	1	—	2	—	—	—
62— 65	6	0	1	5	—	—	—
66— 69	2	2	0	3	2	1	—
70— 73	—	6	2	1	17	3	2
74— 77	—	0	0	1	9	7	1
78— 81	—	2	1	4	10	3	4
82— 85	—	—	—	3	6	2	9
86— 89	—	—	—	0	5	1	7
90— 93	—	—	—	1	3	1	1
94— 97	—	—	—	1	—	3	0
98—101	—	—	—	—	—	—	1
Totals	26	11	4	23	52	21	25
Means	60·3	71·1	71·5	71·7	77·7	80·1	83·7
Standard Deviation	3·33	5·24	5·66	11·40	6·66	8·31	5·82
Coefficient of Variation	5·52	7·37	7·92	15·9	8·57	10·4	6·95

mediate. The numbers in the *lutea*-backcross and in the offspring of selfed *lutea*-backcross are approximately intermediate between *lutea* and the hybrid. The *glox.*-backcross and the offspring of the selfed hybrid are erratic, possibly through the smallness of the series and the restricted parentage, but very likely the hybrid is prepotent.

(*h*) *Number of Stomates of Leaf.* In a given area of the lower epidermis of the leaf the number of stomates is much fewer in *lutea* than in *glox.* All the counting of the stomates was done under the same sized microscope field. In each leaf the number of stomates in 10 different fields was determined and the mean was taken.

TABLE XI.

Length of Corolla.

Grades of length of corolla in 0·1 mm.	*Glox.*	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid × hybrid	Hybrid *Glox.* (♀) × *lutea* (♂)	Offspring of selfed *Lutea*-backcross	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	(*PP* × *YY*) × (*PP* × *YY*)	*PP* × *YY*	(*PY* × *YY*) × (*PY* × *YY*)	*PY* × *YY*	*YY* × *YY*
290—309	—	—	—	1	—	—	1
310—329	—	—	—	0	3	1	1
330—349	—	—	—	1	3	3	3
350—369	—	—	—	0	3	2	9
370—389	—	—	—	0	—	4	3
390—409	—	—	—	0	—	—	1
410—429	—	—	—	3	—	—	—
430—449	1	1	—	5	—	—	—
450—469	1	0	—	5	—	—	—
470—489	3	1	1	5	—	—	—
490—509	5	2	—	8	—	—	—
510—529	1	0	—	3	—	—	—
530—549	4	0	—	—	—	—	—
550—569	0	2	—	—	—	—	—
570—589	0	—	—	—	—	—	—
590—609	0	—	—	—	—	—	—
610—629	1	—	—	—	—	—	—
Totals	16	6	1	31	9	10	18
Means in mm.	51·8	50·6	—	46·3	33·9	35·7	35·6
Standard Deviation	4·06	4·27	—	4·82	1·63	2·09	2·23
Coefficient of Variation	7·84	8·44	—	10·41	4·81	5·83	6·26

In *lutea* the average number of stomates in a given area is 9, and in *glox.* about 26, while in the hybrid, *lutea* is very strongly dominant, in that the average number is only 11: also, in both backcrosses the number is still about 11. Thus, in this character there is no approximation at all to intermediate conditions. See Table IX, p. 218.

(*i*) *Length of Guard-cells of Leaf.* The stomates are fewer, but considerably larger in *lutea* than in *glox.* The lengths of the guard-cells of 15 stomates in the lower epidermis of a mature leaf were determined and the mean was taken as representing the characteristic length of the guard-cells of that plant.

In *lutea* the mean length is 34μ and in *glox.* 25μ and in the hybrid 29μ; thus, the hybrid is exactly intermediate. The *lutea*-backcross (32μ) and the offspring of the selfed *lutea*-backcross (33μ) are also practically intermediate between the hybrid and *lutea.* On the other hand, *glox.*-backcross plants (28μ), although few in number, show no tendency to be intermediate and they do not differ appreciably from the hybrid itself. See Table X, p. 219.

(2) *Flower.*

(*a*) *Length of Corolla.* In the inheritance of floral characters we may find marked dominance on the part of one or the other parent species, and this is the case in the corolla-length. The length of the corolla was measured on the lower surface with the lower lip pressed into the long-axis. Whenever possible at least six of the first formed flowers were measured and the mean was taken as the characteristic corolla-length of the plant.

In *lutea* the mean length is about 36 mm. and in *glox.* it is 52 mm. while the mean length of the hybrid (46 mm.) is not quite intermediate, it being further removed from *lutea* than from *glox.* The *lutea*-backcross (36 mm.) and the offspring of the selfed *lutea*-backcross (34 mm.) are practically indistinguishable from the pure *lutea* species. In like manner, the *glox.*-backcross is closely similar to the pure *glox.* species. See Table XI, p. 220.

(*b*) *Colour.* The flowers of *lutea* were either primrose-yellow or buff in colour with a characteristic marbled pattern of olive on the lower inner surface (Pls. IV and V). A small patch of purple of varying intensity often occurred at the insertion of the filaments of the stamens. The flowers of *glox.* were either white over the general surface, or the corolla possessed a more or less intense purple colouration, but the lower inner surface of the flower in the experimental plants always bore a deep purple spotting of varying extent. The hybrid had a much redder appearance than the parent *glox.*, and this was one of the most characteristic features of all the plants. The hybrids also exhibited a considerable, but variable, amount of primrose yellow, and there was some appearance of *lutea* marbling especially on the sides and sometimes on the lower lip on the inner surface of the corolla, but this marbling was reproduced in light purple and not in olive.

The colours of the flowers have been compared with some care with Robert Ridgway's Colour Standards, in order to obtain a comparative idea of the constitution of the colours in the parents and hybrid, and the result of the diagnosis is given in the accompanying table.

TABLE XII.

Diagnosis of Colours of Flowers.

Generation	*D. glox.*		*D. lutea*					Hybrid	
Character	Purple flower (1)	Purple flower (2)	Yellow flower (1)	Buff flower (2)	Marbling pattern (1)	Marbling pattern (2)	Marbling pattern (3)	Red flower (1)	Red flower (2)
Name of Colour	Light Mallow Purple	Liseran Purple	Primrose Yellow	Deep Colonial Buff	Light Yellowish Olive	Isabel Colour	Ecru Olive	Light Rosolane Purple and Yellow	Tyrian Pink and Yellow
Constituent colours:									
Red °/₀	52	52	—	—	—	—	—	74	74
Violet °/₀	48	48	—	—	—	—	—	26	26
Tone:									
Colour °/₀ ...	77·5	90·5	77·5	77·5	55	55	55	77·5	90·5
White °/₀ ...	22·5	9·5	22·5	22·5	—	—	—	22·5	9·5
Black °/₀ ...	—	—	—	—	45	45	45	—	—
Broken colour scale:									
Colour °/₀ ...	100	68	42	42	42	42	42	68	100
Neutral Gray °/₀ ...	0	32	58	58	58	58	58	32	0
Constituent colours:									
Orange °/₀ ...	—	—	0	25	0	47	25	0	0
Yellow °/₀ ...	—	—	100	75	100	53	75	100	100

In *glox.* the general colouration of the corolla exhibited two tones of purple in different plants; these were, light-mallow-purple and liseran-purple, the latter differing from the former in possessing less white.

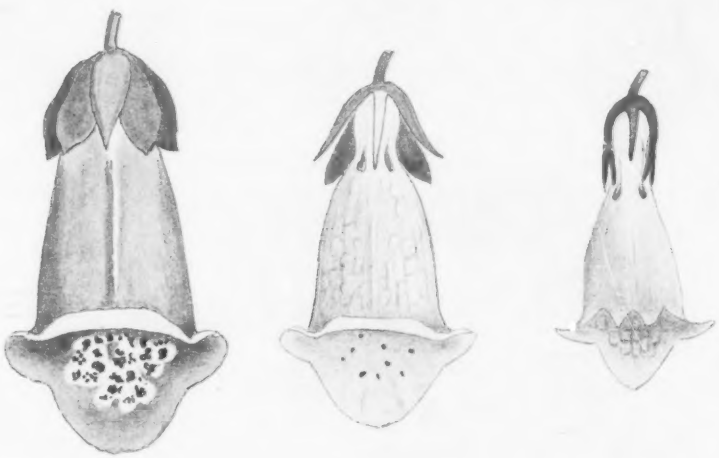
In *lutea* there were two kinds of yellow in different plants; namely, primrose-yellow and deep-colonial-buff, the latter differing from the former in possessing an admixture of 25 °/₀ of orange. The colour of the marbled pattern also varied in different plants, and three types were distinguished; namely, light-yellowish-olive, isabel and ecru-olive, the last two differing from the first in possessing some admixture of orange with the yellow.

The hybrids were obtained by crossing *glox.* with primrose-yellow *lutea.*

The general purple colouration of the hybrids occurred in the same two tones as were found in pure *glox.*, namely, light-rosolane-purple and tyrian-pink. Primrose-yellow derived from the male parent (*lutea*) also occurred in variable amount and was generally quite obvious in those parts of the flower where the purple colouration was absent.

In many plants there occurred a definite mosaic of purple and yellow colouration over the corolla (see Text-fig. 2), the yellow tending to occur in irregular bands or blotches with purple between. In other words, the tissue which carried the purple colouration tended to bear little or no yellow colouration. It was clear that a single pigment (formed in some way by the union of the *glox.* purple and the yellow) was not produced, but either both pigments were present together in a tissue, or one was absent. Probably in any given cell either the one colouration or the other was produced, but not both.

It will be seen that in these purples of the hybrid there is 74 °/₀ of red as compared with 52 °/₀ in those of the *glox.* parent; thus, in the hybrid there is a definite change in the quality of the colour, which is not directly traceable to either parent species. This appearance of new and beautiful colours, which do not seem to arise by the simple mixture of the colours of the parents, is a common feature in species-hybridization of plants.



P *PY* *Y*

Text-fig. 2. Deep-purple *glox.* *P* (♀) × primrose-yellow *lutea* *Y* (♂) yields hybrid *PY*. In hybrid note the great reduction in spotting. Also, in hybrid the purple of *glox.* and the yellow of *lutea* form a mosaic. On the outside of the corolla-tube the shaded streaks and rays are purple while the clear, squarish or rectangular spots between are yellow. On the under lip the radiating streaks are purple and the areas between yellow.

The only other characters studied which appear to exhibit some originality in the hybrid are the shape of the sepals and the prominent serration of the leaf, which is much greater in the hybrid than in either of the parent species.

One fact must be mentioned as showing the complexity of the subject. A homozygous white *glox.* with purple spots was pollinated with a primrose-yellow

lutea, and contrary to expectation the hybrid exhibited a distinct purple or reddish colouration over the general surface of the corolla. This unexpected influence of the yellow foxglove is doubtless associated with the fact that in the great majority of the *lutea* plants used in these experiments there occurred small purple-coloured stains on the corolla at the places where the filaments of the stamens were attached.

With respect to the purple colouration, the *glox.*-backcross tended to be nearer the hybrid parent than the *glox.* parent; but in the case of the *lutea*-backcross, the *lutea* parent was prepotent, since the purple colouration was almost entirely eliminated, and only very exceptionally could a very faint and evanescent tinge of purple be seen on the yellow flowers. The offspring of selfed *lutea*-backcross were all pure yellow.

TABLE XIII.

Spotting of the Corolla.

Grades of spotting in percentages	*Glox.*	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid *Glox.* (♀) × *lutea* (♂)	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	*PP* × *YY*	*PY* × *YY*	*YY* × *YY*
0— 2	—	2	16	10	17
3— 5	1	1	4	—	—
6— 8	0	0	6	—	—
9—11	1	1	1	—	—
12—14	3	0	2	—	—
15—17	2	1	2	—	—
18—20	3	0	1	—	—
21—23	4	1	—	—	—
24—26	1	—	—	—	—
27—29	0	—	—	—	—
30—32	0	—	—	—	—
33—35	1	—	—	—	—
36—38	1	—	—	—	—
39—41	0	—	—	—	—
42—44	0	—	—	—	—
45—47	1	—	—	—	—
Totals	18	6	32	10	17
Means	20·7	9·0	5·0	0	0
Standard Deviation	9·81	7·87	5·23	0	0
Coefficient of Variation	47·1	87·4	104·6	0	0

(*c*) *Spotting of the Corolla.* As elsewhere explained, the degree of spotting was recorded as the estimated percentage area of the spotted region actually occupied by the spots. The distribution of spotting is given in Table XIII, p. 224.

In the experimental *glox.* plants the flowers always bore purple spots, while in *lutea*, spots were never present, but their place was taken by a characteristic marbled pattern. Nevertheless, the spots and marbled pattern are probably quite distinct characters and not homologous in any strict sense.

In the hybrids there is a very marked diminution in the extent and number of the spots (Pls. IV and V).

An intermediate condition would be given by $\frac{1}{2}$ [*glox.* purple spots 20·7 °/₀ + *lutea* 0 °/₀] = 10·3 °/₀, but the mean extent of spotting in the hybrids is only

TABLE XIV.

Length of Sepal.

Grades of length in tenths of millimetres	*Glox.*	*Glox.* backcross Hybrid (♀) × *glox.* (♂)	Hybrid × Hybrid	Hybrid *Glox.* (♀) × *lutea* (♂)	Offspring of selfed *Lutea*-backcross	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	(*PP* × *YY*) × (*PP* × *YY*)	*PP* × *YY*	(*PY* × *YY*) × (*PY* × *YY*)	*PY* × *YY*	*YY* × *YY*
40— 49	—	—	—	—	—	—	1
50— 59	—	—	—	—	—	—	6
60— 69	—	—	—	—	1	3	16
70— 79	—	—	—	—	13	2	28
80— 89	3	—	—	1	19	9	15
90— 99	4	1	—	7	30	12	19
100—109	11	0	—	26	35	5	6
110—119	15	6	—	26	21	1	3
120—129	15	4	1	30	10	—	1
130—139	12	6	1	28	4	—	—
140—149	5	0	1	18	4	—	—
150—159	9	1	—	9	2	—	—
160—169	7	1	—	11	4	—	—
170—179	5	0	—	1	—	—	—
180—189	1	1	—	1	—	—	—
190—199	1	2	—	—	—	—	—
Totals	88	22	3	158	143	32	95
Means	130·6	134·0	124·5	127·2	103·9	89·8	82·4
Standard Deviation	24·88	26·44	8·16	19·70	20·97	11·99	15·66
Coefficient of Variation	19·0	19·7	6·6	15·5	20·2	12·2	19·0

5·1 °/₀. The spotting of the *glox.*-backcross is also not large enough for an intermediate condition, being 9 °/₀ instead of 12 °/₀. Thus the union of *glox.* and *lutea* conspicuously inhibits the production of spotting.

This inhibiting effect of *lutea* on spotting is so marked that in the *lutea*-backcross and in the offspring of selfed *lutea*-backcross I never detected the faintest suspicion of a spot.

(*d*) *Length of Sepals.* The left-hand upper lateral sepal was measured, and some 3 or 4 of the first formed flowers of each plant were taken. The separate means for the different plants were not calculated, but all the individual sepals measured were incorporated in Table XIV on p. 225.

There was no approach to an intermediate condition in the hybrid, the sepal-length being practically the same as in the *glox.* parent, although in *lutea* the

TABLE XV.

Width of Sepal.

Grades of width in tenths of millimetres	*Glox.*	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid × hybrid	Hybrid *Glox.* (♀) × *lutea* (♂)	Offspring of selfed *Lutea*-backcross	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	(*PP* × *YY*) × (*PP* × *YY*)	*PP* × *YY*	(*PY* × *YY*) × (*PY* × *YY*)	*PY* × *YY*	*YY* × *YY*
0— 7	—	—	—	—	—	—	2
8— 15	—	—	—	—	87	28	74
16— 23	—	—	—	—	77	8	19
24— 31	—	—	—	12	3	—	—
32— 39	—	9	1	41	—	—	—
40— 47	1	8	1	56	—	—	—
48— 55	3	2	1	29	—	—	—
56— 63	22	3	—	12	—	—	—
64— 71	23	—	—	3	—	—	—
72— 79	21	—	—	4	—	—	—
80— 87	9	—	—	—	—	—	—
88— 95	7	—	—	—	—	—	—
96—103	1	—	—	—	—	—	—
104—111	2	—	—	—	—	—	—
Totals	89	22	3	157	167	36	95
Means	71·4	43·1	43·5	44·3	15·5	13·3	12·9
Standard Deviation	12·24	8·17	6·53	10·22	5·65	3·33	3·48
Coefficient of Variation	17·1	19·0	15·0	23·1	36·5	25·0	26·9

sepals are much shorter (8 mm.) than in *glox.* (13·0 mm.) (Pls. IV and V). The *lutea*-backcross plants maintained a relatively short sepal-length, but the offspring of the selfed *lutea*-backcross had a mean length (10·4 mm.) intermediate between that of the hybrid and of *lutea*. The *glox.*-backcross exhibited a mean sepal-length (13·4 mm.) greater even than that of the *glox.* parent.

(*e*) *Width of Sepals.* The maximum width of the free portion of the left-hand, upper, lateral sepal was measured. All the sepals individually were incorporated in the table, the means for the separate plants not being taken.

The width of the sepals differs markedly from the length, in that the hybrid is very nearly intermediate in this character between *glox.* and *lutea*.

TABLE XVI.

Ratio $\frac{\textit{Sepal-Width}}{\textit{Sepal-Length}} \times 1000$.

Grades of Ratios ×1000	*Glox.*	*Glox.*-backcross Hybrid (♀) × *glox.* (♂)	Hybrid × Hybrid	Hybrid *Glox.* (♀) × *lutea* (♂)	Offspring of selfed *Lutea*-backcross	*Lutea*-backcross Hybrid (♀) × *lutea* (♂)	*Lutea*
	PP × *PP*	*PY* × *PP*	(*PP* × *YY*) × (*PP* × *YY*)	*PP* × *YY*	(*PY* × *YY*) × (*PY* × *YY*)	*PY* × *YY*	*YY* × *YY*
100—139	—	—	—	—	8	2	1
140—179	—	—	—	—	18	6	14
180—219	—	—	—	3	2	3	8
220—259	—	—	—	8	—	—	1
260—299	—	1	1	7	—	—	—
300—339	—	3	1	8	—	—	—
340—379	—	2	1	2	—	—	—
380—419	1	—	—	2	—	—	—
420—459	2	—	—	—	—	—	—
460—499	7	—	—	—	—	—	—
500—539	8	—	—	—	—	—	—
540—579	2	—	—	—	—	—	—
580—619	3	—	—	—	—	—	—
620—659	3	—	—	—	—	—	—
660—699	2	—	—	—	—	—	—
700—739	2	—	—	—	—	—	—
740—779	1	—	—	—	—	—	—
Totals	31	6	3	30	28	11	24
Means	554·3	326·2	319·5	284·8	150·9	163·1	174·5
Standard Deviation	90·72	27·49	32·67	53·40	22·32	26·72	25·32
Coefficient of Variation	16·4	8·5	10·3	16·8	14·8	16·4	14·5

On the other hand, the *lutea*-backcross plants and the offspring of selfed *lutea*-backcross have nearly the same sepal-width as the *lutea* parent. The *glox.*-backcross plants maintained the same sepal-width as that of the hybrid. See Table XV, p. 226.

(*f*) *Ratio of Sepal-Width to Sepal-Length.* In this case the means of the separate plants were taken. The distributions are given in Table XVI, p. 227.

The mean ratio of the hybrid is nearer to that of *lutea* than to that of *glox.* The mean ratios of the two backcrosses are abnormally low. It is clear that hybridization has introduced a new feature in the sepal, which especially influences the ratio of the length to breadth, and as a matter of fact the long, narrow sepals of the hybrid and *glox.*-backcross are noticeable at the first glance of the flowers. The corolla of the hybrid and *glox.*-backcross is predominantly *glox.* in nature, but the calyx is not so, owing to the shape of the sepals being modified by the relative increase in their length.

5. Correlation of Characters.

There appears to be but little correlation between most of the different characters which had been selected for measurement. The measurements of 23 offspring, derived from the selfing of one plant of a *lutea*-backcross generation, were arranged in correlation tables, and the coefficients of correlation for the three following pairs of characters were found to be very small or practically zero: thickness of lamina and pilosity, thickness of lamina and number of stomates, and thickness of lamina and length of guard-cells. On the other hand, there was a considerable negative correlation between the number of stomates and length of guard-cells. The coefficient of correlation is $-\cdot567$ with probable error $\cdot0529$, indicating a tendency for smaller stomates to be associated with a greater number of them (Table XVII, p. 229).

In the various generations of the plants under observation the following *lutea*-characters occurred together and did not blend with *glox.*-characters: yellow colouration of flower, complete absence of general purple colouration of the corolla, absence of purple spotting, downward flexure of the tip of the young, growing flower-axis.

6. Parental Correlation of Hybrid Generation.

When *glox.* plants are crossed together and offspring are raised any small variation in a measurable character of a parent tends to be inherited by the offspring to a certain degree. This was abundantly demonstrated in my paper on foxgloves in *Biometrika*, July, 1922. A mathematical measure of the relationship is given by drawing up correlation tables between each parent and offspring or between mid-parents [$\frac{1}{2}$ (character of ♀ parent + character of ♂ parent)] and offspring, and calculating the coefficients of correlation. In the case of the foxgloves cultivated by me the coefficients for mid-parents and offspring averaged about $\cdot553$ and the average coefficient of each parent separately and offspring would be about half of this value, that is from, say, $\cdot25$ to $\cdot30$.

TABLE XVII.

Offspring of selfed lutea-*backcross Plant.*

Correlation Table. Number of Stomates and length of Guard-cells.

Length of Guard-cells.

Grades of number of stomates in field with diameter of 0·353 mm.	*86—89*	*82—85*	*78—81*	*74—77*	*70—73*	*66—69*	*62—65*	*58—61*	Totals
9—10	1	—	—	—	—	—	—	—	1
11—12	—	—	—	2	1	1	—	—	4
13—14	—	—	—	2	2	1	—	—	5
15—16	—	—	—	—	2	—	1	1	4
17—18	—	—	1	1	2	1	—	—	5
19—20	—	—	—	—	—	—	—	—	0
21—22	—	—	—	—	1	—	1	—	2
23—24	—	—	—	—	—	1	—	—	1
25—26	—	—	—	—	—	—	—	—	0
27—28	—	—	—	—	—	—	—	—	0
29—30	—	—	—	—	—	—	—	—	0
31—32	—	—	—	—	—	—	—	1	1
Totals	1	0	1	5	8	4	2	2	23

It would obviously be interesting to ascertain whether the intensity of inheritance of such small variations in a measurable character of a parent is the same when the paired plants are of different species. The volume of material for testing this question is meagre, but still it should be sufficient to indicate whether there is any marked difference.

In all, 13 *glox.* plants with small, but definite, variations in their measurable characters were crossed with the pollen from several *lutea* plants and 32 hybrid plants which flowered were raised. Now, the question is, whether the coefficients of correlation for various characters between the *glox.* parents and the hybrid offspring show some approximation to a value of ·27 ?

Seven characters were taken:

(1) Degree of intensity of the general purple coloration of the corolla, coefficient of correlation $r = +\cdot07$.

(2) Percentage amount of purple spotting of the corolla, $r = +\cdot21$.

(3) Ratio of width to length of the corolla, $r = +\cdot04$.

(4) Pilosity of leaf, $r = +\cdot27$.

(5) Length of guard-cells, $r = +\cdot22$.

(6) Indentation depth, $r = -\cdot20$.

(7) Angle of slope of bundles, $r = +\cdot05$.

The first two characters are not recognizable in *lutea*, while the remaining characters are, of course, present in both parents.

Now, the exact figures arrived at for each character cannot be reliable, as the probable errors are large on account of the scarcity of material, but the average (+·09) of the seven coefficients has a somewhat greater value. Thus, this coefficient of ·09 is to be compared with what may be termed the normal parental coefficient of about ·27. It is obvious, therefore, that in these plants there has been a very marked reduction in the parental correlation, i.e. the intensity of parental inheritance for small variations in measurable characters is lessened when two distinct species are crossed.

7. The Chromosomes of *Digitalis*.

Being unaware of any observations on the chromosomes of the different species of *Digitalis* some attempt was made to determine the chromosome-number of the experimental plants. The anthers from young flower-buds were fixed in strong Flemming-without-acetic, Dutoscq picric-solution, and also in corrosive and acetic. They were embedded in paraffin, cut into sections of different thickness, stained with iron-haematoxylin, and in other ways, and examined under oil-immersions. The first named fixative seemed the best, and many karyokinetic figures of the developing pollen grains were found.

Unfortunately the chromosomes of these plants were very small, and a determination of the exact number was found to be exceedingly difficult. The chromosomes were rounded or very slightly elongated, and in *gloxiniæflora* they were about 0·5μ in length and in *lutea* they were rather larger being about 0·7μ. After numerous counts, in which I was assisted by my research student, Mr F. S. Bush, we came to the conclusion that the haploid number for *gloxiniæflora* was sometimes 12 and sometimes 13, for *lutea* 8, for the hybrid sometimes 10 and sometimes 11, for *glox.*-backcross 11 or 12 and for *lutea*-backcross 8 or 9. The lack of constancy in the numbers did not seem to be due entirely to the difficulty in counting, and there was certainly considerable irregularity in the karyokinetic spindles of the hybrid and backcrosses.

It was subsequently ascertained that Hasse-Bessel* (1921) had published an account of certain observations on *Digitalis* in which he gave the haploid number of *D. purpurea* as 24 and of *D. lutea* as 48. Now, these numbers differ so markedly from those obtained from the Natal plants that the matter was referred to Dr Hill, Director of the Kew Botanical Gardens, who suggests from the above evidence that possibly *D. gloxiniæflora* (*Digitalis gloxinioides* Clem.) arose as a sport from *D. purpurea* in which there was a double reduction of chromosomes, thus changing the haploid number (24) of *purpurea* into that (12) of *gloxiniæflora*.

The haploid number (8) of my *lutea* plants appears to be one-sixth of that (48) of the plants examined by Hasse-Bessel, but an explanation of this matter can scarcely be offered without a knowledge of the haploid numbers of *D. ambigua* and of allied wild species.

* Hasse-Bessel, G., "*Digitalis*-Studien II.," *Zeitschrift für induktive Abstammung- und Vererbungslehre*, Bd. xxvii, S. 1—26, 1921. Quoted from J. L. Collins and Margaret C. Mann's Paper "Interspecific hybrids in Crepis," *Genetics*, May, 1923, p. 226.

8. GENERAL REVIEW.

Measurable characters as strongly contrasted as possible in the two parent species were selected for study. These characters were of very various nature, ranging from linear dimensions to physiological attributes. In spite of an intensive study of the hybrid and of the backcrosses I failed to detect, except in the case of colour, any character which exhibited typical Mendelian inheritance.

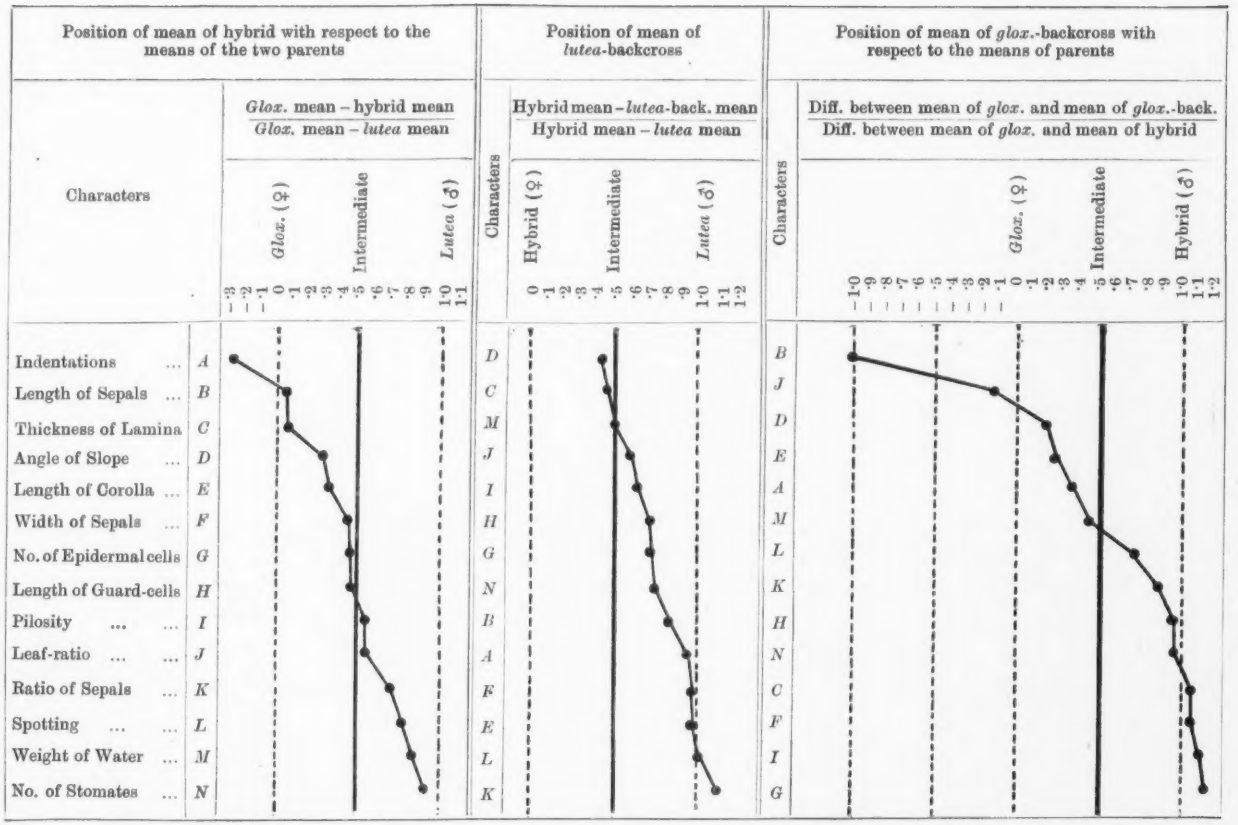
The positions of the means of the various characters of the hybrid with respect to the means of the *glox.* and *lutea* parents are shown graphically in the accompanying table. In the character of the indentation of the leaf (Text-fig. 3, first diagram) *glox.* exceeds *lutea*, but the hybrid exceeds even *glox.* In the length of sepals and in the thickness of lamina the hybrid is closely similar to *glox.* In the angle of slope of certain marginal bundles of the leaf and in the length of the corolla the hybrid is much nearer to *glox.* than to *lutea.* In five characters the hybrid is practically intermediate, and in four other characters the hybrid diverges widely from *glox.* and approaches *lutea.* The influence of both parents is, however, apparent in all the characters; for even in the case of the indentations the general shape of the serrations exhibits clearly a *lutea* influence (Text-fig. 1, p. 214, and Pl. VI).

In *lutea*-backcross the positions of the means of the various characters with respect to the mean of the hybrid parent and that of the *lutea* parent are quite different to what they were in the case of the hybrid and its two parents. Reference to the second diagram of Text-fig. 3, p. 232, shows that in two characters only (angle of slope, thickness of lamina) are the means of the backcross on the hybrid side of the intermediate line. The means of eight characters lie between the intermediate value and the *lutea* value, three characters are very close in their mean values to the *lutea* mean, while in the last character (ratio of sepals) the mean of the backcross extends beyond the *lutea* mean.

Thus, in the *lutea*-backcross the influence of the hybrid parent was not obvious in four out of the 14 characters, namely, width of sepals, length of corolla, spotting and ratio of sepals.

In the offspring derived from selfing the *lutea*-backcross plants the means of the various characters differ considerably from the *lutea*-backcross means. The positions of the offspring means with reference to the means of the hybrid and of *lutea* were plotted in a diagram for comparison with the second diagram of the table, but it could not be shown there without confusion. In one character (number of stomates) the mean of the offspring is higher than that of the hybrid, in five characters the offspring mean is on the hybrid side of the intermediate value, in five characters the offspring mean lies between the intermediate value and the *lutea* value, in one character (absence of spotting) it is identical with *lutea*, while in two characters (length of corolla, ratio of sepals) the offspring mean is beyond the *lutea* mean.

In the *glox.*-backcross (third diagram of table) in two characters (length of sepals and leaf-ratio) the means exceed those of the *glox.* parent, in four characters

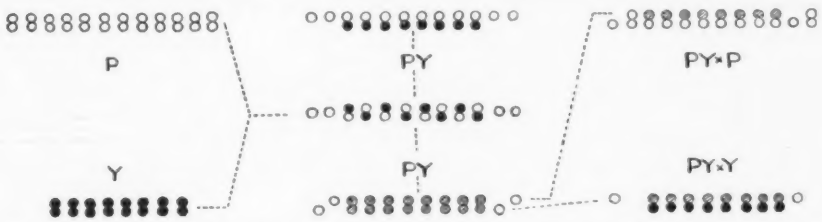

Position of mean of hybrid with respect to the means of the two parents
Characters
Glox. mean – hybrid mean / Glox. mean – lutea mean
Glox. (♀)
Intermediate
Lutea (♂)
Indentations ... A
Length of Sepals ... B
Thickness of Lamina C
Angle of Slope ... D
Length of Corolla ... E
Width of Sepals ... F
No. of Epidermal cells G
Length of Guard-cells H
Pilosity I
Leaf-ratio ... ... J
Ratio of Sepals ... K
Spotting L
Weight of Water ... M
No. of Stomates ... N
Position of mean of lutea-backcross
Characters
Hybrid mean – lutea-back. mean / Hybrid mean – lutea mean
Hybrid (♀)
Intermediate
Lutea (♂)
D C M J I H G N B A F E L K
Position of mean of glox.-backcross with respect to the means of parents
Characters
Diff. between mean of glox. and mean of glox.-back. / Diff. between mean of glox. and mean of hybrid
Glox. (♀)
Intermediate
Hybrid (♂)
B J D E A M L K H N C F I G

Text-fig. 3. Graphs of the hybrid and two backcrosses, showing the relative positions of the means of 14 characters of the offspring with respect to the means of the two parents. In the *lutea*-backcross note that the greater part of the graph lies between the intermediate value and the *lutea* mean, while the other two graphs are more evenly balanced about the intermediate vertical.

the means are on the *glox.* side of the intermediate value, in four characters the means lie between the intermediate value and the hybrid value, and in four characters the backcross means are slightly beyond the hybrid values. Thus the influence of the *glox.* parent was not obvious in six out of the 14 characters, namely, number of epidermal cells, pilosity, width of sepals, thickness of lamina, number of stomates, and length of guard-cells.

The offspring raised from selfing the hybrid were too few in number to give any clear result, but there are indications that the means of the different characters would by no means wholly coincide with the means of the hybrid, just as we found that the means of the offspring of the selfed *lutea*-backcross did not necessarily agree with the *lutea*-backcross means.

These complicated relationships are not inconsistent with the various shufflings of hereditary influences in the different fertilizations, and it is pretty clear that in these particular crossings if a generation of plants approached *glox.*, say, more closely than another generation did, then the former would be found to contain a greater number of *glox.* chromosomes than the latter. If we assume that the haploid number of *glox.* is 12 and of *lutea* 8, then the diploid number of the hybrid would be 20 (12 *glox.* + 8 *lutea*). If these chromosomes in the hybrid are supposed to pair, there would be 8 pairs, each consisting of one *glox.* and one *lutea* chromosome, and 4 solitary *glox.* chromosomes (Text-fig. 4, *PY*). If the arrangement of the chromosomes of the hybrid in the reduction spindle is a matter of chance, then



Text-fig. 4. Diagrams of the most frequent arrangement of the chromosomes in the reduction division of the various generations of foxgloves, on the supposition that the chromosomes are distributed by chance in the formation of the dyaster. *P*=*glox.*, *Y*=*lutea*, *PY*=hybrid, *PY*×*P*=*glox.*-backcross, *PY*×*Y*=*lutea*-backcross.

the commonest distribution of chromosomes in each aster would be 6 *glox.* and 4 *lutea*, but any other division would be possible, such as 2 *glox.* and 8 *lutea* in one aster and 10 *glox.* and no *lutea* in the other. A gamete having the latter constitution, if viable, should according to current views possess no *lutea* influence whatever, unless an interchange of substance, or in some way a mingling of attributes of the members of the pairs of chromosomes, had occurred before separation took place. Now, the general evidence of breeding would show that on crossing a hybrid with a parent species we never obtain an offspring which in all characters is entirely pure with respect to one parent. Thus, we must suppose, on the arbitrary assumption that the chromosomes are the sole bearers of hereditary

influences, that an intermingling of the attributes of the paired chromosomes always takes place during the preparatory changes (synapsis) for reduction.

As to whether we can generally refer characters to the influence of some one or more discrete determinants, factors or genes situated in definite places in specific chromosomes must be regarded as very doubtful. The conception is a difficult one to picture, and would scarcely have been entertained had it not been for the fact that the mode of inheritance of the abnormalities dealt with by Morgan and others in their experiments with *Drosophila* seems capable of being interpreted on this kind of basis. If chromosomes are of such specific character, it may well be asked, what would happen if a gamete in the mixed chromosome arrangement of hybrids and backcrosses failed to obtain all the different kinds of chromosomes, for in that case some of the determinants controlling certain characters would on the theory be entirely absent. Of course, it is easy to say that such gametes are not viable, but it is doubtful if this gratuitous statement is an adequate explanation. In the present observations I can see no clear evidence that the chromosomes are not all alike in constitution and reinforce one another.

In any case, whether the chromosome hypothesis is accepted or not, or whether the original Mendelian interpretation which did not make any assumption with regard to chromosomes is preferred, the question remains: Are ordinary racial, measurable characters dependent on discrete, unchangeable factors transmitted from parent to offspring? If a character depends on a single factor, then the offspring derived from the selfing of hybrids or from backcrosses should exhibit sharply defined segregation and the variation of the offspring with respect to that character would be great. If a character is dependent on several factors, sharply defined segregation would not occur, but marked variability would be present. If a character is dependent on a large number of minute, discrete genes, it is not clear how any statistical observations could either prove or disprove the assumption.

In the present case in no character is there a conspicuous increase in the variability of the backcrosses above that of the hybrid and of the parent species. A measure of variability (standard deviation) was found for the distribution of each character, and this was converted into Professor Karl Pearson's Coefficient of Variation $= \frac{\text{Standard deviation}}{\text{Mean}} \times 100$. The means in the different generations of 13 characters are given in the accompanying table: spotting has been omitted, since this character is absent from one of the parent species, and its inclusion would confuse the comparative results.

Means of Coefficient of Variation of 13 Characters

Glox.	*Glox.*-backcross	Hybrid	Offspring *lutea*-backcross	*Lutea*-backcross	*Lutea*
24·0	27·2	25·5	22·9	27·2	24·5

Thus, in the *glox.*-backcross there is only an increase of 3·2 °/₀ in the variability above *glox.* and in *lutea*-backcross only 2·7 °/₀ above *lutea* and 1·7 °/₀ above the coefficient of the hybrid, while the offspring of the selfed *lutea*-backcross were less variable than any of the other generations. There is here no warrant to assume that the characters are controlled by unmodifiable determinants, unless such determinants are supposed to be numerous and very small, and then the conception tends to become intangible and loses significance.

9. BEARING OF THE PRESENT RESULTS ON THE CHROMOSOME HYPOTHESIS.

After synapsis the members of the pairs of chromosomes of the hybrid may be supposed to become similar, if not exactly alike, and this fundamental change in their nature is indicated in the diagrams of Text-fig. 4, p. 233, by the oblique shading of the chromosomes.

There is, of course, no question that when a sport or abnormality is crossed with the normal form sharply defined segregation of the sporting character frequently occurs in the second generation. For some unknown physiological reason the sporting character will not mingle in many cases with the normal character. Now, in the case of the hybrid and backcrosses of *Digitalis* there is clearly reluctance in some of the characters for the *glox.* character to mingle with the *lutea* character, and more or less marked dominance of one of the parents results; but the condition is not sharply defined, and all grades of dominance may be observed, the mingling of the hereditary influences resulting in a definite bias on one side or the other.

In the *glox.*-backcross the most frequent arrangement of chromosomes, if the distribution is by chance, would be 8 hybrid chromosomes and 2 + 12 *glox.* chromosomes, while in the *lutea*-backcross there would be 8 hybrid chromosomes, 8 *lutea* chromosomes and 2 *glox.* chromosomes (Text-fig. 4). Now, the *lutea*-backcross is predominantly *lutea* in nature, and it will be noticed that correspondingly there are only 2 pure *glox.* chromosomes present.

In the case of certain characters which cannot be regarded as abnormalities, such as colour, it is physiologically improbable that any blending of the hereditary influences, say, for yellow and purple, would be capable of producing a single pigment of an intermediate colour; but in ordinary measurable characters which blend on crossing, the hereditary influences (or factors, if the term is preferred) governing them may be supposed to mingle and become intermediate in nature during synapsis, and by such a simple conception Mendelian and blended inheritance are brought into line. It is indeed probable that the line of division between Mendelian and blended inheritance is not a sharp one as is usually supposed, but that all grades of incomplete segregation or partial blending may occur. We see this principle well brought out in the varying dominance of one or the other parent in the present hybrids and backcrosses.

10. Concise Summary.

1. The hybrid derived from *glox.* (♀) × *lutea* (♂) is matroclinous. It is doubted whether *lutea* (♀) × *glox.* (♂) is so strictly matroclinous.

2. The second generation (F_2), hybrid (♀) × hybrid (♂), was variable, but apparently not excessively so, and on the whole it tended to resemble the hybrid itself.

3. The *glox.*-backcross (hybrid (♀) × *glox.* (♂)) resembled the hybrid, and the reciprocal (*glox.* (♀) × hybrid (♂)) was somewhat similar, although possibly with a weaker habit; but the *lutea*-backcross (hybrid (♀) × *lutea* (♂)) was strongly patroclinous, being very much like pure *lutea.*

4. The cross *glox.* (♀) × *lutea* (♂) produced about 1 °/₀ large seeds and 50 °/₀ small seeds compared with pure *glox.* capsules. *Lutea* (♀) × *glox.* (♂) produced about 0·5 °/₀ large seeds and no small seeds.

Glox.-backcross selfed seemed quite sterile, while some of the selfed *lutea*-backcross plants were quite fertile.

Three or four plants out of the 33 hybrids exhibited some fertility. The offspring of the selfed hybrid were mostly destroyed by accident, but one plant which flowered was quite sterile, although capsules of considerable size were formed with *glox.* pollen.

5. The means of 14 characters in the backcrosses were compared with the means of the two parents, and very varying dominance on the part of one or the other parent was found.

6. Hybridization certainly appeared to introduce new features; the purple colouration of *glox.* (light-mallow-purple or liseran-purple) being converted in the hybrid into light-rosolane-purple or tyrian-pink in which there is a greater percentage of red. The indentation of the leaf and the shape of the sepals are also influenced in a new direction.

7. There was little correlation between most of the characters selected, but a definite negative correlation was noticed between the number of stomates and the size of the stomates.

8. Parental correlation for ordinary small variations ("fluctuating variations") seemed to be considerably reduced by hybridization.

9. The bearing of these observations on Mendelism and the chromosome hypothesis is considered, and it was pointed out that Mendelian inheritance and perfect blending inheritance may be regarded as the two end terms of a series, and that all grades of partial blending or partial segregation lie between, as witnessed by the varying dominance of one or the other parents in the different crosses that were made.

DESCRIPTION OF PLATES.

PLATE I.

Fig. 1. *Digitalis gloxiniæflora* (female parent).
Fig. 2. *Digitalis lutea* Linn. (?) (male parent).
Fig. 3. Typical flowers of the *gloxiniæflora* plants.
Fig. 4. Enlarged view of the flower-stalk of *lutea* showing the characteristic downward flexure of growing region.
Fig. 5. Hybrid plant, *glox.* (♀) × *lutea* (♂); note the large serrated leaves and vigorous habit.
Fig. 6. Flowers of the above hybrid plant; note the marbled pattern within the tube and the great paucity of spots.
Fig. 7. *Glox.*-backcross, hybrid (♀) × *glox.* (♂); note the elongated, strongly serrated leaves.
Fig. 8. Flowers of the above backcross plant; spotting not extensive, but more pronounced than in most of the hybrids.

PLATE II.

Fig. 9. *Glox.*-backcross, hybrid (♀) × *glox.* (♂); note vigorous habit and large shiny leaves. In general aspect the plant strongly resembles the hybrid.
Fig. 10. Flower-stalk of above plant.
Fig. 11. Isolated flowers from two other *glox.*-backcross plants. Spots scanty and marbled pattern not so obvious as in most of the hybrid plants.
Fig. 12. *Glox.*-backcross with pure *glox.* as seed-plant, *glox.* (♀) × hybrid (♂); note the elongated leaves but weak habit of the plant.
Fig. 13. Flower-stalk of above plant. The spotting was abundant, and the colour of the flowers possessed in a well-marked degree the characteristic rosolane-purple of the hybrid.
Fig. 14. *Glox.*-backcross plant with wide leaves and weak serration.

PLATE III.

Fig. 15. *Glox.*-backcross plant with stiff, shiny, crinkled leaves with pronounced serration.
Fig. 16. Flowers of *glox.*-backcross plant showing paucity of spots and elongated narrow sepals.
Fig. 17. *Lutea*-backcross plant, hybrid (♀) × *lutea* (♂), with moderately wide leaves.
Fig. 18. *Lutea*-backcross plant with wide leaves.
Fig. 19. *Lutea*-backcross plant of usual habit and with elongated narrow leaves; note the downward flexure of flower-stalk.
Fig. 20. Flowers from three *lutea*-backcross plants, almost indistinguishable from the flowers of pure *lutea*.
Figs. 21 and 22. Flower-stalks of *lutea*-backcross plants.

PLATE IV.

Flowers of Parents and Hybrids. ½ natural size.

Beginning at the left-hand top corner *lutea* Y_3 (♂) crossed with *glox.* P_1 (♀) gave hybrids (nos. 1, 2) with very tiny spots. Y_3 (♂) × P_2 (♀) gave two hybrids of which no. 1 shows a certain amount of obvious spotting. Y_3 (♂) × P_3 (♀) produced one plant (1) with no spots visible, although the marbling of the yellow foxglove is distinguishable, but marked out in purple. Narrow sepals. Y_3 (♂) × P_4 (♀) produced hybrids (nos. 1—4) with very scarce, small spots near base of flower. Narrow sepals. Y_3 (♂) × P_5 (♀) gave hybrids 1—3: no. 3 bore only small, deformed flowers.

Y_2 (♂) × P_1 (♀) gave one hybrid. The female parent P_1 was richly supplied with spots, but in the hybrid the spotting was very weak. Y_2 (♂) × P_2 (♀) gave three hybrids. Y_2 (♂) × P_3 (♀) gave three hybrids with a variable amount of spotting.

Y_{3s} (♂) × P_1 (♀) gave three hybrids with a fair amount of spotting.

PLATE V.

Upper half of Plate—Hybrids. ½ natural size.

$Y_4(♂) \times P_1(♀)$ gave hybrid with a few tiny spots at base of corolla-tube. Narrow sepals. $Y_4(♂) \times P_2(♀)$ gave three hybrids (in upper horizontally placed rectangle). $Y_4(♂) \times P_3(♀)$ gave four hybrids (in lower rectangle) with variable spotting and narrow sepals.

Lower half of Plate—Backcrosses. ½ natural size.

$Y_3(♂) \times P(♀)$ produced hybrid PY with a few, minute, scattered spots.

This PY (♀) was crossed with an ordinary *glox.* plant, PP (♂), and the flowers of three resulting *glox.*-backcrosses are shown in the rectangle on the left. Spotting present but weak.

PY (♀) was crossed with an ordinary *lutea*, Y_3Y_3 (♂), and the flowers of the resulting 7 *lutea*-backcrosses are shown in the polygon on the right. No vestige of *glox.* spotting is visible; but the marbled pattern of *lutea* is well-marked.

PLATE VI.

Portions of leaves showing the nature of the margin. In many cases the character of the venation seen from below is discernible. Natural size.

Lower portion of Plate below horizontal division—Hybrids.

Beginning at the left upper corner, and passing from left to right, *lutea* $Y_4(♂)$ crossed with *glox.* P_1 (♀) produced offspring P_1Y_4 1; P_2 (♀) crossed with Y_4 (♂) produced offspring P_2Y_4 1; $P_3(♀) \times Y_4(♂)$ produced offspring P_3Y_4 1, 2 and 3; $P_4(♀) \times Y_4(♂)$ produced offspring P_4Y_4 1, 2—5. In the case of offspring P_3Y_3 1 the male parent Y_3 is seen as the first figure on the left in the second horizontal row.

The remaining figures are to be interpreted in a similar manner.

It is clear that the hybrids exhibit a characteristic indentation of the margin, which tends to be very considerably more developed than in either of the parents.

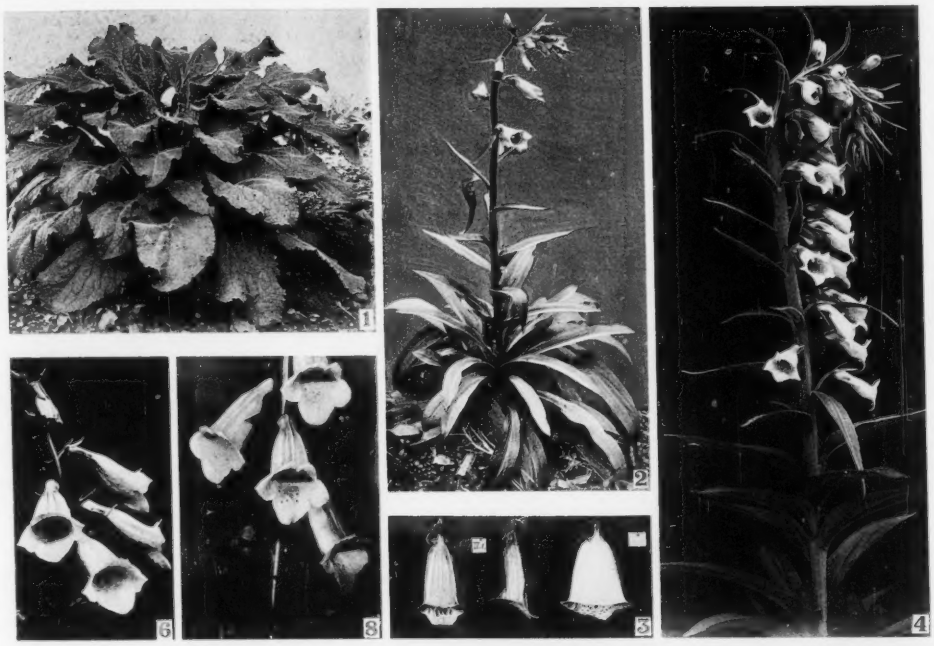
Upper portion of Plate—Backcrosses.

In the middle part of the upper horizontal row of pieces of leaves there is placed a group of 5 leaves of parents. *Lutea* Y (♂) crossed with *glox.* P (♀) produced hybrid PY.

This hybrid PY (♀) was backcrossed with a *lutea* YY (♂) and the leaves of 16 offspring are shown in the excised polygon on the left. Letters *a*—*g* represent the offspring raised from 7 different capsules. The amount of indentation of the leaves tends to exceed greatly that of *lutea*, also, the shape and size of the venation islands differ considerably from what is seen in pure *lutea*.

The hybrid PY (♀) was backcrossed with an ordinary *glox.* PP (♂) and the leaves of the 9 offspring are given in the excised polygon on the right. Both in the indentation and in the venation these leaves tend to resemble the hybrid PY more closely than pure *glox.* PP.

An additional example of a *glox.*-backcross ($PP \times PY$) with parents is shown in the three figures placed in the bottom right-hand corner of the Plate.



Photo, C. Akerman and E. Warren

Digitalis gloxiniæflora, D. lutea, Hybrid and *glox.*-Backcross.

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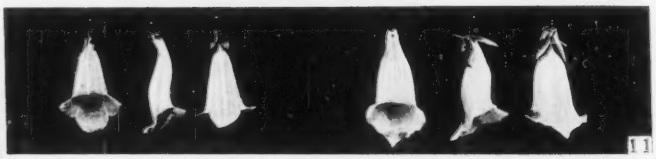

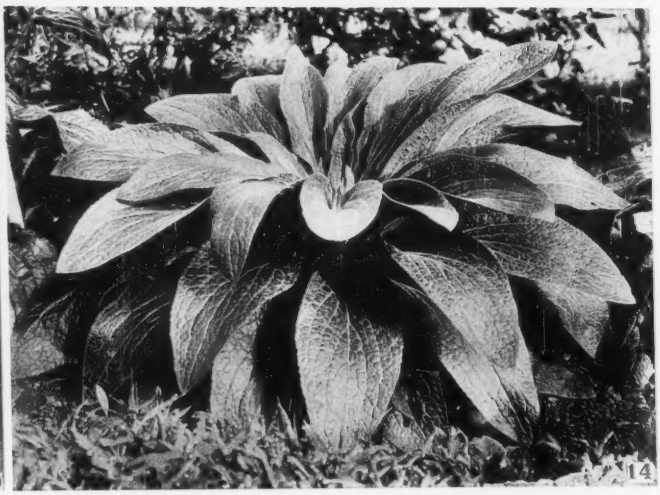

Photo, C. Akerman and E. Warren

Glox.-Backcrosses.

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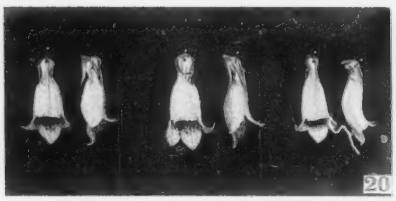
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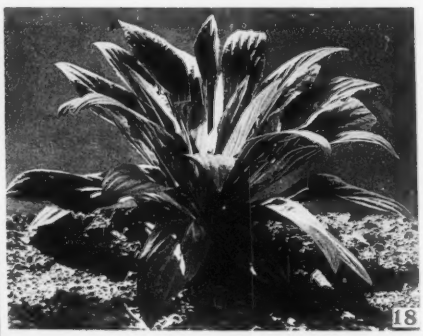
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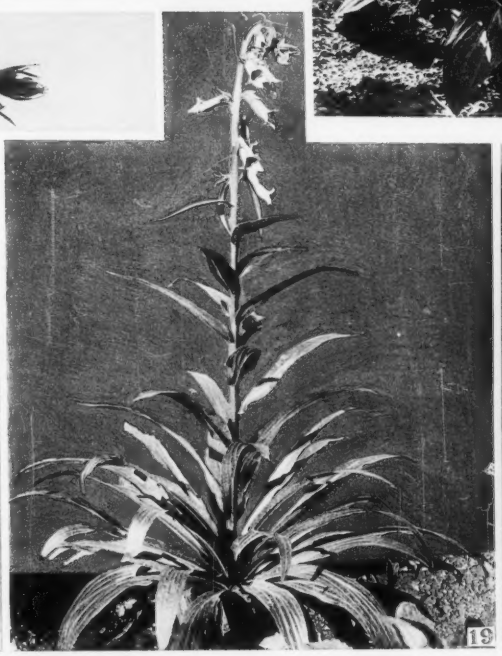
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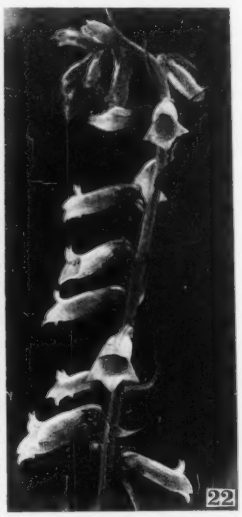
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Photo, C. Akerman and E. Warren

Glox.- and *lutea*-Backcrosses.

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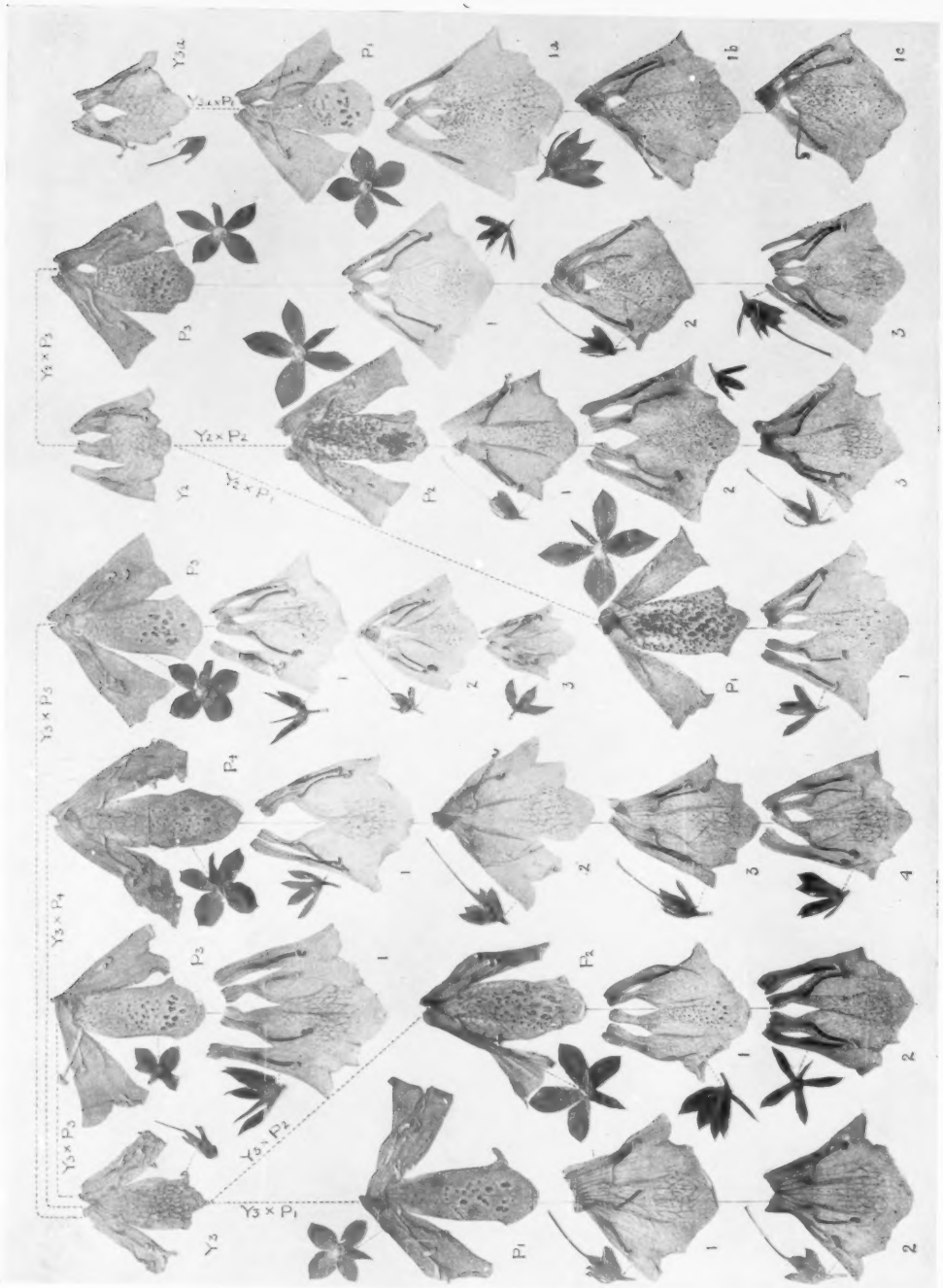

Digitalis gloxiniæflora (♀) × *D. lutea* (♂).

Photo. C. Akerman

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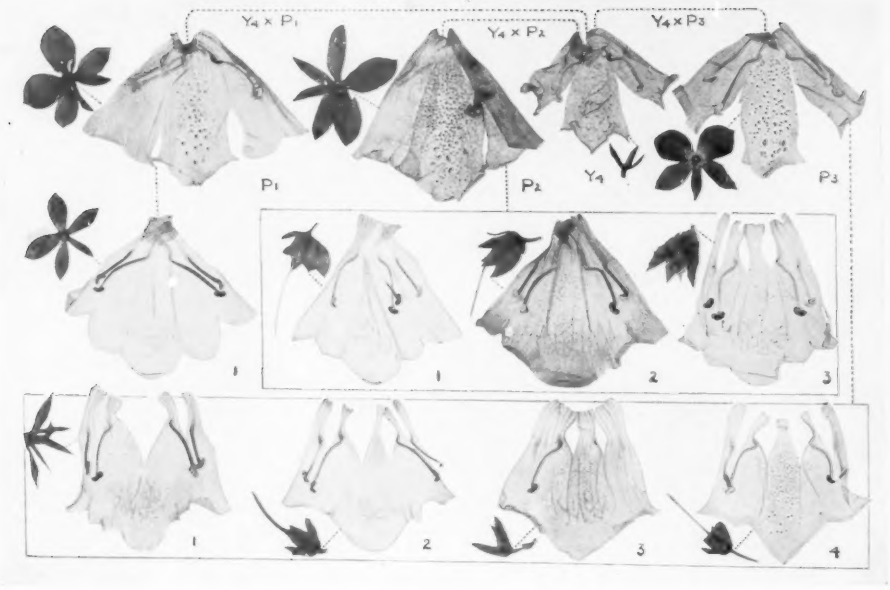
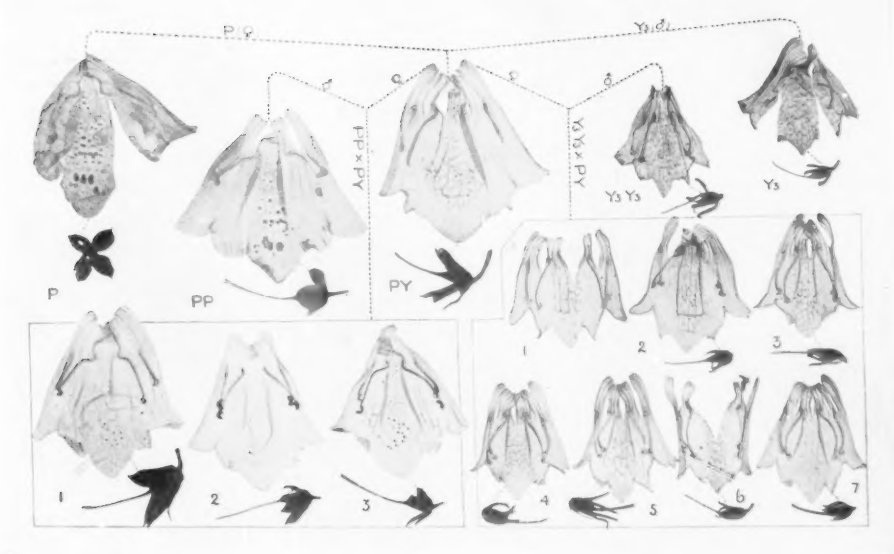


HYBRIDS.



Photo, C. Akerman

BACKCROSSES.

D. glox. (♀) × *lutea* (♂) above; *glox.*- and *lutea*-backcrosses below.

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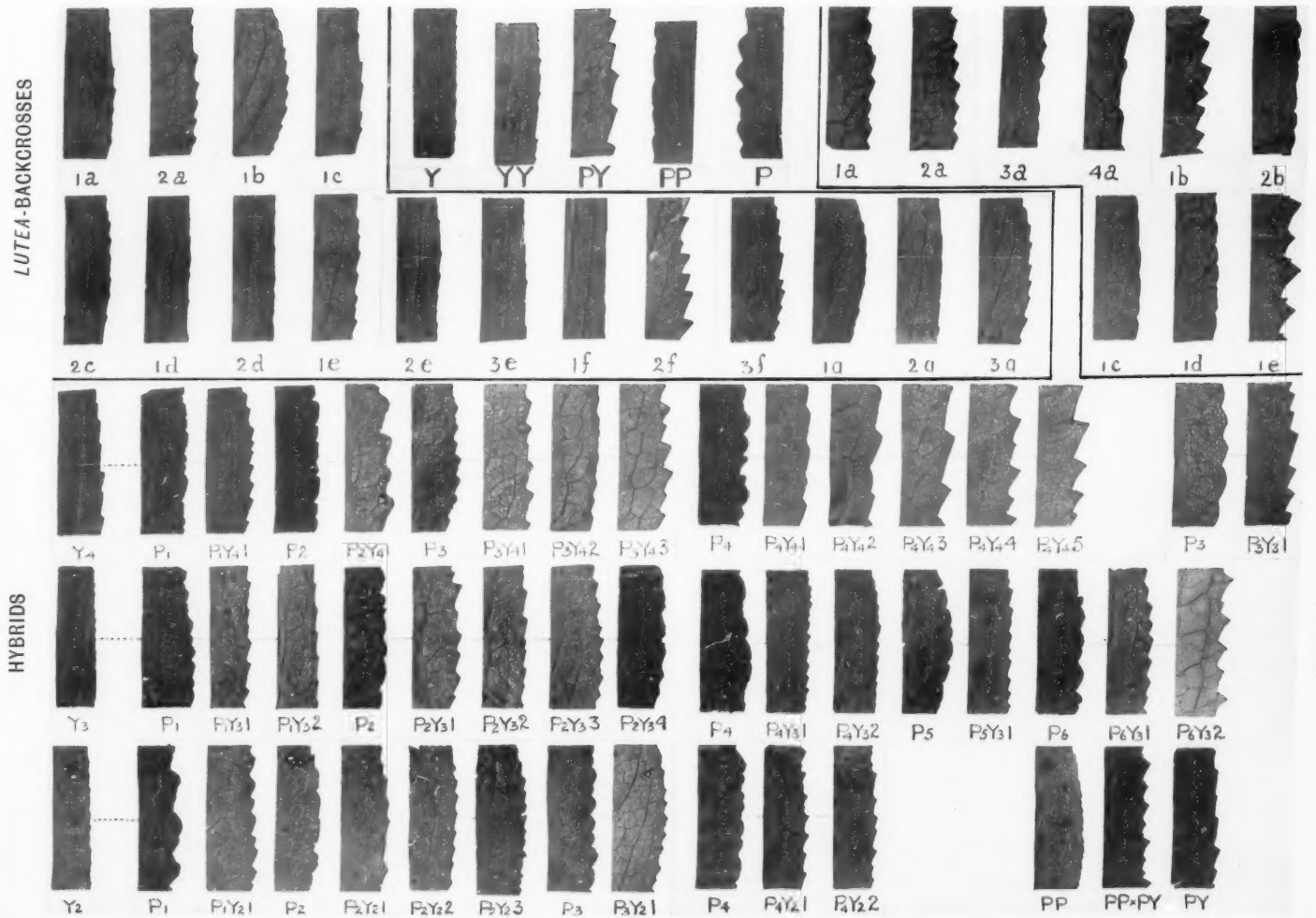
LUTEA-BACKCROSSES
GLOX.-BACKCROSSES
HYBRIDS
HYBRIDS
1a 2a 1b 1c
Y YY PY PP P
1a 2a 3a 4a 1b 2b
2c 1d 2d 1e 2e 3e 1f 2f 3f 1g 2g 3g
1c 1d 1e
Y4 P1 P1Y41 P2 P2Y41 P3 P3Y41 P3Y42 P3Y43 P4 P4Y41 P4Y42 P4Y43 P4Y44 P4Y45 P3 P3Y31
Y3 P1 P1Y31 P1Y32 P2 P2Y31 P2Y32 P2Y33 P2Y34 P4 P4Y31 P4Y32 P5 P5Y31 P6 P6Y31 P6Y32
Y2 P1 P1Y21 P2 P2Y21 P2Y22 P2Y23 P3 P3Y21 P4 P4Y21 P4Y22 PP PP×PY PY

Photo, C. Akerman

Indentation and Variation of Leaves: *D. glox.* (♀) × *D. lutea* (♂) hybrids in lower three rows, 16 *lutea*-backcrosses in left upper division, 9 *glox.*-backcrosses in right upper division.

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ON THE PROBLEM WHETHER TWO GIVEN SAMPLES CAN BE SUPPOSED TO HAVE BEEN DRAWN FROM THE SAME POPULATION.

By E. C. RHODES, M.Sc. (London).

PART I.

1. In the present discussion we shall consider that the samples are distributed in alternate categories, that is to say, the individuals belong to a certain class A or do not belong to this class. We shall suppose that we have two samples of n and m individuals respectively, and that, in the first sample there are r individuals of class A, and in the second sample there are s individuals of this class. We may for the sake of simplicity represent the state of our knowledge in the form:

	Class A	Class Not-A	Totals
Sample 1	r	$n-r$	n
Sample 2	s	$m-s$	m

Further we must point out that although we have referred to these samples as first and second, we attach no meaning of priority to these two words, using them merely to distinguish one sample from the other, and actually the samples are supposed to be drawn concurrently. I propose considering the problem of the possibility of these samples having been drawn from the same population from a novel point of view, which gives a result different from that obtained when the problem is considered from the "classic" point of view. I refer to the usual method where the difference between the proportions of class A occurring in the two samples is compared with its standard-deviation.

2. We will start by supposing that there is an infinite population of individuals separated into the two classes, A and not-A, of which the chance that an individual drawn belongs to class A is p. Then we will suppose that, in a sample of n drawn from this population, the chance that there are between x and $x+dx$ individuals of class A is

$$\frac{1}{\sqrt{2\pi}\,.\,\sigma}\,e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2}dx,$$

where $\bar{x}=np,\quad \sigma^2=npq,$

where $q=1-p.$

We are here assuming that the normal curve can replace a binomial distribution, i.e. that p and $1-p$ are not too small, and that n is large. Further we should

have, in a sample of m, the chance that there are between y and $y+dy$ individuals of class A is

$$\frac{1}{\sqrt{2\pi}\,.\,\sigma'}e^{-\frac{1}{2}\left(\frac{y-\bar{y}}{\sigma'}\right)^2}dy,$$

where $\bar{y}=mp$, $\sigma'^2=mpq$, making similar assumptions as before.

Accordingly we should have as the chance of these two events happening together, viz., the occurrence of between x and $x+dx$ of class A in a sample of n, and the occurrence of between y and $y+dy$ of class A in a sample of m, the expression

$$\frac{1}{2\pi pq\sqrt{nm}}e^{-\frac{1}{2}\left(\frac{(x-np)^2}{npq}+\frac{(y-mp)^2}{mpq}\right)}dx\,dy.$$

Thus for different values of x and y we are led to consider a normal surface in two variates. Of the many ordinates of this surface, one of them will be that for which $x=r$, $y=s$; and further those ordinates standing on an ellipse in the (x, y) plane given by

$$\frac{(x-np)^2}{npq}+\frac{(y-mp)^2}{mpq}=\frac{(r-np)^2}{npq}+\frac{(s-mp)^2}{mpq},$$

will be all equal. We can then find the chance that in our two samples n and m we get the doublet r, s or other doublets less likely. This will be obtained by finding the volume of the surface standing on that part of the (x, y) plane outside the contour ellipse given above, because for these values of x, y such that

$$\frac{(x-np)^2}{npq}+\frac{(y-mp)^2}{mpq}>\frac{(r-np)^2}{npq}+\frac{(s-mp)^2}{mpq},$$

the chance is less than the limiting chance when x is r and y is s. We have then to find the integral

$$\iint\frac{1}{2\pi pq\sqrt{nm}}e^{-\frac{1}{2}\left(\frac{(x-np)^2}{npq}+\frac{(y-mp)^2}{mpq}\right)}dx\,dy,$$

taken from the contour given by

$$\frac{(x-np)^2}{npq}+\frac{(y-mp)^2}{mpq}=\frac{(r-np)^2}{npq}+\frac{(s-mp)^2}{mpq}$$

to infinity.

This integral, by simple transformations, becomes

$$\int_{\chi}^{\infty}e^{-\frac{1}{2}\chi^2}\chi\,d\chi=e^{-\frac{1}{2}\chi^2}\quad\text{.....(i),}$$

where the limiting χ is given by

$$\chi^2=\frac{(r-np)^2}{npq}+\frac{(s-mp)^2}{mpq}\quad\text{.....(ii).}$$

We have therefore found the chance that if two samples n and m are drawn from an infinite population characterised by p, we should get r of class A in the sample of n and s of class A in the sample of m, or other numbers in class A less likely to

occur together. It is this expression I suggest as a basis of argument as to whether the two samples as known are drawn from the same population.

3. It will be seen that the chance obtained depends on χ^2 which involves p; and the question arises as to the value of p. In other words, we do not know our original population from which the samples are drawn, what value shall we give to p? We can proceed in this way.

The chance that in a sample of n we get r of class A, and in a sample of m we get s of class A, is

$$\frac{n!}{r!\,(n-r)!}\,\frac{m!}{s!\,(m-s)!}\,p^{r+s}\,q^{n+m-r-s}.$$

This chance is maximum when p has the value $\dfrac{r+s}{n+m}$ which we will call P.

Thus we may say that of the many populations from which the given samples may have been drawn that which is characterised by P is the most likely. If we say then that this was the population from which the samples were drawn, we have the chance that in the given samples we get the doublet r, s or other doublets less likely is

$$e^{-\frac{1}{2}\chi^2} \qquad \text{.....(iii),}$$

where

$$\chi^2=\frac{(r-nP)^2}{nPQ}+\frac{(s-mP)^2}{mPQ}.$$

Now

$$r-nP=\frac{rm-sn}{n+m},\quad s-mP=\frac{sn-rm}{n+m},$$

so

$$\chi^2=\frac{(rm-sn)^2}{(n+m)^2}\cdot\frac{n+m}{nmPQ},$$

therefore

$$\chi^2=\frac{(rm-sn)^2(n+m)}{nm\,(r+s)\,(n+m-r-s)}.$$

4. It is of interest to point out that, if we were considering the problem of whether the series of numbers

$$r,\ s,\ n-r,\ m-s \quad \text{Total } (n+m),$$

might have occurred in random sampling when a single sample is drawn from a population when the expected or average values were

$$nP,\ mP,\ nQ,\ mQ \quad \text{Total } (n+m),$$

we should find

$$\begin{aligned}&\frac{(r-nP)^2}{nP}+\frac{(s-mP)^2}{mP}+\frac{(n-r-nQ)^2}{nQ}+\frac{(m-s-mQ)^2}{mQ}\\&=(r-nP)^2\left(\frac{1}{nP}+\frac{1}{nQ}\right)+(s-mP)^2\left(\frac{1}{mP}+\frac{1}{mQ}\right)\\&=\frac{(r-nP)^2}{nPQ}+\frac{(s-mP)^2}{mPQ},\end{aligned}$$

that is to say, the χ^2 we have arrived at above. In such a problem as this, of course, we should use the χ^2 Table in the Book of *Tables for Statisticians*, using $n'=4$. The value $e^{-\frac{1}{2}\chi^2}$, (iii), can be obtained from the same Table, using $n'=3$.

Further in the usual method of considering if the two samples as given have been drawn from the same population, when we compare the difference between the proportions of class A with the S.D., if we take this S.D. as $\sqrt{\frac{PQ}{n}+\frac{PQ}{m}}$, we should be comparing $\frac{r}{n}-\frac{s}{m}$ with $\sqrt{\frac{PQ}{n}+\frac{PQ}{m}}$.

If we desire to obtain the chance that the given difference or others less likely would occur we should have the expression

$$2\int_{\frac{r}{n}-\frac{s}{m}}^{\infty}\frac{1}{\sqrt{2\pi}\cdot\sqrt{\frac{PQ}{n}+\frac{PQ}{m}}}e^{-\frac{1}{2}\frac{u^2}{\left(\frac{PQ}{n}+\frac{PQ}{m}\right)}}du,$$

which is

$$\sqrt{\frac{2}{\pi}}\int_{\chi}^{\infty}e^{-\frac{1}{2}\chi^2}d\chi \quad \text{.....(iv),}$$

the limiting χ being

$$\frac{\frac{r}{n}-\frac{s}{m}}{\sqrt{PQ\left(\frac{1}{n}+\frac{1}{m}\right)}},$$

which is the same as that already given above.

The integral (iv) we should of course obtain from the Table of the Probability Integral in the Book of Tables, and it corresponds to the case $n'=2$ of the Table of Goodness of Fit.

5. It will be seen that the proposed suggestion for considering the problem of two given samples being drawn from the same population uses as a basis the chance that in the given samples of n and m, there shall occur r and s of class A respectively as a double event or other doubles less likely to occur together; whereas the usual method uses as a basis of argument the chance that the difference in the proportions of class A should be $\left(\frac{r}{n}-\frac{s}{m}\right)$ or other differences less likely to occur. The two interpretations of the fundamental problem are entirely different, and, naturally, lead to different chances. An example will illustrate the difference. Samples of boys of the same age are considered, from two districts, the character measured being the weight. The two samples may be shown *:

	Light	Heavy	Totals
District 1	142	118	260
District 2	156	189	345

* *On the Relationship of the Condition of Teeth in Children to Factors of Health and Home Environment*, by E. C. Rhodes, 1921, p. 54.

Here we have

$$n = 260,\ r = 142\ ;\ m = 345,\ s = 156\ ;\ P = \tfrac{298}{605};$$

$$\frac{r}{n} - \frac{s}{m} = \cdot 09398\ ;\ \sqrt{PQ\left(\frac{1}{n} + \frac{1}{m}\right)} = \cdot 04106\ ;$$

$$\chi = 2\cdot 29,\ \chi^2 = 5\cdot 24.$$

The chance that in samples of 260 and 345 drawn from a population characterised by P we should get the numbers actually observed in the "light" class or other frequencies less likely to occur together is ·07; obtained by entering the Table of Goodness of Fit for $\chi^2 = 5\cdot 24$, $n' = 3$. The chance that in these samples we should get a difference between the proportions of the "light" class to the whole equal to that observed, viz. ·09398, or a difference less likely is ·02; obtained by entering the Table of the Probability Integral for $x = 2\cdot 29$. Thus as a basis of further argument as to whether these two samples may be regarded as drawn from the same population, we have in the first case the figure ·07, and in the second case the figure ·02. The cause of the difference between these figures may be shown simply by an illustration. If two samples of the sizes indicated in this example are drawn from the population characterised by P, the chance that we get only one individual in the "light" class in each sample is very small indeed, and such a double event would help by its chance to contribute towards the figure ·07, the chance that the double observed or a double more unlikely should happen. But in such a case as this the difference between the proportions of the "light" class to the whole would be ·0095, and this difference being less than that observed would not contribute its chance to the total ·02, which is the chance that a difference as great as that observed and other differences greater (or, what is the same thing, less likely) should occur.

PART II.

The same line of argument as in Part I may be applied to the consideration of two samples which are distributed in more than two classes. If we consider the general case where the samples are given as

							Total
Sample 1	n_1	n_2	...	n_k	...	n_{s+1}	n
Sample 2	m_1	m_2	...	m_k	...	m_{s+1}	m

distributed in $(s+1)$ classes; then the argument proceeds on similar lines. We shall assume as before that there is an infinite population from which samples may be drawn characterised by $p_1, p_2, \ldots p_k, \ldots p_{s+1}$. Assuming normality for the distributions of frequency in each class we have the chance that in a sample of n the frequencies in the respective classes lie between x_k and $x_k + dx_k$, where $k = 1, 2, \ldots s$. (If s of the classes have their frequency fixed the $(s+1)$th class frequency is known.) This chance is

$$\text{Const. } e^{-\frac{1}{2}\left[Sc_{kk}\left(\frac{x_k - \bar{x}_k}{\sigma_k}\right)^2 + 2S'c_{kk'}\frac{(x_k - \bar{x}_k)(x_{k'} - \bar{x}_{k'})}{\sigma_k \sigma_{k'}}\right]} \times \prod_1^s dx_k \quad \ldots\ldots\ldots\text{(v)},$$

where
$$\bar{x}_k = np_k,$$
$$\sigma^2_k = np_k(1-p_k),$$
$$c_{kk} = \frac{\Delta_{kk}}{\Delta},$$
$$c_{kk'} = \frac{\Delta_{kk'}}{\Delta},$$

where
$$\Delta = \begin{vmatrix} r_{11} & r_{12} & r_{13} \dots r_{1s} \\ r_{21} & r_{22} & r_{23} \dots r_{2s} \\ \dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots \\ r_{s1} & r_{s2} & r_{s3} \dots r_{ss} \end{vmatrix},$$

and Δ_{kk} is the minor of r_{kk}, $\Delta_{kk'}$ is the minor of $r_{kk'}$ in this determinant. Further r_{kk} for all values of k is equal to unity, and $r_{kk'} = r_{k'k}$. Also $r_{kk'} = \dfrac{-np_kp_{k'}}{\sigma_k\sigma_{k'}}$; and the summation sign S refers to summation of k from 1 to s, and the sign S' refers to summation of k and k' for all different values of k and k' from 1 to s.

Since
$$r_{kk'} = \frac{-\sqrt{p_kp_{k'}}}{\sqrt{(1-p_k)(1-p_{k'})}} \quad \text{(independent of } n\text{)}$$
we should get, in the similar expression to (v) when considering the sample of m, the same coefficients c, but different σ's.

We should have this expression for the chance that the combined event of drawing a sample of n and getting in the respective classes between x_k and $x_k + dx_k$, drawing a sample of m and getting between y_k and $y_k + dy_k$ (k varies from 1 to s),

$$\text{Const. } e^{-\frac{1}{2}\left[Sc_{kk}\left(\frac{x_k-\bar{x}_k}{\sigma_k}\right)^2 + 2S'c_{kk'}\frac{(x_k-\bar{x}_k)(x_{k'}-\bar{x}_{k'})}{\sigma_k\sigma_{k'}} + Sc_{kk}\left(\frac{y_k-\bar{y}_k}{\sigma'_k}\right)^2 + 2S'c_{kk'}\frac{(y_k-\bar{y}_k)(y_{k'}-\bar{y}_{k'})}{\sigma'_k\sigma'_{k'}}\right]} \times \prod_1^s dx_k \prod_1^s dy_k \quad \text{.....(vi).}$$

These lead as before to the consideration of a multiple correlation surface in $2s$ variates, and we find the total "volume" of this surface outside a limiting "contour." This "volume" will give us the chance that in two samples of n and m we should get the distributions

$$n_1 \; n_2 \dots n_{s+1},$$
$$m_1 \; m_2 \dots m_{s+1},$$

or other distributions occurring together which are less likely. After transformation the integral giving this volume may be expressed in the form

$$\int_\chi^\infty e^{-\frac{1}{2}\chi^2}\chi^{2s-1}d\chi \Big/ \int_0^\infty e^{-\frac{1}{2}\chi^2}\chi^{2s-1}d\chi,$$

where the limiting χ in the integral in the numerator is given by

$$\chi^2 = Sc_{kk}\frac{(n_k-np_k)^2}{np_k(1-p_k)} + 2S'c_{kk'}\frac{(n_k-np_k)(n_{k'}-np_{k'})}{\sigma_k\sigma_{k'}} + Sc_{kk}\frac{(m_k-mp_k)^2}{mp_k(1-p_k)} + 2S'c_{kk}\frac{(m_k-mp_k)(m_{k'}-mp_{k'})}{\sigma'_k\sigma'_{k'}} \quad \text{.....(vii).}$$

To evaluate this χ^2 we follow Professor Pearson (*Phil. Mag.* Vol. 50, p. 157), and obtain

$$\chi^2 = \sum_1^{s+1} \left[\frac{(n_k - np_k)^2}{np_k} + \frac{(m_k - mp_k)^2}{mp_k} \right] \quad \text{.....(viii).}$$

Now, with regard to the best values of p_k to take, we proceed as before, finding the chance that we actually get in a sample the two distributions obtained. It is

$$\frac{n!}{n_1!\, n_2! \ \dots\ n_{s+1}!} \frac{m!}{m_1!\, m_2! \ \dots\ m_{s+1}!} p_1^{n_1+m_1} p_2^{n_2+m_2} \dots p_{s+1}^{n_{s+1}+m_{s+1}},$$

and, as before, we find when this is a maximum, for different p's. We find that the best values of the p's are those given by $p_k = \dfrac{n_k + m_k}{n+m}$, for $k=1$ to $s+1$. Substituting these values in (viii), we have, since

$$(n_k - np_k)^2 = (m_k - mp_k)^2 = \frac{(n_k m - m_k n)^2}{(n+m)^2},$$

$$\chi^2 = \sum_1^{s+1} \frac{(n_k m - m_k n)^2}{nm\,(n_k + m_k)} = nm \sum_1^{s+1} \frac{\left(\dfrac{n_k}{n} - \dfrac{m_k}{m}\right)^2}{n_k + m_k} \quad \text{.....(ix).}$$

This is the same general expression as that given by Pearson in his paper in *Biometrika*, Vol. VIII. p. 250. Thus to find the chance of obtaining the given distributions in the two samples of n and m from the given population, or other distributions less likely to occur together, we find the expression (ix) and enter for this χ^2 in the Goodness of Fit Table for $n' = 2s+1$, where $(s+1)$ is the number of classes in the distributions.

PART III.

1. It will be seen that this method of treatment may be extended to the consideration of the case of more than two samples. Let us consider first the problem of three samples, where the individuals are in alternate categories.

	Class A	Not-A	Total
Sample 1	r	$n-r$	n
Sample 2	s	$m-s$	m
Sample 3	t	$l-t$	l

Suppose we wish to consider the possibility that these samples are drawn from the same population. We will as before assume an original population characterised by p and we have the chance that in three samples sizes n, m, l we get frequency in class A between x and $x+dx$, y and $y+dy$, z and $z+dz$ respectively, is

$$\frac{1}{(2\pi)^{\frac{3}{2}} \sigma\, \sigma'\, \sigma''} e^{-\frac{1}{2}\left\{\left(\frac{x-\bar{x}}{\sigma}\right)^2 + \left(\frac{y-\bar{y}}{\sigma'}\right)^2 + \left(\frac{z-\bar{z}}{\sigma''}\right)^2\right\}} dx\,dy\,dz,$$

where $\bar{x} = np$, $\sigma^2 = npq$, $\bar{y} = mp$, $\sigma'^2 = mpq$, $\bar{z} = lp$, $\sigma''^2 = lpq$,

where $q = 1-p$.

As before we obtain the chance that we get in class A the numbers actually observed (r, s, t) or other combinations less likely, this chance reducing to the ratio

$$\int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2}\chi^2 d\chi \Big/ \int_{0}^{\infty} e^{-\frac{1}{2}\chi^2}\chi^2 d\chi,$$

where the limiting χ is given by

$$\chi^2 = \frac{(r-np)^2}{npq} + \frac{(s-mp)^2}{mpq} + \frac{(t-lp)^2}{lpq}.$$

To evaluate this we require to know the best value to give to p; proceeding as before, arguing from the chance that the triple event as described does happen, viz.

$$\frac{n!}{r!(n-r)!}\frac{m!}{s!(m-s)!}\frac{l!}{t!(l-t)!}p^{r+s+t}q^{n+m+l-r-s-t},$$

we obtain the best value of p is

$$\frac{r+s+t}{n+m+l} \quad (P, \text{ say}).$$

It is probably best to write χ^2 in the form

$$\frac{1}{PQ}\left\{n\left(\frac{r}{n}-P\right)^2 + m\left(\frac{s}{m}-P\right)^2 + l\left(\frac{t}{l}-P\right)^2\right\}.$$

Thus by finding this χ^2 and entering the Goodness of Fit Table for $n' = 4$, we get the chance that the triple event as observed or others less likely would occur. It will be realised that this may be extended to any number of samples. Thus if we have N samples, distributed in alternate categories, we find

$$\chi^2 = \sum_{1}^{N}\left\{\frac{n\left(\frac{r}{n}-P\right)^2}{PQ}\right\},$$

where n is the number drawn in a typical sample, r the observed frequency in class A of this typical sample and

$$P = \frac{\sum_{1}^{N}(r)}{\sum_{1}^{N}(n)}$$

$$= \frac{\text{total number drawn in class } A \text{ in all samples}}{\text{total number drawn in all samples}}.$$

2. A little consideration will show that this can be extended to the case where the samples are distributed in $(s+1)$ classes as in the second part of Part II. We should arrive at an integral

$$\int_{\chi}^{\infty} e^{-\frac{1}{2}\chi^2}\chi^{Ns-1} d\chi \Big/ \int_{0}^{\infty} e^{-\frac{1}{2}\chi^2}\chi^{Ns-1} d\chi,$$

where N is the number of samples. Using the same notation as before we should find that the limiting χ is given by

$$\chi^2 = \sum_{1}^{s+1}\frac{(n_k-np_k)^2}{np_k} + \sum_{1}^{s+1}\frac{(m_k-mp_k)^2}{mp_k} + \sum_{1}^{s+1}\frac{(l_k-lp_k)^2}{lp_k} + \ldots \; N \text{ such terms.}$$

Again we should take as the most probable values for the p's

$$p_k = \frac{n_k + m_k + \ldots \text{ to } N \text{ terms}}{n + m + \ldots \text{ to } N \text{ terms}},$$

where $k = 1$ to $s+1$.

Suppose we arranged our knowledge of the samples obtained in the tabular form

						Total
Sample 1	n_1	n_2	...	n_k	...	n_{s+1}	n
Sample 2	m_1	m_2	...	m_k	...	m_{s+1}	m
Sample 3	l_1	l_2	...	l_k	...	l_{s+1}	l
⋮	⋮	⋮	...	⋮	...	⋮	⋮

showing the distributions in the $(s+1)$ classes of all N samples. It will be seen that the numerator of p_k is the sum of the frequencies in the kth column of this table. If for the moment we care to look at this table as a contingency table in two variates, then the frequency expected in the first cell of the kth column is np_k, and a glance at our χ^2 will show that it is the same as $(n+m+l+\ldots) \times$ mean square contingency, if such a table were regarded as a contingency table and the mean square contingency were obtained. Referring to this number as ϕ^2, we have then :— To get χ^2 arrange the given data as though the observed frequencies formed a contingency table, obtain from this ϕ^2, multiply ϕ^2 by the total of all the samples drawn and so get χ^2. Then the chance that in N samples of $n, m, l, \ldots$ we get the observed distributions or others less likely to occur together is obtained by entering the Goodness of Fit Table for χ^2 and $n' = Ns + 1$, where N is the number of samples and $(s+1)$ is the number of classes in each sample. It will be seen therefore, that, if we choose to regard an ordinary contingency table as representing the results of drawing N samples each distributed in $(s+1)$ classes, we have here an expression giving the chance that these samples may have been drawn from the same population. This point of view helps to supplement the information ordinarily derived from the coefficient of contingency. It is perhaps of interest to emphasise the argument in the preceding pages, that we are here obtaining a chance that a certain number of samples may reasonably be regarded as drawn from the same population. For instance we may be interested to know if the male populations of Leeds, Bradford, Huddersfield and Dewsbury are homogeneous as regards age distribution. In this case we have four samples. Care must however be shown when this line of argument is applied to another problem. I refer to the problem where we have one sample only, but the individuals of the sample are known to possess two attributes in varying degrees. In such a case as this the information would be presented in the form of a contingency table. Suppose such a table has r rows and c columns, and the variate measured horizontally may be referred to as A, that measured vertically as B; then we may ask ourselves, as a means of obtaining an idea as to possible relationship between A and B, whether the r arrays for constant B, regarding them as r samples with frequency in c classes, may have arisen in random sampling from the same population. This would mean the calculation of

χ^2 from the Table and entering the Goodness of Fit Table for $n' = r(c-1)+1$. Alternatively we may ask the other question, considering the c arrays for constant A as c samples with frequency in r classes; are such c samples likely to have been drawn from the same population? In this case we enter for the same χ^2 as before, but now for $n' = c(r-1)+1$. It will be seen that when r and c are different, the resulting P's are different, and naturally so, since they are chances of different events. In the one case we find the chance that certain r samples, distributed in c classes, or other samples less likely have been drawn from a certain population; in the other case we find the chance that certain c samples, distributed in r classes, or other samples less likely have been drawn from another population. These two problems are different and of course lead to different results. We get a definite answer to a definite question:—What is the chance that a certain number of samples as given may have arisen by chance from the same population, or other samples less likely to occur? How far this chance may be used as a basis of argument as to the independence (or otherwise) of two variates, as judged from a single sample shown as a contingency table, is a matter for the individual worker.

ON THE DIFFERENCE AND THE DOUBLET TESTS FOR ASCERTAINING WHETHER TWO SAMPLES HAVE BEEN DRAWN FROM THE SAME POPULATION.

By KARL PEARSON.

I SHOULD like to add a few words to Mr Rhodes' very interesting and important discussion. Mr Rhodes speaks of the Difference method as the "classic" method of distinguishing between significant and insignificant differences, and he rightly so calls it; for it must be remembered that this method has been in use for nearly 150 years, and if we were now to declare it fallacious and replace it by the doublet method a great deal of work—both astronomical and statistical—would have to be redone or the conclusions reconsidered. Now it seems to me that when we are comparing any two distributions we may test the probability of their common origin in a great variety of ways. We may inquire into their means, or their standard-deviations, or their higher moments, or several or all of these. And if, having regard to their probable errors, we find a divergence of significance in any *one* of these, we are at liberty to assert that they are not consubstantial. I mention this matter, because it seems to me that there are many tests which may be applied with equal logical validity to test whether two samples are consubstantial, and that if with two equally logical tests one should lead to a significant difference and the second to an insignificant difference, then we must assert that in a certain respect the two samples have no significant difference, but in another they have a significant difference, and on the basis of this they cannot be considered wholly of the same origin, i.e. are not what we are to understand by the word consubstantial. Thus it might be possible for two samples to show no significant differences in means and standard-deviations, but significant differences in their third moment coefficients. On the basis of these considerations I think it extremely likely that different tests may give different results. The statistician has to apply of such tests a number reasonable for the purpose he has in hand; and will, I should say, always be guided in rejecting or accepting common origin by the most stringent of those tests.

(1) Let us take a particular case. We have two means m_1, m_2 of two samples, we can test the improbability of the doublet (m_1, m_2), or the improbability of the difference $(m_1 - m_2)$ on the hypothesis that the samples are consubstantial. If these two methods can be shown to be equally logical, and in any actual case the chance of more improbable doublets occurring is great, but the chance of more improbable differences occurring is small, I should be inclined to say that the difference method being a more stringent test is to be given the preference.

If σ_1, σ_2 and n_1, n_2 be the respective standard-deviations and frequencies of any two samples, then Mr Rhodes would take

$$\text{const.} \times e^{-\frac{1}{2}\left(n_1 \frac{m_1^2}{\sigma_1^2} + n_2 \frac{m_2^2}{\sigma_2^2}\right)} dm_1 dm_2$$

for the probability of his doublet lying between m_1, m_2 and $m_1 + dm_1$, $m_2 + dm_2$ and he proceeds very properly to measure the volume of his frequency surface lying outside the ellipse

$$n_1 \frac{m_1^2}{\sigma_1^2} + n_2 \frac{m_2^2}{\sigma_2^2} = \chi^2$$

in terms of his whole volume.

But it is equally open to those who take the "classical" view to test the distribution of $m_1 - m_2$. This does *not* involve the taking of the doublet surface above as a preliminary stage; it is quite easy to show that if two variates x and y obey the normal law of distribution, then their sum or difference also obeys that law. For, if the odd moment coefficients of x and y are zero, and the even are given by

$$_x\mu_{2s} = (2s-1)(2s-3)\ldots 1 \times \sigma_x^{2s},$$
$$_y\mu_{2s} = (2s-1)(2s-3)\ldots 1 \times \sigma_y^{2s},$$

it follows by simply raising $z = x \pm y$ to any power and summing that the odd moments of z vanish and the even are given by

$$_z\mu_{2s} = (2s-1)(2s-3)\ldots 1 \times (\sigma_x^2 + \sigma_y^2)^s,$$

which results involve the normal distribution of z.

It seems therefore to me equally logical to inquire how much of this curve for $m_1 - m_2$ lies outside a given value of $m_1 - m_2$ from the simple knowledge of the standard-deviation $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ which is indeed the old test. Both tests may, I hold, with equal validity be applied, and the more stringent will be the safer guide.

(2) Mr Rhodes in replacing in his *particular* case the binomial by the normal curve has neglected terms of the order $\frac{1}{\sqrt{n}}$ and $\frac{1}{\sqrt{m}}$ as compared with unity. By taking as his P that most probable on the basis of his two samples he deduces

$$\chi^2 = \frac{(rm - sn)^2 (n+m)}{nm\,(r+s)\,(n+m-r-s)}.$$

This he obtains by replacing our

$$\sqrt{\sigma_x^2 + \sigma_y^2}, \text{ i.e. } \sqrt{\frac{r\,(n-r)}{n^3} + \frac{s\,(m-s)}{m^3}},$$

by $\sqrt{PQ\left(\frac{1}{n} + \frac{1}{m}\right)}$, where $Q = 1 - P = 1 - \frac{r+s}{n+m}$. This change is perfectly legitimate, but it is unnecessary; it involves the hypothesis of using the most probable value of P. Now P only differs from r/n or s/m by terms of the order of their standard-deviations, i.e. of order $\frac{1}{\sqrt{n}}\sqrt{\frac{r}{n}\left(1 - \frac{r}{n}\right)}$ and $\frac{1}{\sqrt{m}}\sqrt{\frac{s}{m}\left(1 - \frac{s}{m}\right)}$, and these containing the factors $1/\sqrt{n}$ and $1/\sqrt{m}$, are terms of the order of those which have been neglected in replacing the binomials by normal curves. As long as this step is taken, we seem at liberty to compare $r/n - s/m$ with

$$\sqrt{r\,(n-r)/n^3 + s\,(m-s)/m^3}$$

in the "classic" manner, and need not trouble about introducing the best value of P at all*.

(3) In the same manner Mr Rhodes when he comes to $(s+1)$-fold samples considers the "bevy"

$$n_1, m_1;\ n_2, m_2;\ \ldots\ n_{s+1}, m_{s+1}$$

and asks what is the probability of a more improbable bevy being drawn. But it is equally legitimate to ask what is the probability of a more improbable set of differences

$$n_1 - m_1,\ n_2 - m_2,\ \ldots\ n_{s+1} - m_{s+1}$$

being drawn. Both methods of approaching the problem depend upon the computation of χ^2, and the use of the tables of goodness of fit. But Mr Rhodes' "bevy" method requires us to look up P for $2s$ independent variates, while in the difference method† there are only s independent variates. In Mr Rhodes' case we enter the Table‡ with $n' = 2s+1$, in the difference method with $n' = s+1$. Suppose now our sampled population consisted of material divided into fifteen categories and that $\chi^2 = 29$. Then the bevy method leads us to enter with $n' = 29$ and gives $P = \cdot 41$, the difference method with $n' = 15$ and gives $P = \cdot 01$. We should conclude accordingly that on the bevy hypothesis the samples might easily be consubstantial, but that on the difference hypothesis this was very unlikely.

Now it seems to me that if we tested two samples in two different ways—say, to take a concrete case, we tested for equality of means and equality of standard-deviations—and found the means insignificantly different but the standard-deviations very significantly different, we should conclude that they could not be consubstantial. Accordingly in the present case wherein we apply two different, and, I think, equally logical tests, we are bound to decide our judgment by the more stringent, i.e. the one with the smaller P. But since the doublet or bevy test always gives a greater n', it will always give a higher P, and accordingly the "classic" method is the safer method to use, if we desire to have confidence in our judgment that the two samples may be reasonably treated as consubstantial. Had the doublet or bevy test given a lower value of P, I should have said that it must be used in preference to the difference test. Mr Rhodes' paper is serviceable because it shows us, while really justifying the long-established classical method, that we have rather slipped into its use by luck§ than knowledge; there has never been a proof that it is the "best" or most stringent test.

* It is quite reasonable to suggest that $\sqrt{\frac{1}{n}\frac{r}{n}\left(1-\frac{r}{n}\right)+\frac{1}{m}\frac{s}{m}\left(1-\frac{s}{m}\right)}$ is as "good" an approximation to the *unknown* $\sqrt{PQ\left(\frac{1}{n}+\frac{1}{m}\right)}$ as $\sqrt{\frac{r+s}{n+m}\left(1-\frac{r+s}{n+m}\right)\left(\frac{1}{n}+\frac{1}{m}\right)}$ is under the conditions of approximation Mr Rhodes has previously adopted.

† *Biometrika*, Vol. VIII. p. 252.

‡ *Tables for Statisticians and Biometricians*, Part I, p. 26.

§ I take it that the early writers on the subject desired to find out whether two populations were "different," i.e. were not samples from a single population, and thus without further ado discussed the *differences* of their constants—a verbal but lucky guide.

With Mr Rhodes' process of testing a multiplicity of samples I must confess that I should feel unhappy. I should much prefer to apply the difference test to every pair of samples, or, if that labour were too great, to compare each sample with the sum total of all the samples—the general observed population. Mr Rhodes' process would lead to very high value of n', and I should be very doubtful of its adequate stringency.

(4) Again with Mr Rhodes' comparison on p. 247 between the problem of N-samples and a contingency table, I am by no means content. I do not think this point of view "helps to supplement the information ordinarily derived from the coefficient of contingency." On the contrary I think it will confuse at any rate the beginner. The contingency table is a sample from a population, and a sample of such a nature that its marginal totals vary from sample to sample. In Mr Rhodes' case of N-samples one marginal total (and one only) has its constituents constant. The case does not correspond to a true contingency table at all, but to what I have elsewhere* dealt with under the heading of *partial* contingency, namely cases of contingency in which linear relations are imposed on certain of the cell contents, every such relation reducing the degree of freedom by unity. Mr Rhodes' N-samples written as a contingency table are a *partial* contingency table in which, if there are p cells, the degrees of freedom are not $p-1$ as in the ordinary contingency table but $p-N$. The distinction is an important one, as non-observation of it has led before this into errors in the use of the goodness of fit tables. Mr Rhodes has set a most interesting discussion afoot, which I should like to generalise as follows: We can test whether two samples are consubstantial in a variety of ways, is it possible to find a better test—by which I mean a more stringent test—than the "classic" method of examining the distribution of differences?

* *Biometrika*, Vol. XI. p. 145 *et seq.*

THE SKULL OF ROBERT THE BRUCE, KING OF SCOTLAND, 1274—1329.

By KARL PEARSON, F.R.S.

The king ansuer has maid thaim nane,
Bot menyt hys hand-ax shaft sua
Wes with the strak brokyn in twa.
BARBOUR'S *The Bruce*, XII, 96.

ROBERT THE BRUCE VIII was the seventh lineal descendant of Robert de Brus (or de Bréaux) I who came over with the Conqueror and was rewarded by him with 94 manors in England, among others that of Danby in Cleveland, where his castle is still in existence, symbolic of the rugged sturdiness of the stirp. The third Robert was the founder of the Scottish branch, holding the fief of Annandale. Of the seven Roberts who preceded King Robert, nearly all were stalwart fighters, wealthy and even generous men, but scarcely of the stuff out of which great statesmen even in those days were made. The fifth Robert, who died in 1245, married Isabel daughter of David, the son of Henry of Scotland and brother of William the Lyon. On this marriage was based the claim of the eighth Robert to the crown of Scotland. Robert the Fourth had also married into the Scottish Royal blood, having for wife a daughter of William the Lyon himself. The sixth Robert married a daughter* of Gilbert de Clare of Gloucester; the seventh Robert a Scotswoman, Marjory, Countess of Carrick†, the mother of our Robert the Bruce. Thus our Bruce was a mixture of Scottish and Norman blood, as his allegiance was divided between Scotland and England by his fiefs in both countries. We cannot therefore look upon Bruce as of pure race; on the father's side he should have been a Norseman, but on the mother's side he came from the Celtic blood of Galloway. To judge him by his skull we should say that he was a man of great muscular strength and energy; the frontal would not popularly indicate great intellectual power, but the acts passed in his reign seem to suggest that besides urging the efficient training in arms and the provision of arms for the nation, he had a considerable sense of the importance of justice between class and class and the need of restraining the great nobles from oppressing the commonalty. Among the people he went by the name of the "Good King Robert."

After Bruce's assassination of John Comyn and Comyn's uncle in the church of

* Isabel (b. 1226). Her mother was Isabella, daughter of William Marshall, Earl of Pembroke.

† She was a daughter of Walter, the Steward of Scotland, and this was the first link between Bruce and Stewart blood. I presume he was Walter the Steward of Scotland who died in 1246 and was the great-grandfather of Walter the Steward who married Bruce's daughter Margaret. Thus Margaret and her husband were second cousins and Bruce already had Stewart blood in his veins. It is owing to these marriages that H.R.H. the Prince of Wales bears the titles of Earl of Carrick and Great Steward of Scotland.

the Friars Minor in Dumfries and after his assumption of the crown (1305) he suffered many hardships, but slowly pulled the Scots together, until he won the freedom of Scotland by the great victory of Bannockburn in 1314. The full story of Bruce does not concern us here, but there is little doubt that beyond his great strength, he was a man of marked character, who could determine a great policy, and had the power to carry it through not only by brute force, but by foresight and apt choice of lieutenants. Riding his pony, or "litill palfray," he could fell

> Schyr Henry the Boune*, the worthy
> That wes a wycht knyght, and a hardy,

with one stroke of his battle-axe, and make no other reply to the remonstrance of his friends at the risk he ran, than to regret the broken shaft. But there were other sides to his nature; he could encourage ship-building, lay out his palace gardens, and reform the law. Like most of the Scottish kings, and nobles, and not a few of the like classes in other countries, he had a very considerable progeny born out of wedlock. He was probably a man of strong passions, of strong hatreds and firm friendships. His murder of John Comyn in an ecclesiastical building was not incompatible with strong religious feeling, nor with a vow to visit the Holy Sepulchre, nor with his desire on its non-fulfilment that Douglas should carry his heart there.

In the later years of his relatively short life Bruce is said to have suffered from "leprosy," which Dr Mackay somewhat quaintly describes as "contracted during the hard life of his earlier struggles." Leprosy could scarcely be due to a hard life unless such life brought a man into contact with lepers. Except for the date of his death, 1329, we might more readily imagine it a case of sporadic syphilis, which appears in early times to have been confused not infrequently with leprosy. Chalmers in 1844 had some suspicion that Bruce's disease was not leprosy, for he writes that Bruce died "at Cardross near Dumbarton, on the 7th June, 1329, from a severe disease, then termed leprosy." (*Hist. and Statist. Account of Dunfermline*, Vol. I. p. 148.)

That Bruce's body was brought from Cardross and buried before the high altar in the Abbey Church of Dunfermline seems quite certain†. But a new church was built in 1250 and the old church appears to have become then a vestibule church. There is therefore some doubt as to the positions of the earlier and later *Locum sepulturae Regum*. The matter has been discussed at length by Dr Ebenezer Henderson in his two books‡, and the very good plan of the Abbey by Frances C. Eeles§, together with the work of David Macgibbon and Thomas Ross‖, seem to leave little doubt that the body was found before the high altar of the church

* Sir Henry de Bohun. See *The Bruce*, XII. 29.

† "Sepultusque est rex apud monasterium de Dunfermelyn in medio chori, debito cum honore." Fordun's *Scotichronicon*, Vol. II. p. 292 (see Note § p. 256). See also Barbour's *The Bruce*, XX. 285–7.

‡ *The Royal Tombs at Dunfermline*, Dunfermline, 1856, and the *Annals of Dunfermline, from the Earliest Authentic Period to the Present Time*, A.D. 1069 to 1878, Glasgow, 1879. The latter deals with the tomb on p. 130.

§ See p. xxxi of Dr Erskine Beveridge's *Burgh Records of Dunfermline*, Edinburgh, 1917. The plan is enlightening for the relationships of the three churches.

‖ *Ecclesiastical Architecture of Scotland*, Edinburgh, 1896. See Vol. I. pp. 230—258. This gives an excellent description of the Abbey and the Abbey Churches, and shows that much is still left of the older buildings.

as it existed at Bruce's death. Peter Chalmers in his *Historical and Statistical Account of Dunfermline* (Vol. I. 1844, Vol. II. 1859) gives much information in a discursive way which it would not be easy to find elsewhere, but the reliability of which may be open occasionally to question*. When Edward I in 1304 burnt Dunfermline Abbey, the great church seems to have escaped. It is not so certain whether that church also escaped in 1385 when according to Froissart, Richard II and his lords leaving Edinburgh went to Dunfermline where is a large and fair abbey of black monks in which the Kings of Scotland have been accustomed to be buried. The King was lodged in the Abbey, but after his departure the army seized it and burnt both that and the town†. This does not directly state that the great church was destroyed, although the English army was not likely to be desirous of sparing Bruce's tomb. On the other hand more modern historians deny that Richard II ever crossed the Forth‡. It is accordingly possible that Bruce's monument escaped foreign foes. Henderson holds that the shrines, *tombs*, crosses, relics and images of Dunfermline Abbey were destroyed and the greater part of the buildings razed to the ground by the reforming bigots of 1560. Thus Robert Lindsay of Pitscottie writes§:

Upoun the 28 day thairof [March 1560] the wholl lordis and barones that war on thys syd of Forth passed to Stirling and be the way kest doun the Abbey of Dumfermling.

The "lordis and barones" were armed with a general order or warrant, and "in their mistaken zeal"—a somewhat charitable method of expression—they appear faithfully to have done "their taske til ye leter"—or rather somewhat beyond it‖!—The warrant runs:

To our traist friendis—Traist friendis, after maist harty commendacioun, we praye you to faill not to pass incontinent to the Kirk of Dunfermline and tak doun the hail imagis thereof, and bring them furth to the kirkyard, and burn them openley and sicklyk cast doun the altaris and purge the kirk of all kynd of monumentis of idolatrye. And this ze fail not to do as ze will do us singulare emplesair, and so commitis you to the protection of God. (Signed) Ar. Argyle. James Stewart. Ruthven.—(See Henderson's *The Royal Tombs at Dunfermline*, p. 25.)

Three names of ill omen in Scottish History! There is added a postscript: "Fail not, bot ye tak guid hyd that neither the dasks windochs nor durris be ony wayis hart or broken either glassin wark or iron wark." But what if the "glassin wark" and "windochs" contained the images of saints, and what if the lead of the roofing was desirable for bullets? Anyhow the second church seems shortly after this

* Thus on Plate IV, p. 71 of Vol. I we are given an engraving of a coin of Robert Bruce, which is very unlike the coins reproduced on our Plate VII. Either it was from a very different die, or the engraver has thought it right to add a strong jaw and a chin which inspire some respect! Cf. also Pinkerton's engraving of the Bruce coin in his edition of Barbour's *Bruce*, Vol. I. p. 3. In his second volume Chalmers provides a plate of fragments from Bruce's tomb, but no evidence is given that they are such, or that indeed they are parts of a tomb at all.

† *Histoire et Chronique de Messire Jehan Froissart.* Ed. Denis Sauvage, Paris, 1574. T. I. p. 275. See my *Note* p. 272.

‡ See Fernie's *History of Dunfermline*, pp. 11, 132, 133 and Henderson's *Annals of Dunfermline*, pp. 142–3.

§ *The Chronicles of Scotland*, Vol. II. p. 555, Ed. Dalzell, 1814.

‖ It must be remembered always that the reforming Lords were expropriating lords and had every reason for leaving the ecclesiastical buildings uninhabitable and worthless, in case their former owners desired to return.

visitation to have perished. The question I have been unable to answer is whether Father Time, Richard the Second, or Moray's "lordis and barones" destroyed the monument to Bruce in the Abbey Church. As that monument was almost certain to have included an effigy of the king, a traditional portrait may have existed up to the time of the Scottish Reformation, if indeed the "lordis and barones" were really the despoilers of the tomb of their greatest king as they were the defamers of their most noteworthy queen. Bruce before his death during his long illness gave an order to sculptors in Paris to prepare a monument for him, and we know from the still extant accounts that this was erected after his death. That it must have been of marked importance and splendour can hardly be doubted. It was brought to Scotland by way of Bruges, and the Chamberlain's Rolls refer to considerable sums paid for the gold purchased in Newcastle and York to gild it. Fragments of black and white marble and some stone carvings showing signs of gilding have been preserved at Dunfermline and in the Museum of the Society of Antiquaries*, and are *said* to be from Bruce's tomb. In the same Museum is a small head cut in soft bluish stone, apparently "calm" stone, the same kind of stone as many of the ornaments presumed to be from the tomb are reported to be made of. On Plate VIII the reader will find a representation from four aspects of this stone. It will be seen that, although the head has been badly damaged, it bears a crown. It has been suggested that this is the head of the king's effigy from the tomb. This seems to me improbable; not only do the large curls suggest a child†, but the main effigy of the monument would be likely to be of greater size; it is more reasonable to suppose it represented Bruce's son, David II, who was crowned in his father's lifetime, attendant at his father's tomb. However this may be, the head is not helpful for the portraiture of Bruce, nor have we any real information as to the design of and the effigies associated with the monument. We might not unnaturally expect to find not only Bruce, but his two wives who both died before him‡ and his two children—Margaret and David—represented. If so, not a fragment of the larger figures has survived. It is possible that the one thing from the tomb which has been preserved is an epitaph, which is provided by Fordun§:

> Hic jacet invictus ROBERTUS, Rex benedictus,
> Qui sua gesta legit, repetit quot bella peregit.
> Ad libertatem perduxit, per probitatem,
> Regnum Scotorum; nunc vivat in arce polorum.

Sir Henry Jardine in his *Report* favours the view that the monument was either pillaged at the time of the Reformation or destroyed by the falling ruins of the church. But he gives no arguments for this opinion.

* There are also fragments of the cloth of gold in which the body was wrapped.

† Chalmers (*loc. cit.* Vol. I. p. 149), following Jardine in his *Report*, suggests that the curls are similar in form to those on Bruce's coins. But not only are the coins *not* portraits of Bruce, but the French sculptors would hardly work from coins.

‡ Bruce's first wife was Isabella, daughter of the Earl of Mar; his second wife was Elizabeth, daughter of Aylmer de Burgh, Earl of Ulster, who died in 1327 (Fordun). Some works erroneously give her name as Mary (e.g. H. G. George's *Genealogical Tables*, 1904); the armorials sometimes as Isobell.

§ *Scotichronicon cum Supplementis et Continuatione Walteri Boweri.* Ed. Goodall. Tom. II. p. 293. This brief epitaph is followed by a much longer one.

In 1817 that part of the Abbey Church of Dunfermline in use as the parish kirk having become ruinous a new building was designed, which extended over the area of the royal tombs. In the course of building operations the workmen on February 17, 1818 struck a vault in line with the centre of the ancient choir and in front of where the high altar had formerly stood. This vault covered by two large slabs was pronounced to be a royal vault. It was reclosed and more formally opened on November 5, 1819 after the erection of the new kirk, in the presence of the King's Remembrancer, Sir Henry Jardine, of several Barons of the Exchequer, of Dr Munro, Professor of Anatomy in Edinburgh and of Dr Gregory, His Majesty's First Physician for Scotland; it was decided by them to be the tomb of Robert Bruce*.

The vault or grave was about seven feet long, of regular masonry, situated within a larger vault. It contained a human skeleton, wrapped up in two separate coverings of lead. At the time the grave was first opened, there was at the top of this lead covering, something which had the appearance of a rude crown. Under this lead was a covering of linen cloth interwoven with gold thread, but which, on the second opening, was found in a state of great decay. The whole being removed from over the body, the skull was taken up, and found in a most perfect state. The *os hyoides* was entire, and several cartilages of the larynx were visible, from having, it is supposed, been ossified. The whole teeth in the under jaw were entire, and in their places; but there were four or five in the upper jaw wanting, with a fracture of the jaw-bone in front, probably occasioned by a blow which the king had received in one of his military adventures†.

But the most remarkable circumstance which we observed upon examining the skeleton, was the state of the sternum, which we found had been sawed asunder longitudinally from top to bottom, the most satisfactory evidence that it was the body of King Robert Bruce, as it proved beyond a doubt that it had taken place previous to his interment, in order to get at the heart, which he had directed to be carried by Douglas to the Holy Land, and which the ignorance of the Anatomists of those days had made them perform, in order to enable them to comply with their Soveraign's last commands.

We now turn to a singular matter which is described in the following paragraph:

The workmen, in the course of their operations a few days afterwards, found among the rubbish of the vault a plate of copper, which had escaped notice at the previous examination, having engraved upon it a cross bearing the inscription "*Robertus Scotorum Rex.*" Above the inscription is the figure of a crown; and beneath it a crosslet with four stars or mullets inserted in the angles. The letters of the inscription resemble those on the coins of BRUCE. Upon the whole, there cannot be conceived more satisfactory evidence of any fact of the kind, than that the skeleton thus discovered was that of King ROBERT BRUCE‡.

* Jardine's *Report* lays no doubt on the Tomb being that of Robert Bruce, but then he and others of that date accepted the coffin-plate as genuine. Chalmers (*loc. cit.* Vol. I. pp. 146—150) sums up the arguments, I think, fairly conclusively in favour of Bruce. Extracts from Jardine's account of the opening of the tomb are given in the *Transactions of the Society of Antiquaries of Scotland*, Vol. II. pp. 435—455, 1822. Dr Gregory wrote a more humorous account of the proceedings, and detailed how a friend who accompanied him stole a metatarsal. See *Quarterly Journal of Science, Literature and Arts*, Vol. IX. p. 138, 1820. Chalmers (*loc. cit.* Vol. II. pp. 190–1) states that several individuals (himself included!) procured and still possess relics of Robert Bruce's *body*, leaden covering, shroud or Tomb.

† See our *Norma facialis*, Plate I.

‡ *Transactions of the Phrenological Society*, Edinburgh, 1824, pp. 247–8. This Society was instituted in 1820, on February 22nd. It is therefore not possible that the cast of Bruce's skull, discovered on November 5, 1819, was taken on the Society's initiative. In the paper: "Remarks on the Cerebral

Unfortunately this identity-clinching coffin-plate—accepted by the excavators and many subsequent writers—was the production of two practical jokers who wished to convince the good folk of Dunfermline that the tomb must be that of Robert Bruce. The copper plate is represented on our Plate VII. As Chalmers says (*loc. cit.* Vol. I. p. 150) it is "a well executed and appropriate plate bearing the appearances of antiquity." The culprits who designed and carried out the forgery were a young artist, Thom, a young brother of the new kirk architect, Bush, and one Nimmo, the son of an Edinburgh printer, who had in 1821 to flee from Edinburgh to Paris for being concerned with a scurrilous newspaper *The Beacon*. An amusing account of the matter is given in the *Proceedings of the Society of Antiquaries of Scotland*, Vol. XII. pp. 466 *et seq.*, 1878. See also the Secretary's remarks in Vol. VIII. pp. 360 and 413, 1870. The coffin-plate is now in a case of "Fakes" in the Society's Museum. The chief evil done by the escapade and its long concealment was the reaction from certitude to a really unjustifiable conviction in some minds that the tomb was not that of Bruce.

We now come to the step which saved Bruce's physiognomy for posterity:

The Skull after being prepared was given to Mr Scoular, an artist of talent brought for the purpose, who proceeded to make a cast of it in Paris Plaster, which he executed with great skill (Jardine's *Report*).

W. Scoular was a pupil and assistant of Chantry, and there is no doubt of the high order of his work on this cast.

The skull and skeleton were reported upon by Robert Liston, at that time surgeon in Edinburgh, but clearly Liston had very little craniometric interest, and Bruce's skull was reinterred in 1819—the year of its exhumation—within a few weeks and without any fitting craniometric study. Liston gives a drawing of the skeleton and a few not adequately defined measurements of the long bones, of which, perhaps, the most important are those of the femur as 17·5 inches and the tibia as 15·5 inches*. Assuming this to be the maximum length of femur and the tibia to include the spine, the lengths would be 44·45 cms. and less spine 38·41 cms. respectively. The reconstruction of the stature from the femur is 64″·95, from the tibia 66″·9 and from both 65″·8. Our values indicate that Bruce

Development of King Robert Bruce, compared with his Character as appearing from History" by William Scott (Writer to the Signet) there are three drawings of a cast of Bruce's skull by William Douglas. These are the *normae verticalis*, *lateralis* and *occipitalis*; each about the size of a square inch. They are very poor, and it is quite impossible to judge from them whether they were taken from the cast of the skull discussed in the present memoir. The paper has singularly little craniological value; there are nine measurements given, obviously of a very crude character. Thus "Philoprogenitiveness to lower Individuality" is said to be 8 inches and "Secretiveness to Secretiveness" is given as 6 inches; these correspond, as nearly as anything in the phrenological system can, to maximum length and maximum breadth, and would give for them 203 mm. and 152 mm. against our 198 and 154 respectively. I am unable to test any of the other seven measurements, because they are said to be taken from "the middle of the surface of each organ"; it is impossible to determine the boundary of an area not defined by any recognizable anatomical characters of the skull, and how the "middle" of such an area as, say, *Cautiousness* is to be determined, if its boundary indeed were ascertainable, remains an enigma.

* Liston's report will be found in the *Transactions of the Society of Antiquaries of Scotland*, Vol. II. pp. 450—455.

was a short man of about 5′ 6″, which may account for his choice of a pony as war-steed*.

Liston remarks that four or five teeth were wanting in the upper jaw and that the jaw-bone in front had received a considerable fracture which would no doubt have occurred in some of Bruce's early, bold and hazardous adventures. We have already seen that Sir Henry Jardine was of the same opinion (our p. 257). We can hardly suppose that a surgeon of Liston's eminence would confuse a post-mortem fracture of the upper jaw with a fracture obtained in early life with time for subsidence of the alveolar process. The cast seems to favour Liston's surmise of an ante-mortem injury. Yet among the individuals whom Chalmers reports in 1859 (*loc. cit.* Vol. II. p. 190) as still possessing fragments of Bruce was one "known to possess two of the front teeth or scissors and part of the alveolar process, along with some of the substance found in the small leaden box, which was at a little distance from the tomb, but not being in it, considered to be the bowels of some person of rank." It seems most probable that bone fragments would be pretty plentiful on the site of the excavations and that when Bruce's defect was noticed some such fragment might easily be passed off as that of his missing upper jaw. If the fragment were what it claimed to be we have only another instance of the manner in which the mob is allowed access to excavations of the highest historical importance, and by its ignorant and ruthless spoliation to destroy evidence of high archaeological value. The present owner, if there be one, might now perhaps be willing that his fragment should be compared with the cast, and if it be genuine the defective upper jaw could be completed.

Liston does not even mention that the cast was taken of the skull, but it may be that we owe W. Scoular's engagement to his initiative, and if so we must pardon Liston's lack of measurements for the sake of the excellent reproduction thus provided. The Scoular cast seems to me a work of the highest order of merit, and provides us with a veritable cranial portrait, from which we see the Bruce emerging before our very eyes. The cast must have been made quickly, for the pedestal bears the words Interred 1329—Reinterred 1819. Thus unless the skull was really retained as in the case of Sir Thomas Browne, Scoular had not very long for his task. The original cast appears to be that in the Anatomical Department of Edinburgh University. There is a much inferior cast from this cast in the Royal College of Surgeons, England. I owe my opportunity for studying and photographing the cast to the kindness of Professor Arthur Robinson, for which I am most grateful. The one drawback to the cast for craniometric purposes is the fact that it is placed upon a pedestal to which it is firmly united, so that no examination and measurements of the base of the skull are feasible. Thus while experience of many crania shows approximately where the opisthion lies in reference to the border line of the

* Liston's method of estimating the stature seems to me very unsatisfactory, while Gregory's statement that the femur may be taken as $\frac{1}{4}$ of the stature is quite erroneous. Both make Bruce 5′ 11″, a tall instead of a short man. Jardine probably follows Liston when he writes: "King Robert was a man of about 5 feet 11 inches to 6 feet in stature." The populace likes its national heroes to be big physically; it is noteworthy that many of them were in reality *small* men!

mastoid in the *norma lateralis*, still the occipital sagittal arc and chord and the total height of the skull must remain largely conjectural. Whatever objections may be taken to measurements on a cast—and they are not few or slight—Scoular's work is still so extraordinarily well done and he had a skull of such marked individuality to work upon, that there can be no doubt of the value of a study of this particular cast. I feel quite convinced that a great sculptor could from it restore the personality of Bruce to the Scottish nation and provide an ideal far more correct than the fancy portraits of the 17th and 18th centuries. I can imagine a time, when public opinion being sufficiently educated, it shall be looked upon not as a desecration, but as a solemn duty reverently to exhume and study the crania of the departed great with a view of adequately correcting portraiture, or of supplying it where it is deficient. No doubt some critics will assert that the present writer is singularly wanting in moral sense, if he maintains that a study of Shakespere's skull so far from being ghoulish would lead to a higher appreciation of the personality of our national hero than can ever be provided by the bust over his tomb or the conflicting evidence of the poor contemporary portraiture.

We may now turn to the examination of Bruce's skull itself. Table I provides the measurements, which it has been feasible to take either accurately or approximately on the cast*. For purposes of comparison I have added our Laboratory measurements on the heads of Jeremy Bentham and Sir Thomas Browne. In the fifth and sixth columns are given the means and variations of English crania of the 17th century; in the seventh column the deviations of Bruce's characteristics from this English type and in the last column a rough appreciation of Bruce's degree of individuality.

The contours of Bruce's skull are inserted here as Figs. 1–3; they can be compared usefully with the tissues of the 17th century English provided in *Biometrika*, Vol. VIII. pp. 143—147.

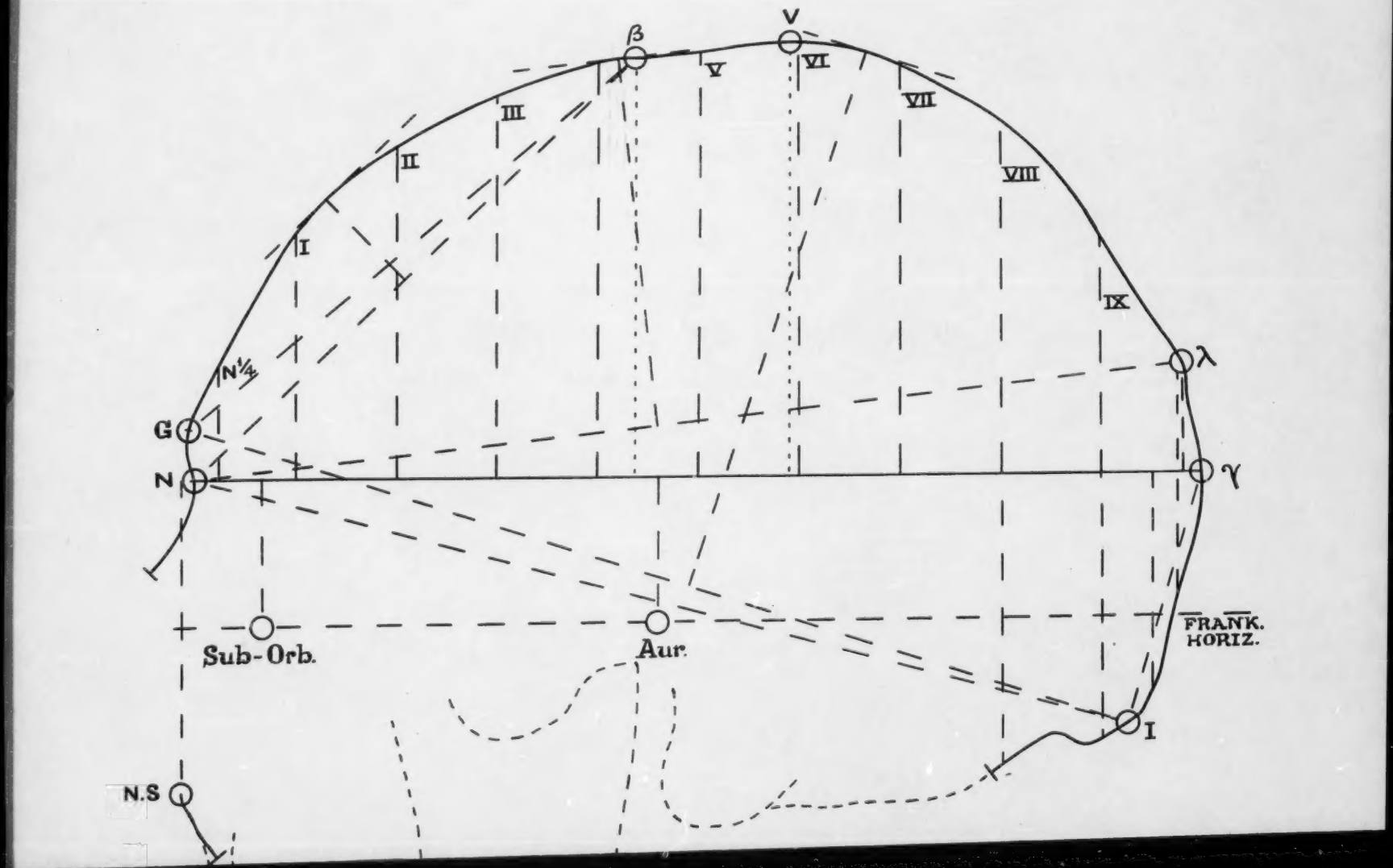
The following table will indicate the degree of flattening of the forehead in the case of Bruce.

Individuals	Frontal Angle ψ†	Frontal Index‡
Eskimos	51°·9	23·0
Guanches	48°·8	22·7
Thurtell, the murderer	45°·4	23·6
17th century English ...	47°·6	22·6
Robert the Bruce ...	43°·5	18·4
Sir Thomas Browne ...	39°·1	12·0
Henry Stewart (Darnley)	37°·3	19·1

* I have to thank my colleague, Mr G. M. Morant, for much aid in this matter. I have occasionally added a few of the more doubtful measurements to his results.

† Angle between nasio-bregmatic chord and the horizontal, $N\gamma$, nasio-gammatic line.

‡ Ratio of subtense of frontal arc to length of frontal or nasio-bregmatic chord; of course multiplied as usual by 100.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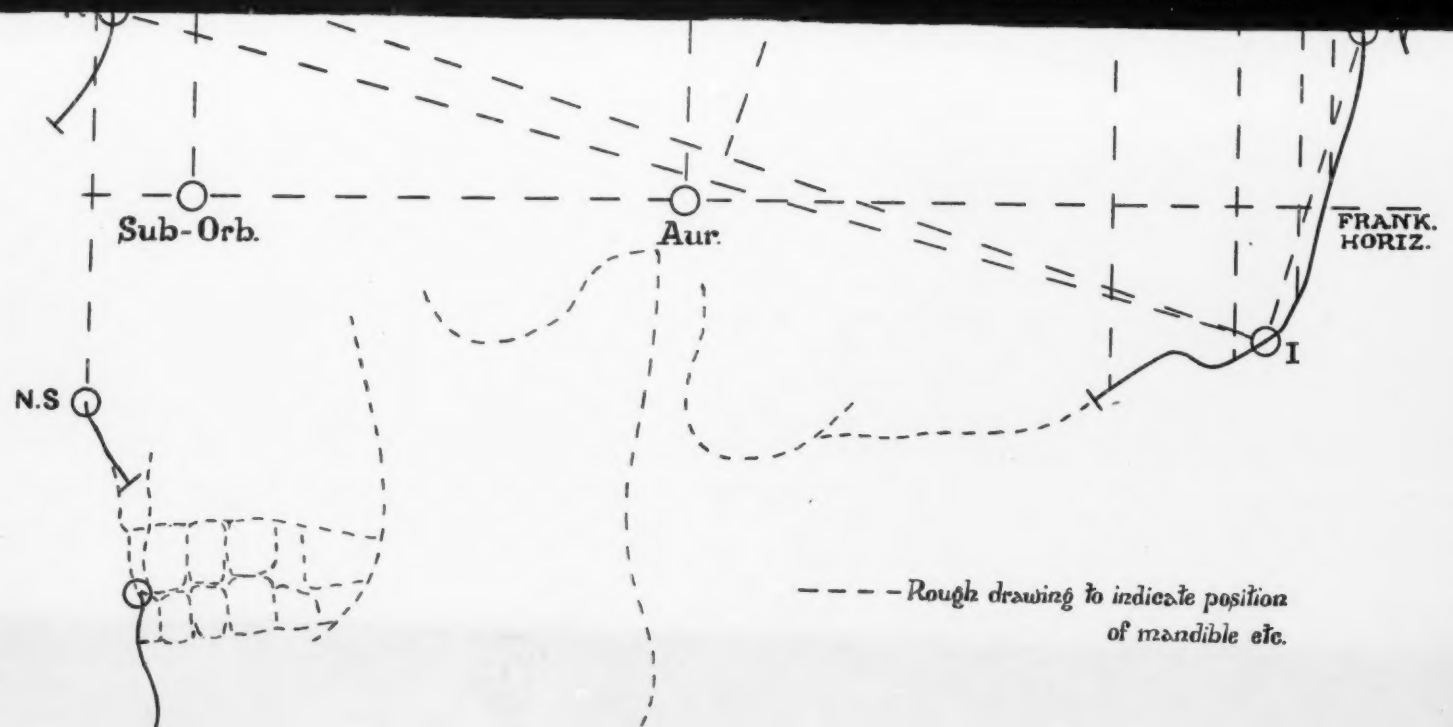
Sub-Orb.
Aur.
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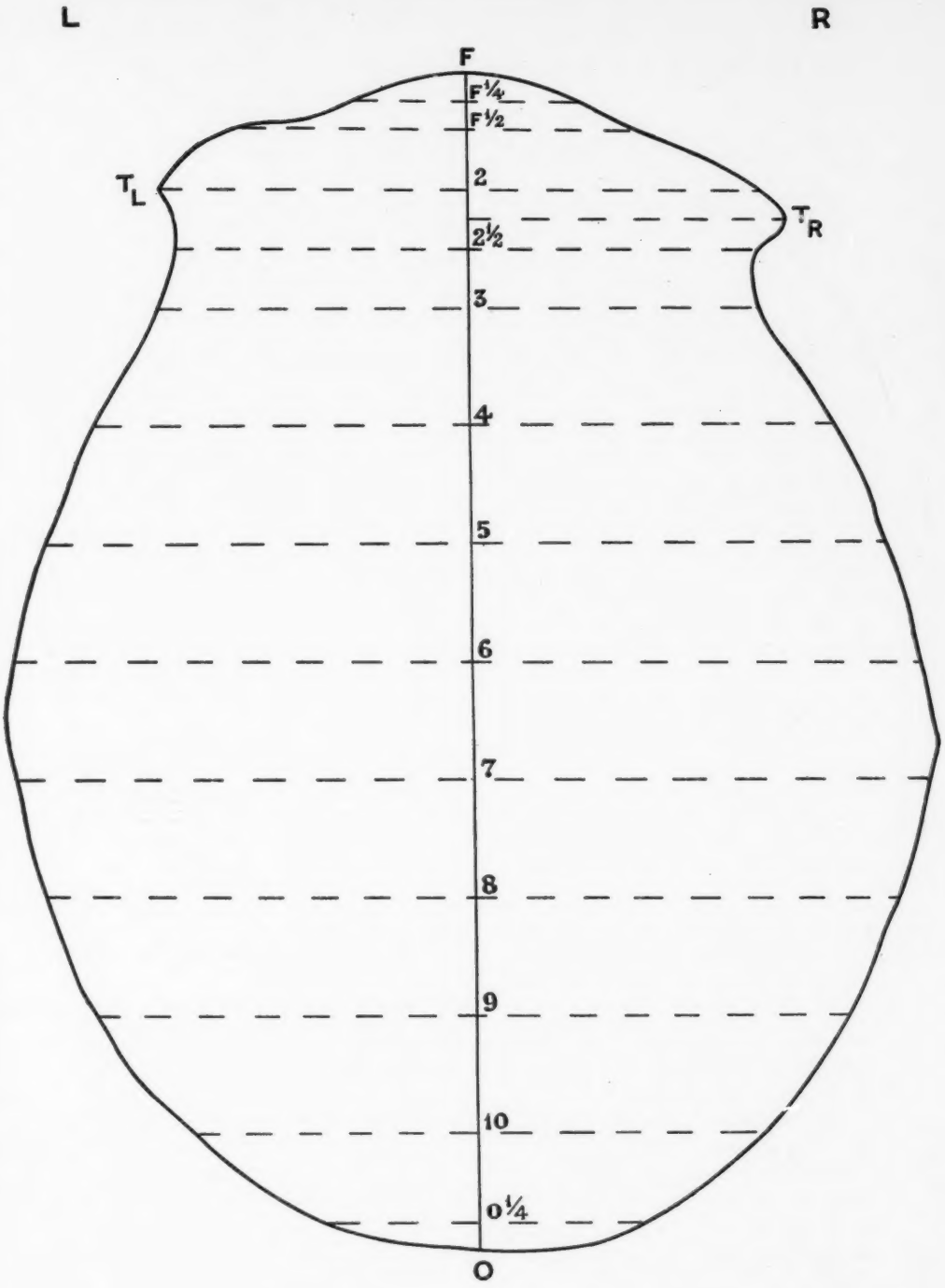
Rough drawing to indicate position
of mandible etc.

Rough drawing to indicate position of mandible etc.

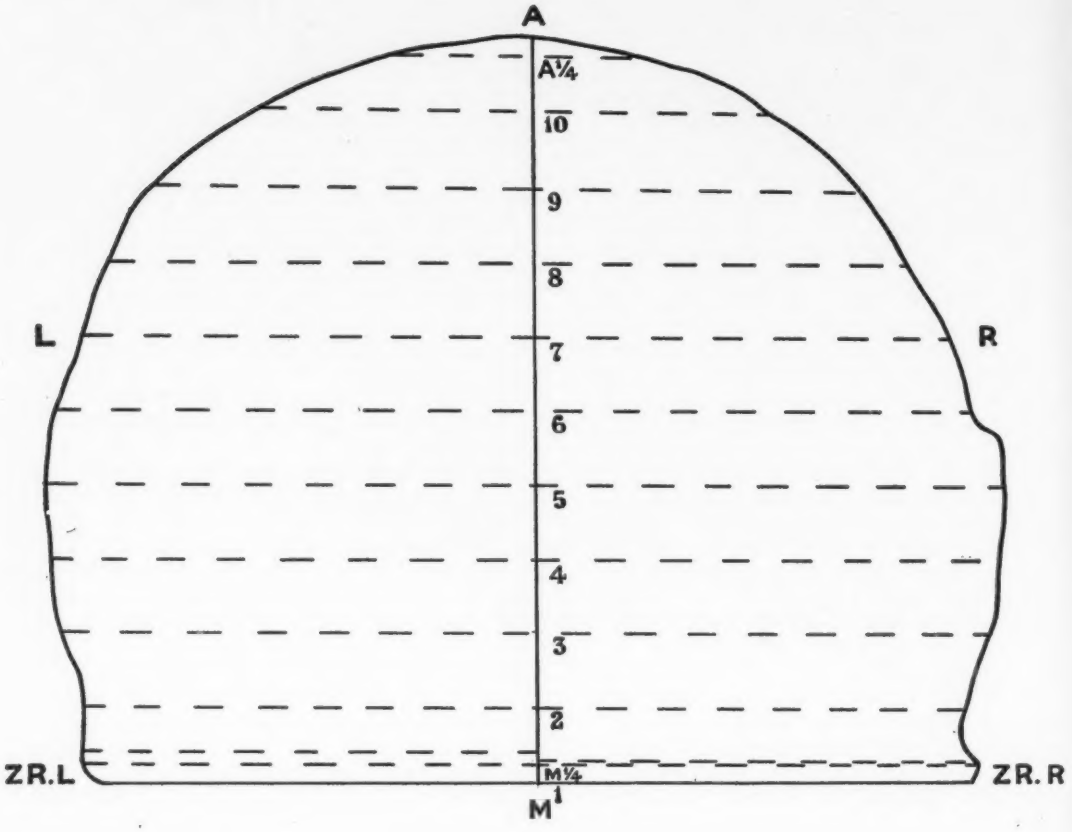
Sagittal Contour of the Cast of the Skull of Robert the Bruce

Fig. 1.



Horizontal Contour of the Cast of the Skull of Robert the Bruce

Fig. 2.



Transverse Vertical Contour of the Cast of the Skull of Robert the Bruce

Fig. 3.

It will be seen that the flatness of Bruce's frontal bone and its small angle with the horizontal (ψ) are far below the average English values, being only exceeded by those of Sir Thomas Browne and in the case of the frontal angle by Bruce's descendant, Lord Darnley.

The outstanding features of Bruce's skull are its great parietal and bizygomatic breadths, its large orbits, its want of harmony in the proportion of breadth to height exhibited not only in the 100 B/H index, but also in the facial index, and the relatively long nose. The mandible is remarkably massive, the chin one of great strength and protuberant, the ramus broad and edges vertical, there being as yet no senile shrinkage at the angles. The teeth are large and to some extent irregular,

Measurements of the Cast of the Skull of Robert Bruce, and comparison with those of Jeremy Bentham, Sir Thomas Browne and the Average English Skull.

Character	Jeremy Bentham	Sir Thomas Browne	Robert Bruce	17th century English		Ratio of Bruce—English Mean to Standard Deviation	Expected Frequency
				Means	Standard Deviations		
C	1475	1509	1595	1476·9	122·37	+ ·96	—
F	—	190	191	187·4	6·17	+ ·58	—
L	192	194·6	198	189·1	6·27	+1·42	—
B	153	143·7	154[a]	140·7	5·28	+2·52	1 in 167
B′	105	87·4	98·0	98·0	4·20	0	—
H	135	121·9	132·0	132·0	5·56	0	—
OH	118·5	102·6	112	112·1	4·28	− ·02	—
LB	105	102·7	—	101·6	4·13	—	—
Q	335	297·0	325	307·9	11·40	+1·50	—
S	380	380	389·3?[b]	377·1	13·69	+ ·89	—
s_1	—	129	134·5	129·3[d]	5·55[d]	+ ·85	—
s_2	—	123	125	128·7[d]	7·61[d]	− ·49	—
s_3	—	128	129·8?[b]	120·5[d]	8·48[d]	+1·10	—
U	560	549	552	524·3	15·02	+1·84	1 in 29
G′H	73 ?	73·1	72·2?	70·2	3·86	+ ·52	—
GH	144??	—	132	—	—	—	—
GB	—	90·9	110?	90·9	5·07	+1·79	—
J	133?	132·3	149?	130·1	5·57	+3·39 ?	1 in 2860?
NH, *R*	55?	52·2	56·0	51·2	2·60	+1·85	1 in 30
NH, *L*	„	52·2	56·4	—	—	—	—
NB	31??	22·6	25·9	24·3	2·16	+ ·78	—
O_1R	—	43·7	46·8[c]	43·0	2·02	+1·88	1 in 33
O_1L	—	42·8	46·2	43·1	1·81	+1·71	—
O_2R	—	36·3	38·3	33·4	2·22	+2·21	1 in 71
O_2L	—	36·7	39·0	33·5	1·88	+2·93	1 in 500
GL	89?	98·3	—	95·9	4·49	—	—
P∠	93°?	84°·7	91°	86°·1	3·92	+1·25	—
100 *B*/*L*	79·7	73·7	77·8	74·3	3·26	+1·07	—
100 *H*/*L*	70·3	62·7	66·7	70·0	3·22	−1·02	—
100 *B*/*H*	113·3	117·8	116·7	106·3	5·11	+2·04	1 in 50
100 *G′H*/*GB*	—	80·4	65·6?	76·5	5·39	−2·02	1 in 45
100 *NB*/*NH*, *R*	56?	43·3	46·3	47·6	4·58	− ·28	—
100 *NB*/*NH*, *L*	„	43·3	45·9	„	„	—	—
100 O_2/O_1, *R*	—	83·1	81·8	77·7	4·66	+ ·88	—
100 O_2/O_1, *L*	—	85·7	84·4	77·9	3·78	+1·72	—
Oc. I.	—	55·8	57·2?	58·52[d]	2·15[d]	− ·61	—
Mandible							
w_1	—	120·8	138	120·0[e]	—	—	—
w_2	—	100·2	114·6	99·4[e]	—	—	—
$g_op_og_o$	—	202·5	234?	201·0[e]	—	—	—
rb′, *L*	—	32·0	37·0	37·3[e]	—	—	—
rb′, *R*	—	32·0	40·9	—	—	—	—
m_2p_1, *R*	—	—	28·8	—	—	—	—
m_2p, *L*	—	—	29·6	29·2[e]	—	—	—

a. The greatest temporal breadth is 157 ; there may have been a slight post-mortem displacement of the right temporal squama. *b*. The opisthion is inaccessible ; the positions of the bregma and lambda are not very certain. *c*. The exact margins of the orbits are somewhat doubtful. *d*. Unfortunately the separate arcs were not taken on the Whitechapel crania, but are here supplied from the Liverpool Street crania. *e*. The values here given are for Tibetans *B*-type, the mandibles of which are those of a massive jawed race. The values of w_1 and w_2 for the 17th century English are about 115·5 and 98·6 mm. respectively. Both these results and those for the Tibetans *B*-type suffice to indicate the massive character of Bruce's jaw.

C=Cranial capacity ; *F*=Ophryon-occipital length ; *L*=maximum length ; *B*=maximum parietal breadth ; *B′*=minimum frontal breadth ; *H*=basio-bregmatic height ; *OH*=auricular height ; *LB*=skull-base ; *Q*=transverse arc ; *S*=total sagittal arc, s_1=frontal, s_2=parietal, s_3=occipital sub-arcs ; *U*=horizontal circumference ; *G′H*=upper face height ; *GH*=total face height ; *GB*=facial breadth ; *J*=bizygomatic breadth ; *NH*=nasal height ; *NB*=nasal breadth ; O_1=horizontal, O_2=vertical orbital diameter ; *GL*=profile length ; *P∠*=profile angle ; *Oc. I.*=occipital index ; the three cephalic, upper face and orbital indices are adequately described by their symbols ; w_1=maximum width at condyles, w_2 at angles ; $g_op_og_o$=mandibular horizontal arc, from gonion to gonion through the pogonion ; *rb′*=minimum breadth of ramus ; m_2p_1=chord from alveolar margin at mid 2nd molar to the same at 1st premolar. *R.* and *L.* refer to right and left sides. Fuller descriptions of methods of measurement will be found in *Biometrika*, Vol. I. p. 430, Vol. VIII. p. 311 and Vol. XV. p. 8. The capacity of Bruce's skull was determined by the Lee-Pearson formula, *Phil. Trans.* Vol. 196 A, p. 253.

the alveolar margins appearing to have receded during life. The upper incisors and a portion of the upper jaw have been broken away possibly by a blow from battle-axe or spear. The mastoids are extremely massive and I doubt if I have ever seen a skull so extraordinarily developed in the region of the inion. The *norma occipitalis* is a most striking one, bringing out the great breadth and general solidity of the skull. The frontal sinuses are markedly developed* and the forehead retreating. The very considerable length of the skull is dwarfed in the *norma verticalis* by the greater relative breadth, but it comes out well in the *normae laterales*, where the contour from bregma to vertex and thence to the lambda is most striking. What Bruce missed in the frontal he more than regained in the post-coronal regions. Studying heads like those of Bruce, Darwin† or Sir Thomas Browne, one begins gravely to doubt whether "high-browedness" has been justly correlated with intellectual capacity, or at any rate with strength of character and imaginative power. One may even question whether the "high-brow" should not lead us to suspect that with the infantile forehead some of the mental characters of infantilism may have been retained. Among men of action, men of great imagination, or men of marked intellectual power high-brows are not by any means the universal rule; it would not surprise me to find a higher percentage of them among minor poets or artists of the cubist school.

Bruce's skull suggests a man of most exceptional muscularity and strength, with a bull-neck and ardent passions‡. And yet none can assert that its massiveness is of a purely animal type, its development in the region of the obelion, its lateral and posterior aspects show developments which at any rate general impression associates with strong mental qualities, i.e. they are distinctly human as opposed to anthropoid characters.

Bruce had great personal prowess and large powers of organisation and imagination. His skull is not disappointing; it is a fit one for a strong leader of those

* The zygomatic ridges except in the immediate neighbourhood of the orbits are not as massive as might be anticipated. The cast lacks the left zygomatic ridge, whether broken off in the skull or more recently from the cast is not clear.

† John Morley once described Darwin as "high-browed," but such an adjective does not seem to me to accord well with his portraits.

‡ If we were to trust phrenology we must conclude that Bruce's *Amativeness* and *Philoprogenitiveness* were most markedly developed. According to Barbour Bruce always got warning of proposed treachery,

> And mony tyme, as I herd say
> Throw wemen that he wyth wald play,
> That wald tell all that thai mycht hér;
>
> See v. 531 *et seq.*

Thus Bruce like Caesar made his Amativeness serve political ends! Our worthy Archdeacon was a child of his times, or he might have found as much difficulty in reconciling this conduct of Bruce with his hero-worship as he did in the case of Bruce's slaughter of "Schyr Ihone the Cumyn" and "Schyr Edmund" "in the freris at the hye Awter"—not of course because he assassinated them, but because he "gave na gyrth to the Awter," i.e. disregarded the Friars' privilege of giving sanctuary. It must also be noted that his son King David Bruce was equally amative, but although he lived to be forty left no offspring—possibly owing to the sins of his father. This Bruce characteristic reinforced by Tudor blood may be noted throughout the Scottish Stewart kings. It culminated, perhaps, in Darnley, who as Ruthven expressed it, at a critical moment turned "effeminate," thereby meaning amative.

days. If we admit the justice of that rather vague phrase—it is disharmonic—, but is that not what we might anticipate from a union of Nordic and Celtic blood? Are not those two elements in combination the source of Bruce's success? He was able by Celtic imagination with a certain dash of slimness to win the Scottish nation to his side, and by aid of Nordic physique and persistency to be triumphant over his opponents. It was a grave loss to Scotland that the Nordic element was weakened in his descendants by too great an infusion of Stewart blood*.

Bruce died when he was 55 and the usual account given is that he suffered much pain in his later years, which incapacitated him for action in the field, and that this was due to the leprosy of which he died. I have already referred to the possibility of syphilis rather than leprosy being the source of his troubles. Due weight is not always given to the manner in which disease changes its nature in the course of centuries, and the nature of the syphilitic epidemic at the end of the 15th century was at least very different from the syphilis of today†. There is little doubt that syphilis was sporadic in Europe long before the great outbreak of that century, and that probably its symptoms were more those of the 15th century than those of modern syphilis. The cast of Bruce's skull seems to indicate that the original was to some extent "pitted" and these pits have been brought to light in the cast by the settlement of dust in them. It would be difficult in the absence of the original skull to assert definitely that the "pitting" exists in it; they might correspond to flaws in the casting, or to injuries to the cast, as some markings certainly do. We have, however, also to remember that Bruce's skeleton was found wrapped up in two coats of lead in a sealed vault, so that there was relatively little chance of post-mortem action on the skull. It is even possible that the peculiarities of the teeth and of the defective upper jaw might have to be viewed from a standpoint other than that of war-time injury. We must also note

* Since writing this monograph my attention has been drawn to Sir Arthur Keith's singularly graceful *Henderson Trust Lecture* III, published in the summer of this year. I fear, however, that I must protest very strongly against his method of *orientating* crania, which produces in all cases a very unnatural receding position of the chin and renders any appreciation of a receding forehead impossible. Further considering what we actually know of Bruce's ancestry for eight generations the following words are open to question: "one has but to look at the skull to recognise in him the kind of men who began to settle about the eastern shores of England and Scotland towards the close of the neolithic age—about 2000 B.C.—bringing with them continental forms of food-vessels or beakers, and hence often spoken of as the 'beaker' men." As far as I can trace them the bulk of Bruce's ancestors certainly did not settle on the eastern shores of England and Scotland in 2000 B.C. If on the other hand all Norsemen (including Normans), all Celts (Welsh and Scottish), all Anglo-Saxons are 'beaker' men, then some explanation is required of the great diversity of skull-shape amongst these groups.

My objections are not to Sir Arthur choosing any line he pleases for measuring characteristics of the brain, but that in *orientating* his very artificial line to the horizontal he gives, I hold, a very erroneous idea of the physiognomy of his subject, and so absolutely disregards the needs of comparative portraiture. As minor criticisms, I may remark, that a comparison of his Figs. 1 and 10 seems to show that he would have reached somewhat different results by orientating from Sir Thomas Browne's right or left profile, and that a "rectangularisation" for his purposes on the nasio-inional line of Schwalbe would have surmounted this alternative and, as far as I can judge, have been an equally good "brain floor." I pass over the difficulty—to the biometrician at least—of what is meant by "bisecting" either the fronto-malar or masto-parietal sutures (note his Fig. 2!).

† This matter will be more fully discussed in a paper shortly to appear in *Biometrika*.

Liston's statement: "There is a kind of mark on the right side of the sagittal suture, most probably the consequence of a severe injury and of subsequent exfoliation." But without the original skull no real stress can be laid on these points; it did not seem, however, proper to pass them entirely by. Considering the habits and ideas of those days there does not seem any greater slur on Bruce for having incurred syphilis than leprosy. Bruce would not have been the first king to be a leper, but then he would not be the only king who has been syphilitic. If we have to consider the probabilities—or better improbabilities—of Bruce's incurring either disease, we are met at once by the difficulty that while we know that leprosy was fairly common, we do not know to what extent syphilis was confused with it and screened by it. Both are about equally probable if we suppose them as originating in the hardships Bruce had to undergo. On this point we cannot pretend that the cast (nor indeed possibly the skull itself) can provide any definite information, until the whole subject of "pitting" in mediaeval crania has been more thoroughly investigated. In two crania of historical importance, where in one case leprosy and in the other more than a rumour of syphilis has been asserted, this pitting occurs. In neither case was it at all likely that the crania had been subjected to post-mortem "root action."

Barbour does not give us much enlightenment; leprosy could hardly be produced by lying in the damp*:

> This malice of endfundeyng
> Begouth, for throw hys cald lying;
> Quhen in hys gret myscheiff wes he,
> Hym fell that hard perplexité. (xx. 75–78.)

When we turn to the portraiture of Bruce the chance of discovering anything of value is very slight; it is not to be expected that paintings would be made in Scotland before 1329. The only possibility is that there may have been a real portrait in the monument prepared for his tomb by the French masons during his life, and that this semblance of Bruce had become traditional, or again that at Dunfermline or Stirling some traces of portraiture remained for 300 years. I can bring forward no definite evidence of this.

Assuming that an effigy of Bruce once existed on his monument we may ask what chances there were of an iconographic tradition. Where could Jamesone in the early or de Witt in the late seventeenth century have picked up their traditions? According to John Pinkerton speaking of the Taymouth Bruce: "It is said that the prototypes of this and some other Royal Portraits by Jamesone were old limnings in the palace at Dunfermline†." I have not succeeded in finding

* On the word *endfundeyng* consult Skeat's not very conclusive note *The Bruce*, p. 609. Raynaud's Disease is worth considering as resulting from cold and just possibly might then be confused with leprosy. It would account for Bruce's severe pains.

† *The Scottish Gallery or Portraits of Eminent Persons of Scotland; many of them after Pictures of the celebrated Jamesone at Taymouth and other Places*, London, 1799, *sub capite*, Robert Bruce. The writer on Pinkerton in the *Dictionary of National Biography* states that Pinkerton's works are of value because of the portraits of Scottish celebrities there engraved for the first time from pictures in private collections. This applies to the engraving of Bruce, but there are to my knowledge other cases, where Pinkerton has reproduced as a portrait what is no portrait at all.

any other reference to these "limnings in the palace at Dunfermline." There may be evidence for them, or they may be a product of Pinkerton's imagination, for he possessed a veritable Scottish capacity for the production of literary and artistic antiques. A second source of possible inspiration were the thirty-eight Stirling heads, or circular oak panels originally on the roof of the Presence Chamber in Stirling Palace, erected by James V about 1529 and taken down in 1777. Blackwood published in Edinburgh, 1817, a book with no author's name entitled:

Lacunar Strevelinense, a Collection of Heads etched and engraved after the carved Work which formerly decorated the King's Room in Stirling Castle.

The book has a short introduction and the writer judging from their appearance assigns names to about half-a-dozen heads, but to none does he give that of Bruce.

The oak carvings to judge by the reproductions were rather noteworthy work, but if there was a Bruce among them it did not serve as a type for Jamesone. That Jamesone painted a picture of Bruce for Sir Colin Campbell at Taymouth Castle is demonstrated by an extract from the "Black Book of Taymouth*":

> Item, the said Sir Coline Campbell gave unto George Jamesune, painter in Edinburgh, for King Robert and King David Bruysses, Kings of Scotland, and Charles 1st King of Great Brittane, France and Ireland, and his majesties Quein and for nine more of the Queins of Scotland their portraits, quhilks are set up in the halls of Balloch the sum of twa hundreth thrie scor punds.

The date of this payment seems to have been about 1635. When Mr John Bulloch saw the Jamesone pictures at Taymouth Castle before 1885, the picture of King David Bruce was still extant, but that of King Robert Bruce was no longer among the 19 Jamesone's pictures preserved there†. That it had been there in 1795 is certain for Pinkerton then engraved it for his *Scottish Gallery*. Pinkerton tells us that Johnston in 1796 made "an exquisite Drawing in colours" of Jamesone's Bruce. This drawing and the original picture ought to be searched for diligently; both can hardly have perished. As to Jamesone's picture itself assuming it adequately reproduced by the engraving, we should have anticipated that Robert would be clothed in mail or ring armour instead of plate, which only begins to appear in Italy about 1328. Pinkerton gets over this by remarking that Robert Bruce highly encouraged the commerce between Scotland and Flanders, whence he might be supplied with a painter and with his plate armour which it is supposed was invented in Germany and hence called German plate and German rivets. But it may be questioned whether this pleading is really necessary. Jamesone need not have troubled himself much about the armour, and the question

* See that excellent work: *George Jamesone, The Scottish Vandyck*, by John Bulloch, Edinburgh, 1885, p. 130. Balloch is an ancient name for Taymouth. Bulloch believes that all Jamesone's pictures of early Scottish Kings, notwithstanding his antiquarian tastes, were fanciful. Walpole's *Anecdotes of Painting in England*, Vol. II. pp. 243—251 gives some account of Jamesone and the extract from the Vellum MS. cited above on p. 247.

† A letter dated Ardmaddy, July 21, 1924, from Mr F. Morrison replying to my inquiries as to the picture says that the Marchioness of Breadalbane has informed him that no such picture had to her knowledge existed at Taymouth. Nor does this picture appear to have been removed with certain of the Taymouth Jamesones to Langton House with the part of the Breadalbane Collection inherited by Lady Elizabeth Pringle.

is whether he had any iconographic tradition—more probably carving than painting —for the features of Bruce. The armour does not settle the question of whether Jamesone at Dunfermline, Stirling or elsewhere had seen any prototype of his Bruce.

There is still another series of Scottish Kings at Holyrood Palace. These were painted by Jacob de Witte a half-century later than the Jamesone Bruce. The contract for the pictures has survived to this day, and is between Hugh Wallace, his Majesty's Cashkeeper on the one part and "James de Witte, Painter, Indwellar in the Cannogate" on the other. The said James "binds and obleidges him to compleatly draw, finish and perfyte the Pictures of the haill Kings who have Reigned over this Kingdome of Scotland, from King Fergus the first King to King Charles the Second, our Gracious Soveraigne who now Reignes Inclusive, being all in number One hundred and ten, ... *and to make them like unto the Originalls which are to be given to him**." This wholesale portraiture of kings, many of whom are mythical†, was to be done in two years at a cost of £120 per annum. Nor are the still remaining century of pictures very edifying. No. 161 of the catalogue is Robert I, the Bruce, and whatever the original given to de Witte may have been it certainly has not any relationship to Jamesone's Bruce. Some of de Witte's pictures seem to have been based on the work of Joannes Jonstonus: *Inscriptiones historicae Regum Scotorum* published in 1602 at Amsterdam, but this work does not contain a portrait of Robert Bruce, only an "inscription‡." The de Witte picture representing Bruce is a full length portrait ("in large Royall posture"—according to the contract!) with legs slightly apart in half armour and shirt of mail, the left arm carries a targe, and the right arm passing across the body holds in the hand a baton pointing downwards; a crimson cloak is draped from the right shoulder. On the head is a steel cap bordered by a coronet; a good deal of brown beard and hair more or less obscures the shape of head and jaws. The face is three-quarter looking to left. There is a background of trees and rocks. G. Cameron's engraving of Bruce *circa* 1820, see our Plate XVI (*a*), seems to have been based on this picture, or on the same prototype, if there was one, as de Witte's.

If we turn from the 17th century paintings to the 18th century engravings we have only two to be noted. The first was cut by L. P. Boitard who was probably a son of the better known father. The print represents the bust of a man of between 60 and 70 in profile, crowned, with sword in right hand, grey and bearded, with a high forehead. The only resemblance to the skull of Bruce is to be found in a somewhat lengthy nose. The whole is in the traditional "Frederick Barbarossa" style. As far as Bruce is concerned it might be thought wholly fictitious: see Plate IX. Beneath is a vignette of Bruce murdering Comyn. There is a copy in the Print Room of the British Museum, a proof before finishing. It is in the "Mary Queen of Scots Album" No. 229, p. 167. I am not aware that this engraving

* *Official Guide to...Holyroodhouse*...By Sir Herbert Maxwell, 1923.

† King Fergus is supposed to have ruled B.C. 330.

‡ Brussius Regni instaurator ac pene novus conditor in omni Fortuna invictus (p. 52). Jonston starts pictorially with David Bruce, but I am not certain where he could find a prototype even for him!

was issued in any book, but I have succeeded in tracing its original. In John Leslie, Bishop of Ross's *De Origine, Moribus et rebus gestis Scotorum Libri Decem*, Romae, 1578 occurs a pedigree, on p. 240, of Scottish Kings. The "root" of this pedigree is a medallion of some 3″ diameter containing a portrait of Robert Bruce. The bust has a crowned head, carries a sword in the right hand, there is a long beard which slopes inwards from the lower lip and excludes the possibility of an emphasised chin. The man is sixty if not sixty-five years of age. This is the source of Boitard's inspiration: see our Plate XV. We carry it back at once 200 years. That is to say, *possibly* within 18 years of the destruction of the Dunfermline monument, and nearly 60 years anterior to the Jamesone picture, but alas! it accords far less with the skull, and impresses one—like the de Witte picture—as a product of the non-historical imagination. It does not seem to me to have any link with the latter, however; indeed far removed as they are the de Witte is closer to the Jamesone. Whence Leslie obtained his Bruce it appears impossible to conjecture. At the same time some of his pedigree medallions have an element of real portraiture; thus on p. 462 there is a fairly reasonable medallion of Mary, Queen of Scots* and Leslie had considerable historical and antiquarian tastes. If he had some traditional conception of Bruce to work on, it was not that of Jamesone or of the skull. The other cut of about the same year was engraved by E. Harding and published by E. Herbert†. Below are the words "From the original at Taymouth." It represents a bearded man half length and three-quarter face, in armour, holding a battle-axe in his left hand‡ and with a steel cap on his head: see our Plate X. This engraving appeared as we have seen in Pinkerton's *Scottish Gallery* of 1799, and was made a year or two earlier. There is a good copy in the Print Room, British Museum, "Royal Portraits, Kings of Scotland" 1868, 8—8—1977§. The Harding engraving suggests a more receding forehead than the Boitard, and it might have a low frontal index. The beards in both obscure any appreciation of the chin. The Boitard portrait seems so fanciful that I have not thought it worth while placing a tracing of the skull upon it. I have, however, done so with the Jamesone portrait after obtaining a photograph of the cast adjusted to about the same aspect: see Plates II and XI. I took lip-line to nasion to give a common base line of measurement. The result is not as bad as might have been expected. The breadth of face is defective, the right ear is much out of position, the forehead above the left eye is impossible, as also the left cheek-bone—the jowls of Bruce must have been very prominent—and the nasal septum is somewhat out of position. The jaws are not unfitting, and considering

* The Mary medallion has a young James medallion attached to it, but Darnley is represented by a blank circle; whether this arose from Henry Stewart not being a *persona grata* to Leslie or because the latter had no portrait of him it is impossible now to decide.

† Some copies have E. Harding for E. Herbert, repeating Harding.

‡ This might possibly be interpreted as evidence that it had been copied *without reversing* from a picture or effigy in which the battle-axe would be in the natural hand. It would be interesting to know in which hand the Taymouth picture gave the axe.

§ A second, rather poorer copy, is in the "Mary Queen of Scots Album" on the same page as the Boitard.

how little attention, even in the present day, artists often give to the skull behind the features they portray, we might have had a worse result. Of course the suggestion of the picture is that the skull cap fits close to the steel cap. That it certainly does not, but there is not enough discordance for one to say straight off that the Jamesone portrait may not be based on a faint tradition of some sort.

It occurred to me that if such a tradition existed it might have arisen not necessarily from the monument, but from a coin of Bruce's. By the courtesy of the Keeper of the Coins and Medals at the British Museum I was able to obtain casts of three Bruce coins. They are shown enlarged on Plate VII. A lantern slide was then made of a coin; this was thrown on a sheet of paper and drawn. The drawing was then reduced so that the length of nasion to lip-line in the drawing accorded with that of the photograph of the right profile of the skull. The rough tracing of this placed on the drawing shows that the coin has a purely conventional figure, having no correspondence with Bruce's proportions: see Plate XII. All features are impossible. The forehead is infantile instead of receding, the mandible has no correspondence in the slope of its base, the mastoid is in front instead of behind the ear, and above instead of below it. If the tracing of the skull be tilted so as to bring the mandible into a more nearly horizontal position, the flesh over the nasion becomes absurdly thick. As for the eye it is remote from the orbit, but this is an artistic (?) convention even in the case of modern designers' profile portraits. On a penny postage stamp or a half-crown of his present Majesty the King the distance from nasion to outer border of orbit is about one-third of the distance from nasion to auricular passage, whereas in the profile of a skull it is nearer one-fifth: see our Plate XVI (*b*). In the Bruce coin it is more than two-thirds! The profile on the coin has not only no resemblance to Bruce, it is absurd as a human figure at all. The coins therefore, as indeed I was warned by the British Museum authorities, may be discarded for our present purpose.

Another source of possible information with regard to Brucian portraiture may be sought in the seals of King Robert. Pinkerton in his *Iconographica Scotica, or Portraits of Illustrious Persons of Scotland,* London, 1797, gives a seal of King Robert from a charter dated in the ninth year (1315) of his reign. The Bruce sits on a highly ornamental throne or settle, he is robed without armour; he holds a sceptre in the right, the orb with cross in the left hand. He is crowned, and has wavy hair without beard*. The face is no portrait, and is largely the product of Pinkerton's imaginative engraver. Much better reproductions of the actual seals are provided by Walter de Gray Birch in his *History of Scottish Seals,* Stirling, 1905. The royal seals therein bear little resemblance to those of Pinkerton! Birch says that Bruce had two seals. The first seal is given in his Nos. 20 and 21, which are somewhat obscure. The obverse suggests a short stout man with a very big head, enthroned; the reverse a man in armour with sword and targe on horseback, the face entirely hidden in a cylindrical helmet. This also suggests a stout short man, as the Bruce, I hold, must have been: see our p. 258. The second seal is said to

* There is no serial number to pages or plates in Pinkerton and so no exact reference can be given!

be of 1318. On the obverse (No. 22) the King is seated on a throne with serpent heads at the arms (like those on the seal of his son, David II), he is robed and crowned, with sceptre in right hand and the left raised to his breast. The head and chin are strong, but the real features are not ascertainable. In the reverse (No. 23) we have as in the first seal the king in armour on horseback with face hidden. Neither side of this second seal suggests the short stout man of the first seal. The curious reader who will examine these seals in Birch's work will soon satisfy himself that the seals contribute no more than the coins to the idea that a traditional portraiture might have been derived from them. It seemed idle to reproduce such seals here.

A last possibility occurred to me, a traditional portraiture might have been preserved in the Armorials. To test this it was needful to compare two good armorials together. Through the courtesy of Mr Gilbert Ogilvy I was able to have photographs taken of the Seton Armorial in his possession and Dr Dickson also permitted photographs of an Armorial (MSS. No. 31. 4. 2) in the National Library of Scotland (formerly the Advocates' Library) to be provided. In each case the Bruce was given twice, once with each wife. The fact that in the latter MS. James VI was represented as a child* indicated the date at which the last entry had been made. A glance at our Plates XIII and XIV will at once convince the reader that there was no traditional Bruce common to the heraldic artists. The Bruce of the Seton Armorial is certainly a short man with an ample nose and a man of reasonable age, but he could have given no hint to Jamesone. The wild and bearded old man of the Advocates' Library Armorial might give a hint, a very slight one, to Ross's artist, but his threatening presentation of the sword was more likely to cause the obvious fear in the face of the "Duke of Hullesteris Dochter" than impress the English at Bannockburn. It is noteworthy that in this Armorial a thistle is placed in the hand of both wives of Bruce; according to Birch (I. p. 42) the first appearance of the thistle as a Scottish emblem occurs on the seal of Bruce. Are we to seek its origin in Galloway or Danby Dale?

Thus the Armorials like the other representations fail us. The above discussion will have sufficed to convince the reader that we have no valid portrait of Bruce, that our only accurate knowledge of the physiognomy of the man is to be gained from his skull. Even the aged dream dreams and I should like to see a national monument to Bruce at Westminster, an effigy based on the skull as only a great sculptor could conceive it. But it should be the gift of Englishmen only to the united nations. It was the Norman element in Bruce, quite as much as the Celtic, which carried him and Scotland to victory at Bannockburn. His blood, with Tudor blood and Stewart blood, still runs in our common royal line. The union of our nations needs no artificial cement, but it would be a graceful act for Englishmen to present Scotsmen with what at present they lack, a real characterisation—which I hold is still feasible—of one of their great heroes. If we can claim justly to share

* Not in the least like James VI as a child in the portrait of him in the National Portrait Gallery, London. This is further evidence, if such were needful, that no attempt at portraiture was made by the armorial artists.

Washington with America, we have every right to share Bruce with the Scots. A great portrait of Bruce is still possible, and if fitly executed would go a long way to justify the value of craniological study for the portraiture of national worthies.

I have to thank numerous correspondents for aid in the details of this paper by replying to my many queries, and I must mention in particular, Mr James L. Caw, Director of the Scottish National Portrait Gallery, and Mr J. Corsan Morton, the Keeper, Mr Francis Caird Inglis, Miss Rosaline Masson, of Edinburgh, and Dr J. F. Tocher, of Aberdeen.

Note to Froissart's Statement, p. 255.

The passage in Froissart is clearly corrupt and the geography is impossible. In Buchon's edition (*Collection des Chroniques Nationales Françaises*, T. IX. p. 147, Paris, 1824) the passage runs:

> Si se départirent le roi et les seigneures de Haindebourc [Édimbourg] et chevauchèrent vers Donfremelin [Dunfermline], une ville assez bonne où il y a une belle et assez grosse Abbaye de noirs moines; et là sont ensevelis par usage les rois d'Ecosse. Le roi d'Angleterre se logea en l'abbaye, car ses gens prirent la ville, ni rien ne leur dura. A leur département elle fut toute arse, abbaye et ville, et puis cheminèrent outre vers Strumelin [Stirling] et passèrent au dessus de Strumelin la rivière de Tay qui cuert à Saint-Jeanston [Perth].

A somewhat clearer version is given by Denis Sauvage. The Breslau MS. reads Strumelin, or Stirling, instead of Donfremelin. The *Book of Pluscarden* (Ed. F. J. H. Skene, Edinburgh, 1880, p. 246) says Richard pillaged and burnt down many churches and monasteries and other sacred places such as Melrose, Dryburgh, Newbattle, also destroyed the whole town of Edinburgh, and the whole of Lothian, and pillaged and burnt the monastery of Holyrood. Fordun (*Scotichronicon* Libr. XIV. cap. 50) says that in 1385 Richard II burnt down with fiery flames God's temples and holy places, to wit the monasteries of Dryburgh, Melrose and Newbattle and the whole town of Edinburgh, with the Church of St Giles. Walter Bower, Fordun's continuator, notes that Richard would have burnt Holyrood Abbey, but for the intervention of his uncle the Duke of Lancaster. Guthrie and Tytler do not accept Froissart's statement that Richard crossed the Forth, and it seems improbable that the Scottish Chroniclers would have omitted Dunfermline Abbey from their list had it really been sacked.

LIST OF THE PLATES

Plate I. The skull of Robert Bruce, *Norma facialis*.

Plate II. The skull adjusted three-quarter face for comparison with the Jamesone picture.

Plate III. The skull: *Norma lateralis*, left profile indicating the massive ramus and missing zygomatic arch.

Plate IV. The skull: *Norma lateralis*, right profile.

Plate V. The skull: *Norma occipitalis*, indicating the remarkable breadth, the great development of inion and the occipital ridges, and the emphasised attachments for the neck-muscles. Note the signs of "pitting" on the skull.

Plate VI. The skull: *Norma verticalis*.

Plate VII (*a*). Facsimile of the "faked" coffin-plate found near the tomb of Robert Bruce from the cast.

(*b*) Enlarged obverses of three coins of Robert Bruce, from casts taken from the originals in the British Museum. By the courtesy of the Keeper.

Plate VIII. Photographs of the head of a royal personage supposed to be from the monument to Robert Bruce at Dunfermline. By kind permission of the *Society of Antiquaries of Scotland*, in whose Museum the fragment is preserved.

Plate IX. Engraving by L. P. Boitard of Robert Bruce, 1797. This is based on the pedigree medallion in John Leslie's *De Origine...Scotorum*, 1578. Beneath is a vignette of Bruce assassinating John Comyn. From a copy in "Queen Mary's Album," a collection of various engravings in the Print Room, British Museum.

Plate X. Robert, the Bruce, from an engraving of 1797 by E. Harding after the picture by George Jamesone formerly at Taymouth Castle. Issued in John Pinkerton's *The Scottish Gallery*, 1799. From a copy in the series "Royal Portraits" in the Print Room, British Museum.

Plate XI. Rough tracing of the three-quarter face photograph of the skull superposed on an enlarged drawing of the Harding engraving. The latter is clearly not an accurate portrait of the Bruce, who had a broader face and a less domed head than is indicated by the steel cap or the forehead, but there may be some faint tradition behind it.

Plate XII. Rough tracing of the left profile of the skull adjusted nasion to lip-line placed upon enlarged drawing of the coin-obverse. The profile on the coin is not only not a portrait of the Bruce, but an impossible human being; it is a numismatic conventional symbol for a king.

Plates XI and XII are much reduced from the original drawings.

Plate XIII. Representations of Bruce in the Seton Armorial. Photographed by the courtesy of the present owner, Gilbert Ogilvy, Esq., of Winton Castle.

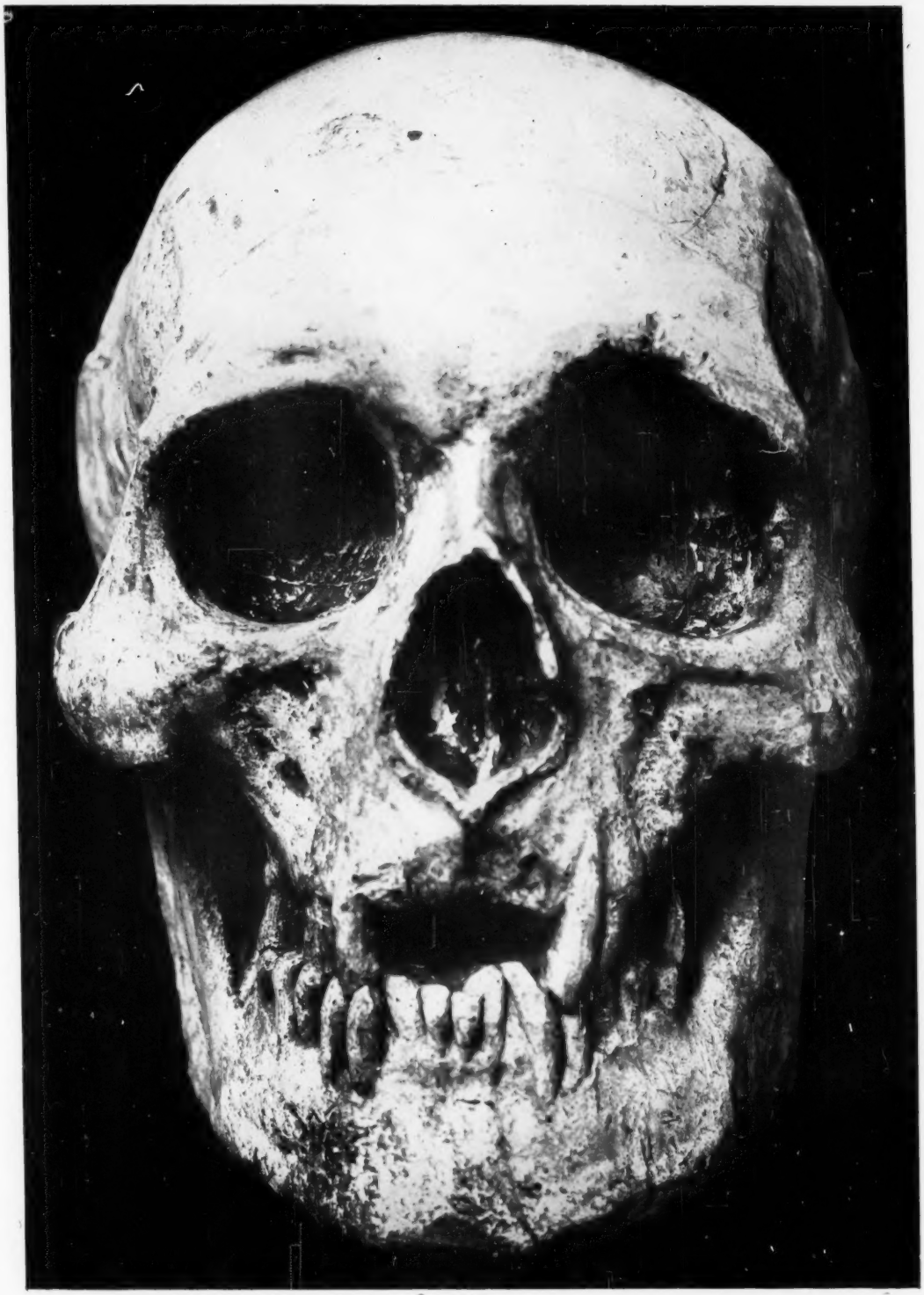
Plate XIV. Representations of Bruce from an Armorial (MSS. No. 31. 4. 2) in the National Library, Edinburgh. By kind permission of Dr Dickson.

Plate XV. Medallion of Robert Bruce from a pedigree of the Stewart Kings in John Leslie, Bishop of Ross's *De Origine...Scotorum*, Romae, 1578.

Plate XVI (*a*). G. Cameron's Engraving of Robert Bruce from the de Witte picture in Holyroodhouse.

(*b*) Profiles of His Majesty the King from life and from a coin to show that the positions of orbit and ear upon coins are nearly as conventional at the present day as in the time of Bruce.

I owe the photographs in Plates I to VII to Mr Egon S. Pearson; the drawings in Plates XI and XII are due to Miss Ida McLearn, while Mr Francis Caird Inglis, of Calton Hill, Edinburgh, made for me the excellent photographs of the difficult subjects in Plates VIII, XIII and XIV.



The Skull of Robert Bruce. *Norma facialis* (about natural size).

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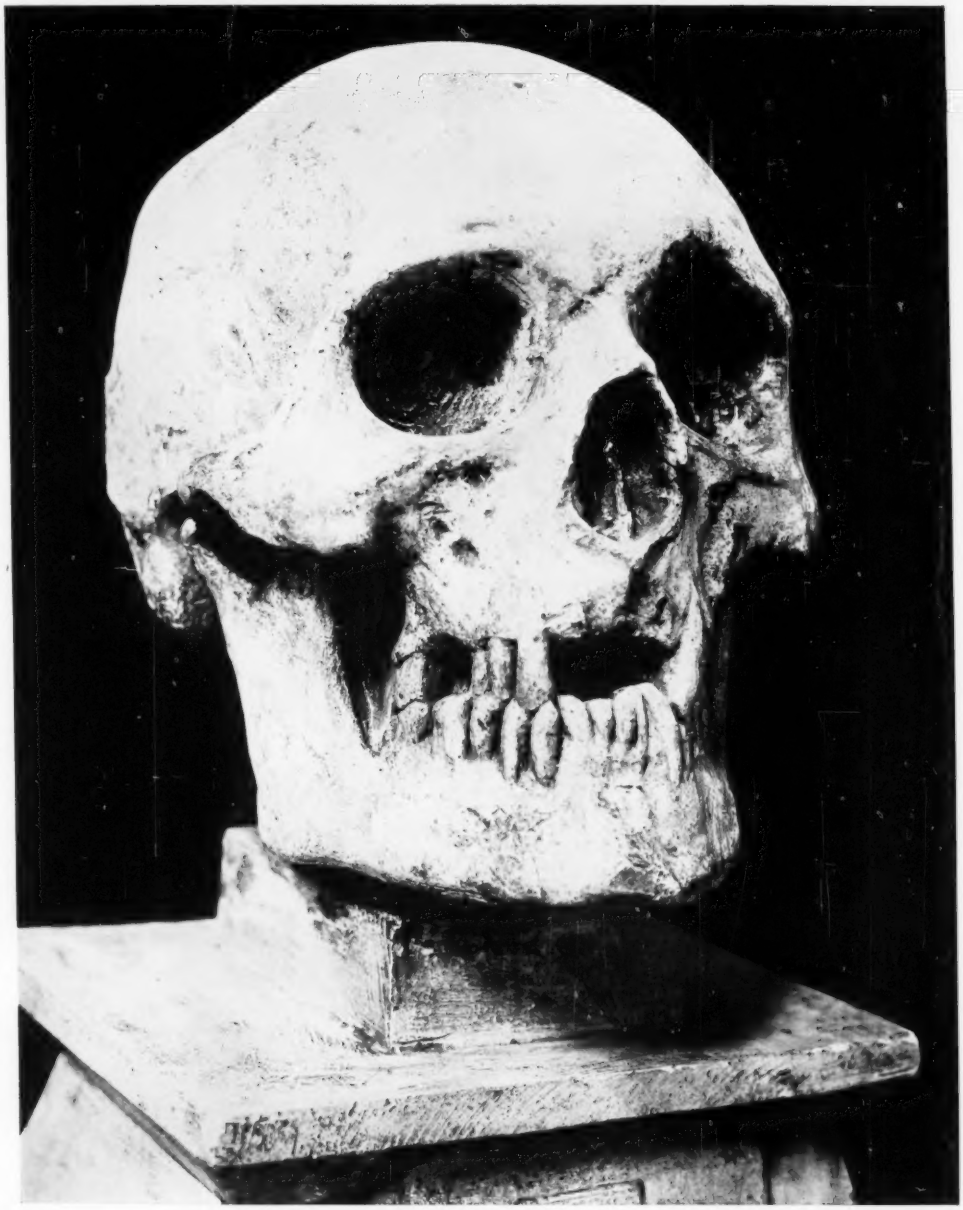

The Skull of Robert Bruce, three-quarter face. Cf. Plates X and XI (circa $\frac{2}{3}$ linear).

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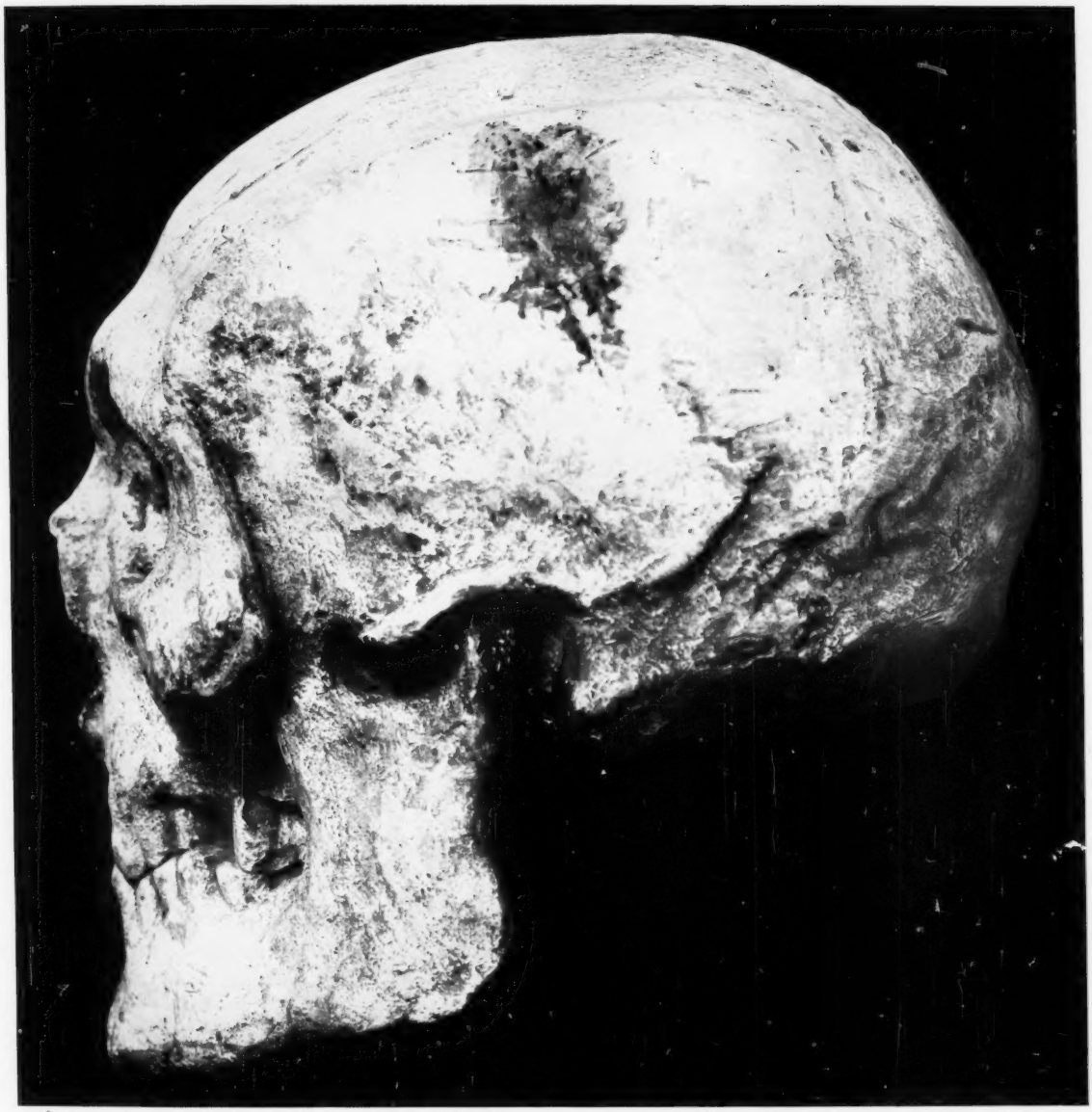

The Skull of Robert Bruce. *Norma lateralis* (Left Profile) (circa $\frac{1}{3}$ linear).

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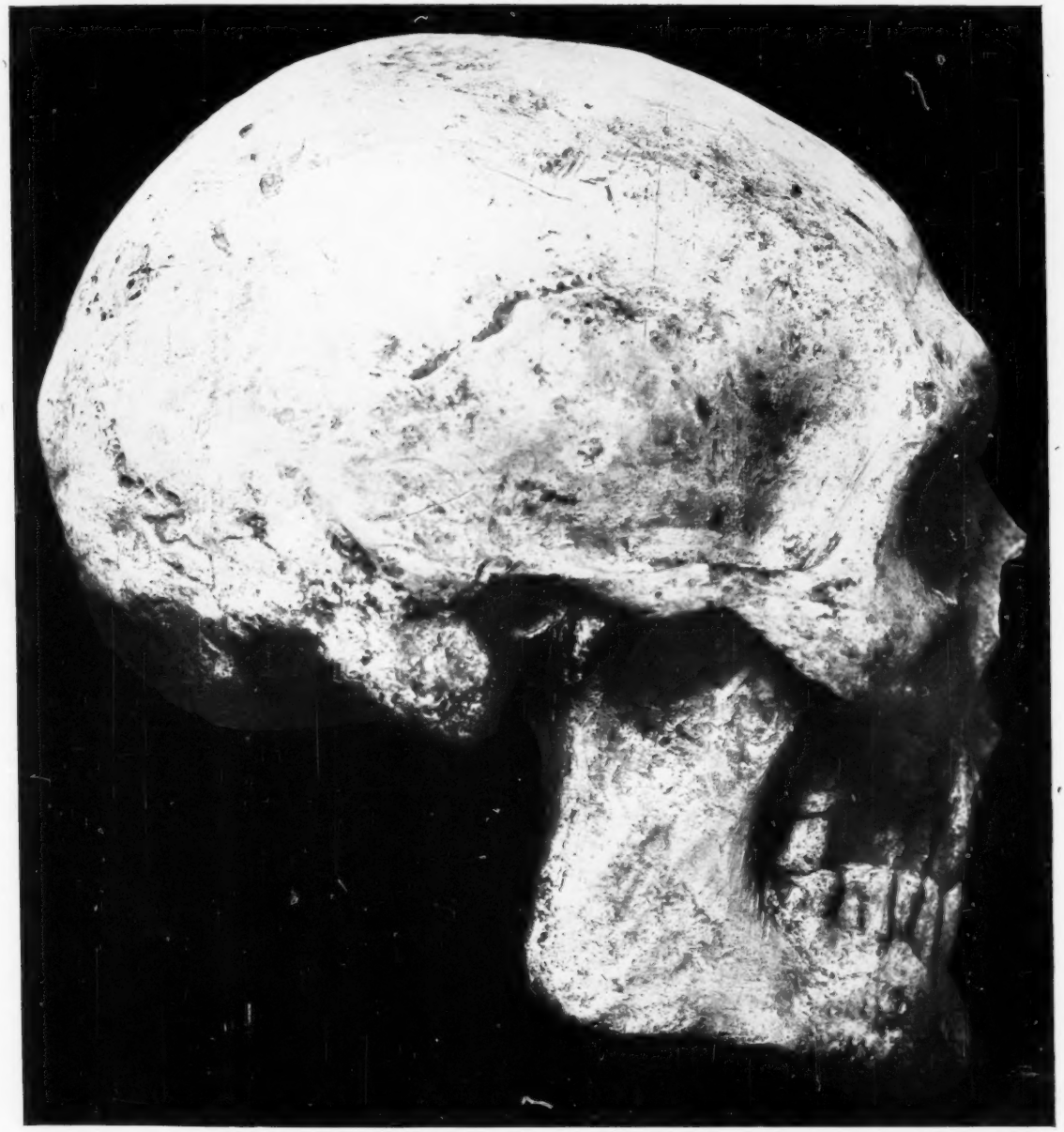

The Skull of Robert Bruce. *Norma lateralis* (Right Profile) (circa $\frac{1}{3}$ linear).

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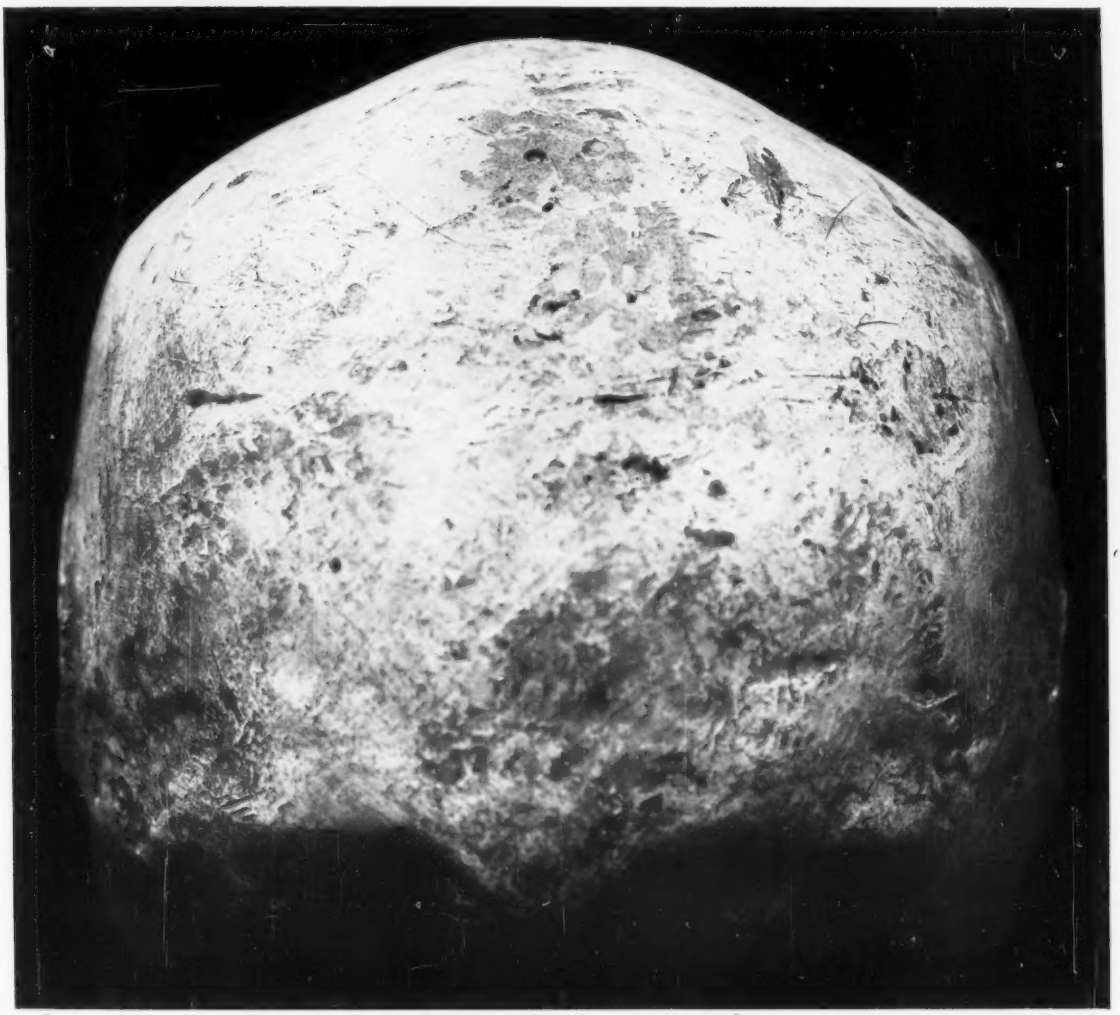
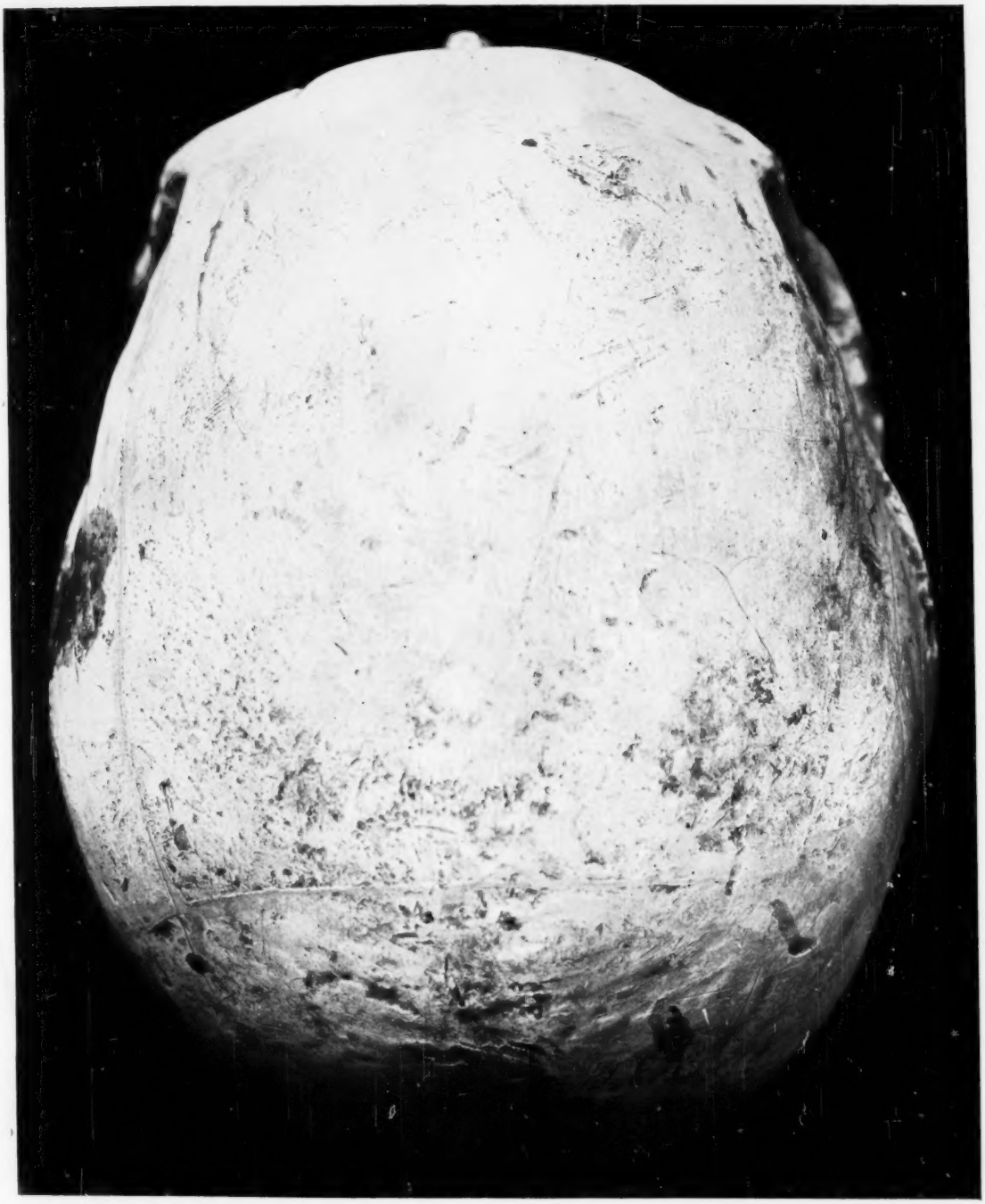


The Skull of Robert Bruce. *Norma occipitalis* (natural size).



The Skull of Robert Bruce. *Norma verticalis* (natural size).

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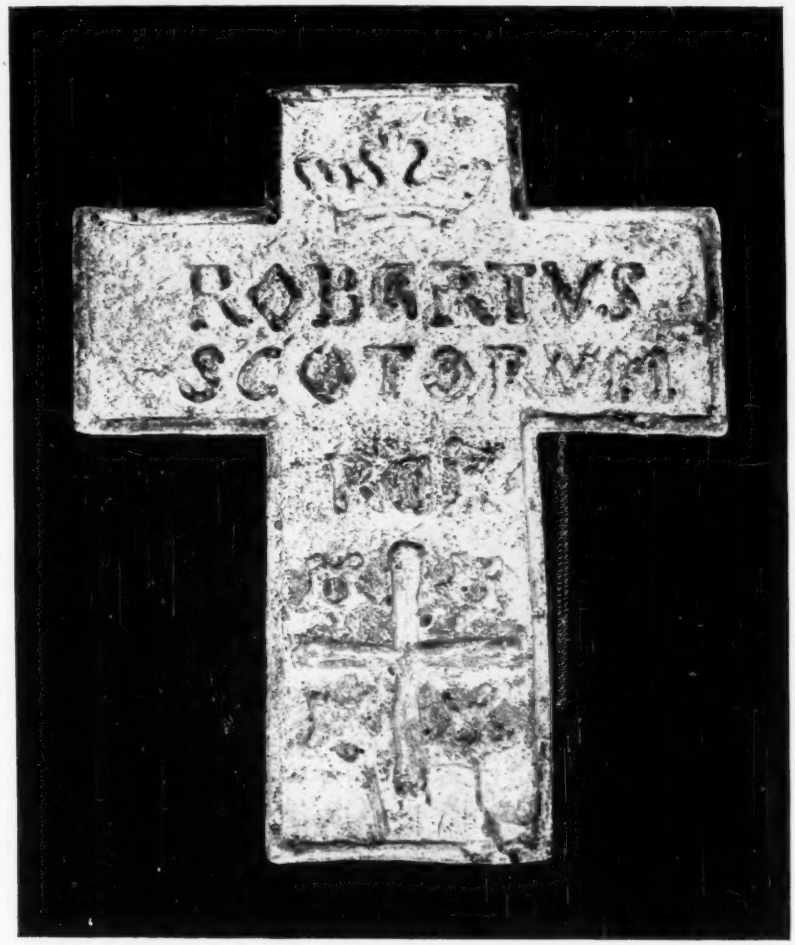

(*a*) "Faked" Coffin-plate of Robert Bruce now in the Museum of the Society of Antiquaries of Scotland.

(*b*) Photographs of Casts of Coins of Robert Bruce in the Department of Coins, British Museum (enlarged 2 : 1).

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Photographs of the carving of a Royal Personage, supposed to be from the Bruce Monument at Dunfermline.
Museum of Society of Antiquaries of Scotland.

Engraving by L. P. Boitard, 1797; it follows the medallion in John Leslie's *De Origine ..Scotorum*, 1578.

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From the Original at Taymouth

Engraving after George Jamesone's picture formerly at Taymouth Castle, executed by E. Harding.

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Three-quarter face outline of skull fitted to Harding's engraving of the Jamesone picture, indicating that the picture is not a true portrait, but might have some faint traditional basis.

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Outline of skull profile adjusted to coin, indicating that the head on coin is not in any way a portrait.

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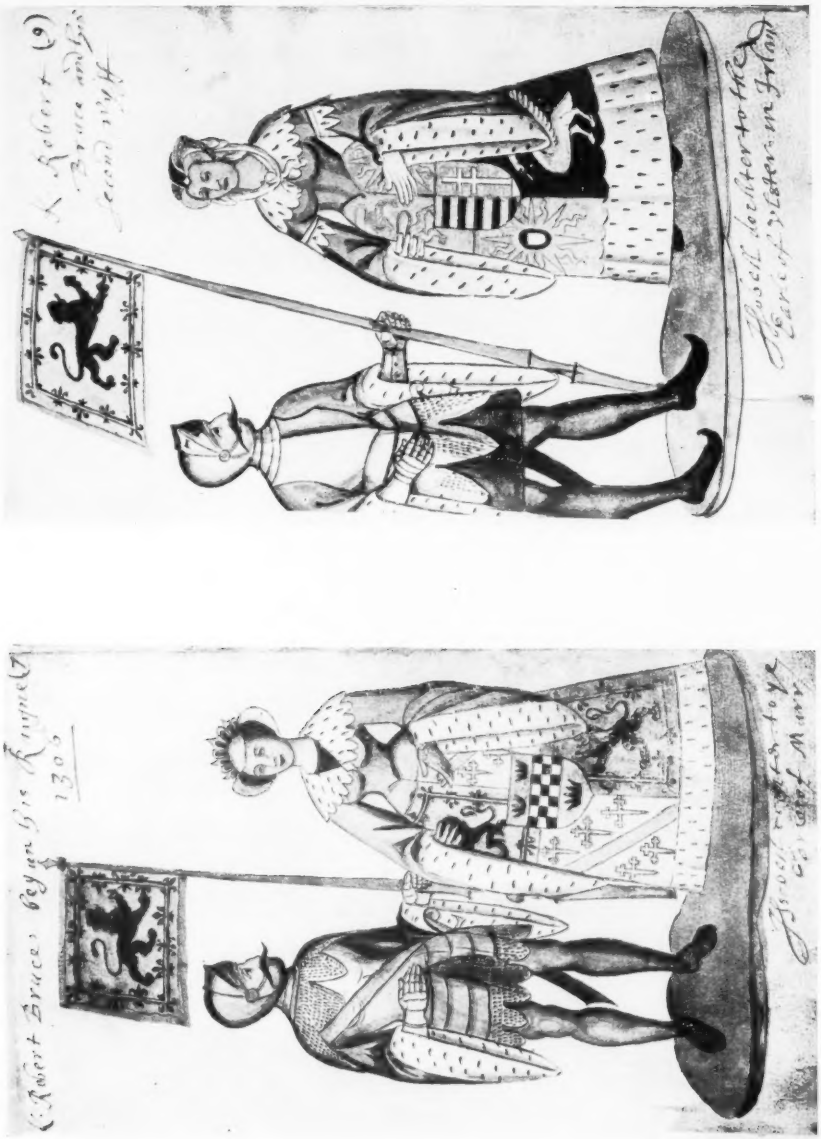

Representations of Bruce in the Seton Armorial. By kind permission of Gilbert Ogilvy, Esq.

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Representations of Bruce from an Armorial (MSS. No. 31. 4. 2) in the National Library, Edinburgh.
By kind permission of the Chief Librarian, Dr Dickson.

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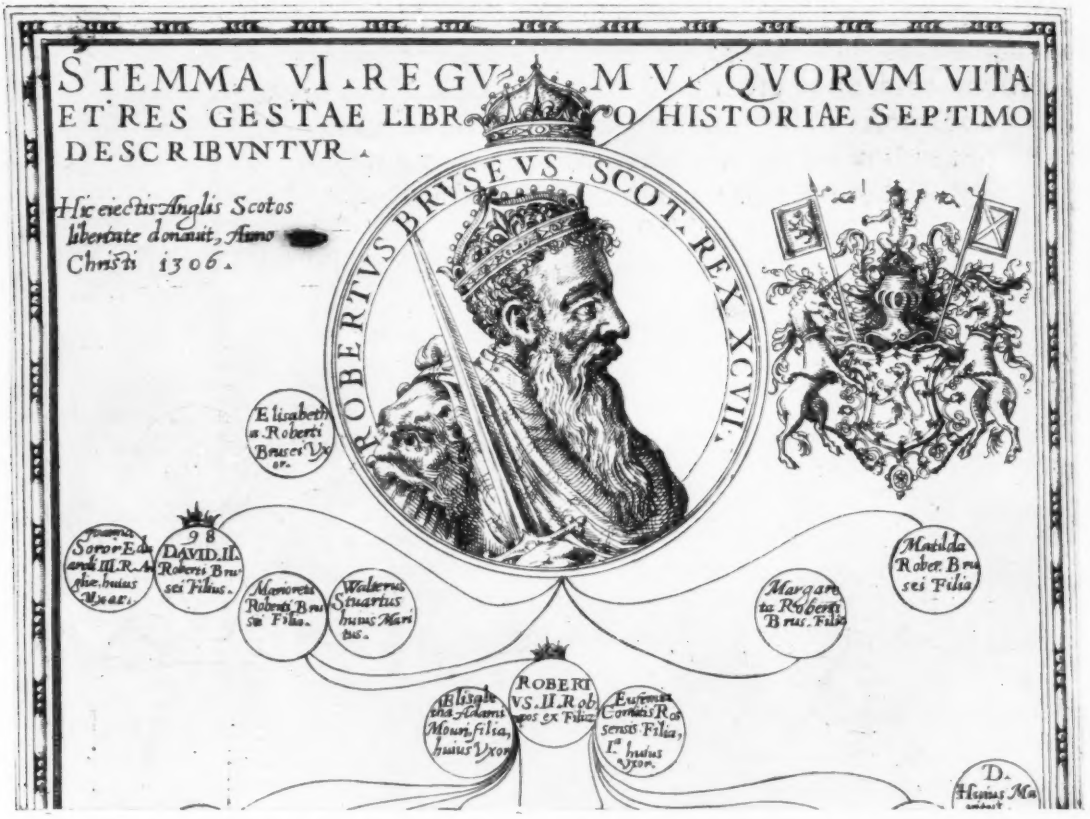

Medallion of Robert Bruce placed by John Leslie, Bishop of Ross, at the top of his pedigree of the Stewart Kings in his well-known work: *De Origine, Moribus et rebus gestis Scotorum*, Romae, 1578. It is the original of Boitard's engraving some two centuries later (see our Plate IX), but whence Leslie drew his inspiration is unknown. The Bruce's massive chin and receding forehead are not in the least indicated.

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Robert Bruce.

(*a*) G. Cameron's Engraving from the de Witte picture of Bruce in Holyroodhouse.

(*b*) His Majesty the King in photographic and numismatic profile. To show how conventional is still the position of the orbit and auricular passage in modern coins.

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A PEDIGREE OF CONGENITAL DISLOCATION OF LENSES *.

By C. H. USHER, M.B., Ch.B.

This short pedigree contains seven individuals with congenital dislocation of lenses in three consecutive generations. In each case both lenses are dislocated. The first case (II. 5), a male, is the only member affected in a sibship consisting of five males and two females and there were two miscarriages. His parents had good vision. His mother showed no evidence of dislocation of either lens and so far as is known his father had normal eyes. The remaining cases are all descendants of this man. He had nine children and five of them had congenital dislocation of their lenses, namely, three males and two females, one of the males (III. 9) has an illegitimate son (IV. 4) who also has congenital dislocation of both lenses. The mother of the first case, Mrs O. (I. 2), married when she was sixteen, and when pregnant with II. 5, at the age of seventeen, she was weak and unwell. She is now (1921) at the age of 79 both healthy and active, her vision is good and the fundus is normal in each eye. Her husband (I. 1), a millwright, married at the age of 29, and died at the age of 59 from cancer of the stomach. He and all of his known relatives, other than his descendants from I. 2, had good sight. His father, a farmer, was killed at the age of 70; his mother who died aged 91 always had good vision. He had three sisters and five brothers.

In the cases of congenital dislocation of the lens that occur in this pedigree there is no uniformity in the direction in which the lenses are displaced. Sym-

* The following account is abstracted from *The American Encyclopedia of Ophthalmology*, Vol. IV. p. 2871.

Ectopia lentis. Congenital dislocation of the lens is a comparatively rare anomaly, one in five thousand cases (10 cases in 50,000, patients of Knapp's).

Martin recorded its occurrence in five successive generations comprising ten individuals. Hereditary tendency noticed also by Graefe, Dickson, Williams, Sattler and others.

The direction of the dislocation is usually symmetrical in the two eyes. In the great majority it is upwards, inclined slightly out or in.

Other positions assumed are down and in, outwards, down and out, the frequency being in the order named.

Exceptionally, one lens may be dislocated in one direction and its fellow in another (asymmetrical). In two recorded cases the direction of the right lens was down and out, the left down and in, in another case the right down and in, the left up and in (Dorsch).

The suspensory ligament as a rule is completely absent in the region corresponding to the aphakic area. The lenses are frequently smaller than normal. Iritis and cyclitis or adhesions resulting from previous uveal inflammation have been observed, excited no doubt by the mobility of the lens.

Associated other defects (1) coloboma of iris, (2) corectopia, (3) aniridia, (4) nystagmus. Stroud reports congenital dislocation of both lenses and corectopia in six members of a family of seven. All members of the family had brown irides that reacted well to light.

Lewis found sixteen cases in six generations of one family. From A. R. Gunn's pedigree "it affects both sexes in equal numbers," etc.

metrical dislocation of the lenses inwards occurs in three cases (III. 7, III. 9, IV. 4), symmetrical dislocation outwards in one case (III. 13), and in another case (III. 10), the dislocation is outwards in one eye and upwards and inwards in the other eye. In case II. 5, where each eye received an injury at different dates, it is known that in the right eye the dislocation was outwards before the injury occurred, in the left eye, however, the original position of the lens is not known for no examination was made before this eye was injured.

The corneal opacities connected with the iris by pigmented bands in cases III. 7, and III. 10, are possibly congenital in origin. In the former case they are situated symmetrically.

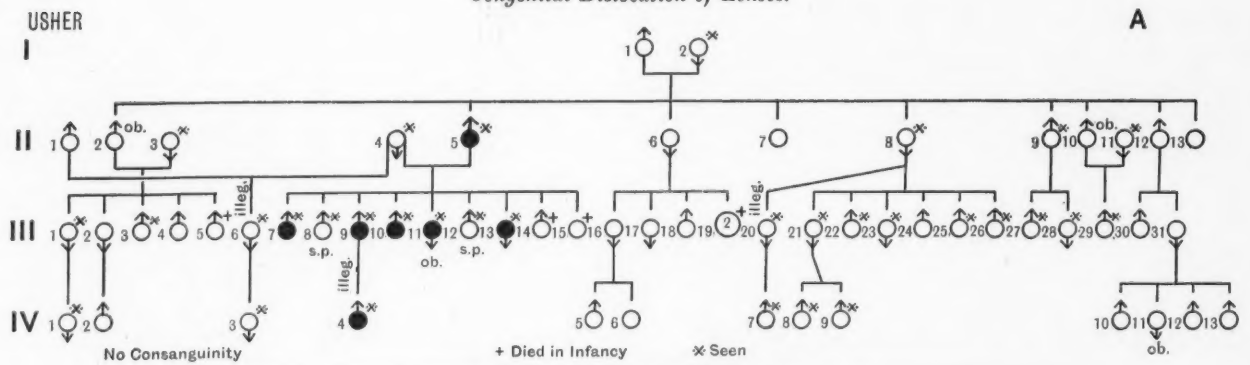
As regards the etiology of the abnormal position of the lenses in II. 5, it is tempting to attribute it to the bad health and weakness of his mother during his intra-uterine life. The eye defect arising originally in II. 5, has been transmitted directly through III. 9, to IV. 4. Mrs O. (I. 2), had seven children and two miscarriages. The children all lived to adult life and had families. Of her seven children only one, her second child, had dislocated lenses. When pregnant with this child at the age of 17, as already stated, she was weak and unwell. She has no dislocated lenses and her husband had good vision. Guyer and Smith have given an account of the transmission through successive generations of eye defects originally produced in animals by means of lens sensitized fowl serum*.

Particulars of the individuals in the pedigree are given as follows:

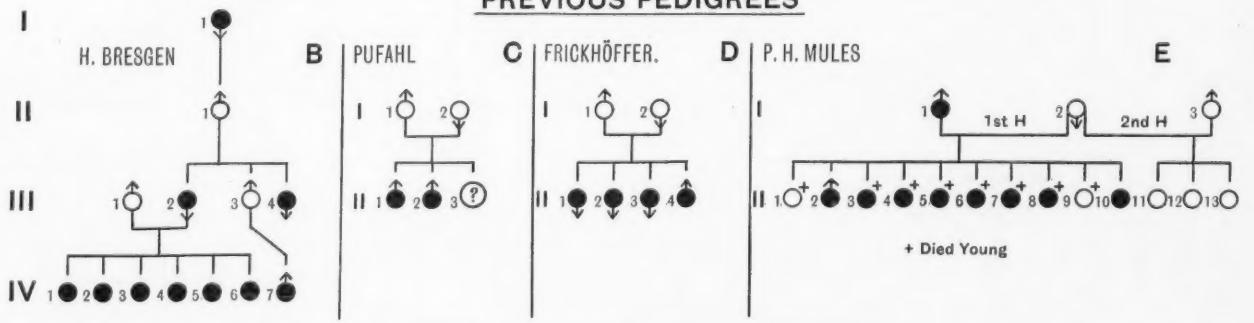
II. 5, John O., age 33 (1899), carter, was born at full time. He stated that he had dislocated lenses as long as he could remember. His left eye had troubled him for two days, and had been red off and on since it was injured with a stone two years previously. On examination: L. V. counts fingers at 9″; T + 1; lens partially dislocated into A. C., inner part of lens lies in front of iris; eye is much congested. R. V. = $\frac{1}{60}$, with + 10 D. sph. = $\frac{6}{60}$; iridodonesis; P. small, circular, contracts to light; lens dislocated outwards, its inner edge being visible at centre of P.; T. n.; fundus n. excepting a congenital crescent below O. D. He was seen again in 1910 on account of the right eye which had been struck with a horse's tail. The condition at this time was as follows: R. V. with + 9 D. sph. = $1\frac{6}{24}$; after a mydriatic upper edge of lens visible when he looks down; iridodonesis of whole of iris; fundus n.; on outer part of cornea a linear, 3 mm., grey opacity and several smaller opacities. L. V. = hand-movements; cornea staphylomatous below; T. n.; iridodonesis; eyeball shrinking and squared. Has been a healthy man and is well nourished. Urine, acid, 1024, no albumin, no sugar. In 1913 he died from cancer of the stomach.

III. 7, James O., age 19 (1910), labourer, with short upper lip, retreating brow, long Hebrew-like nose, red lips and black hair, said he never had good sight. On examination: R. V. = < $\frac{6}{60}$, with + 10 D. sph. and + 3 cyl. = $\frac{6}{18}$. L. V. = $\frac{2}{60}$, not improved by lenses. On each cornea at its upper and inner part is a nebulous opacity

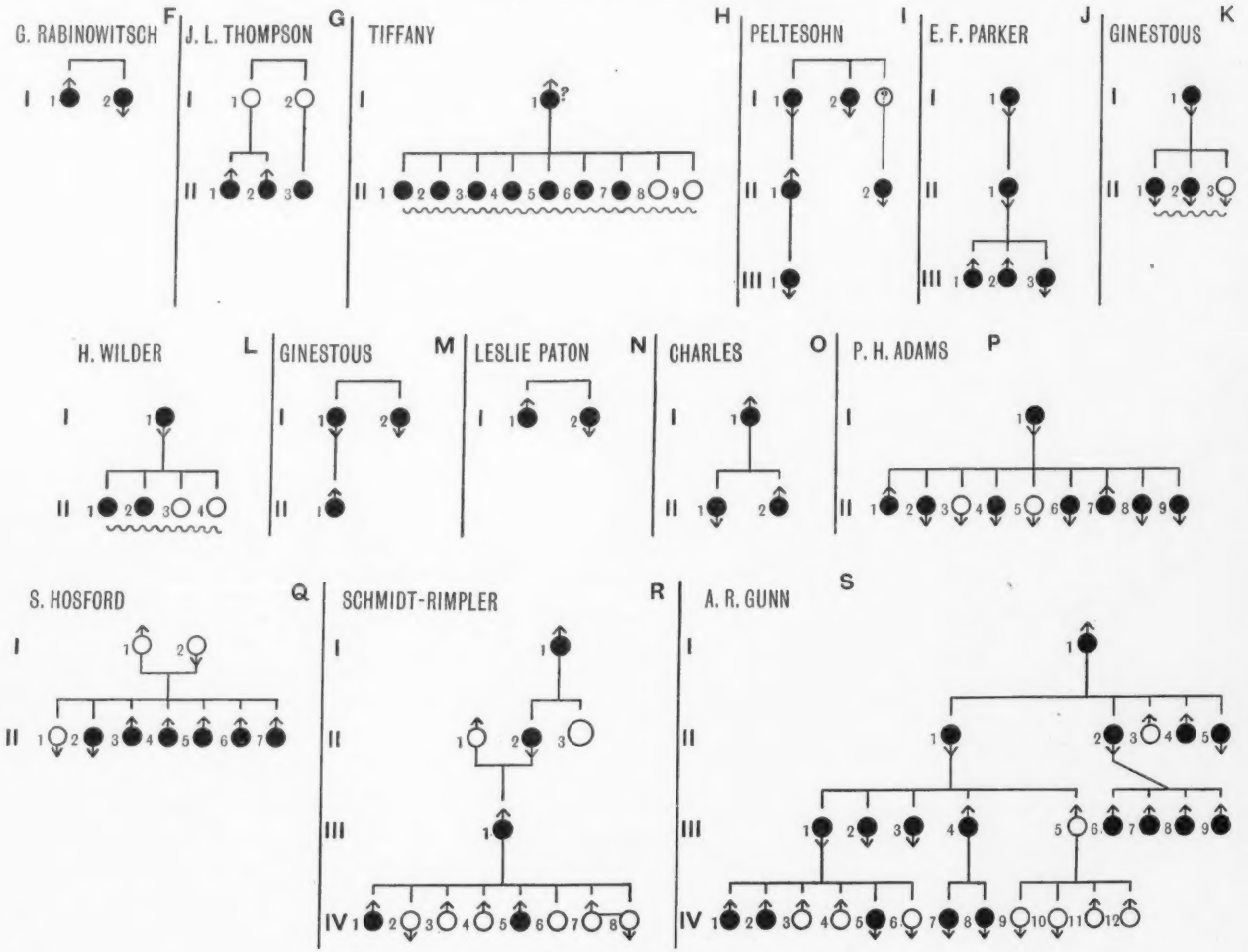
* Guyer, M. F. and Smith, E. A.: "Studies on Cytolysins. II. Transmission of induced eye defects." *The Journal of Experimental Zoology*, Vol. xxxi. No. 2, p. 171, Aug. 20, 1920.



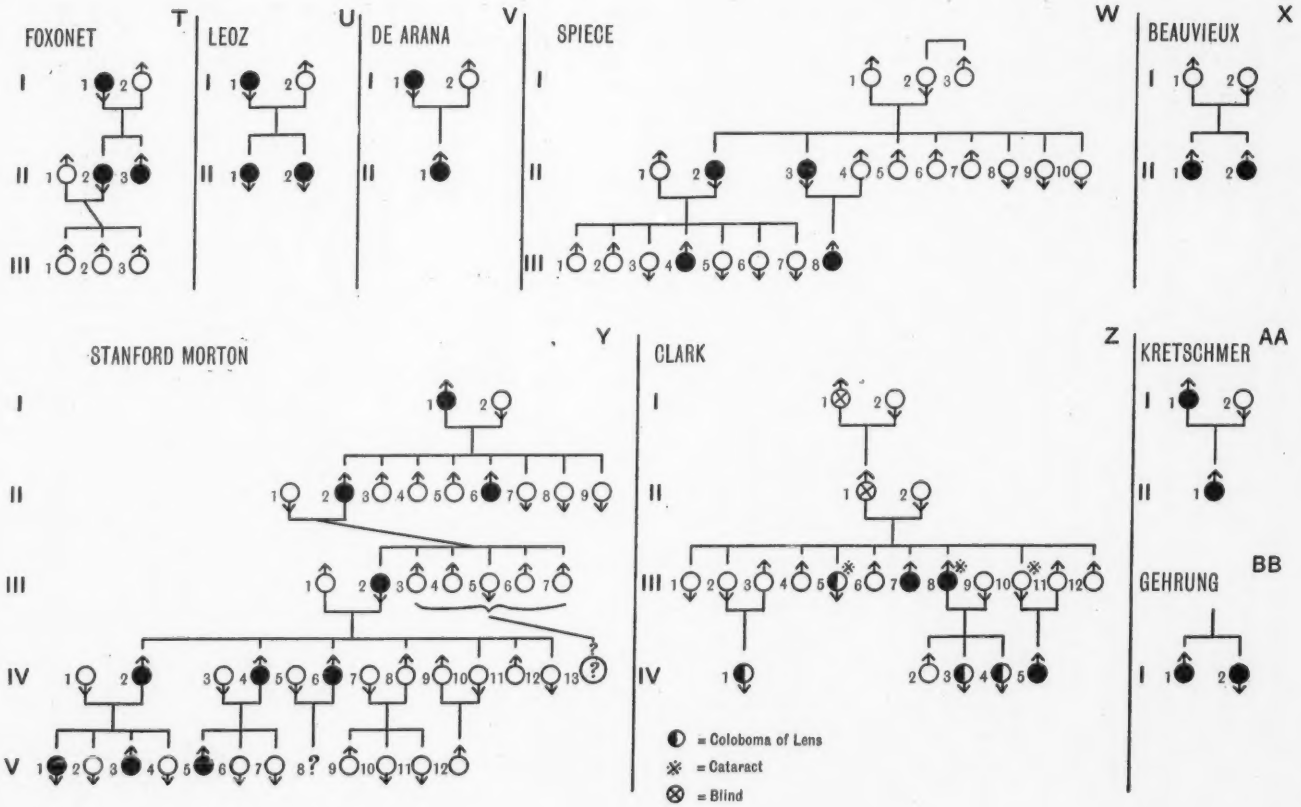
PREVIOUS PEDIGREES



Where mate is not given he, or she, may be assumed to be normal.



Where mate is not given he, or she, may be assumed to be normal.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AA
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BB
= Coloboma of Lens
= Cataract
= Blind

extending from near the centre of the cornea to the periphery, these occupy about ¼ of the cornea and merge into the sclerotic, at other parts the edge of the opacity is well defined; pigmented bands pass forwards from the iris surface to the opacity; iris stroma in each eye appears to have been partially separated from the posterior layers and pulled forwards towards the corneal opacity; pupils, circular, contract to light; iridodonesis; lenses dislocated inwards, edge of lens visible at centre of P.; oph.: no fundus details seen.

III. 9, William O., age 15 (1908), combmaker, has never seen well. R. V. with + 9 D. sph. = $\frac{6}{60}$. L. V. with + 13 D. sph. = $\frac{6}{36}$. Pupils circular; iridodonesis in each eye; lenses displaced inwards; some fine floating vitreous opacities in each eye; fundi n.; left eye diverges upwards and outwards.

III. 10, George O., age 15 (1910), rope maker. R. V. with + 11 D. sph. = $\frac{6}{36}$. L. V. with + 13 D. sph. = $\frac{6}{60}$. R. lens dislocated outwards, edge of lens visible at centre of P.; iridodonesis; fundus n.; a bluish grey opacity with well defined upper margin is present on the lower third of cornea, thick pigmented bands pass to it from the circulus iridis minor, none of the bands are attached to the margin of the pupil; upper part of iris is brown and has a smooth surface, whilst lower part has a bluish grey colour and rough surface, with some large crypts; P., slightly pear shaped with narrow end downwards, contracts readily to light. Left eye has an opacity in upper third of cornea to which extensive bands pass from the anterior surface of the iris; with the exception of one or two, the bands are not attached to pupil margin. The iris appears to be connected with the whole extent of the corneal opacity; iris is evidently much thinned; P., almost circular, contracts well to light; lens dislocated upwards and slightly inwards; no clear view of fundus obtained.

III. 11, Bella O., died at seven years of age, had dislocated lenses.

III. 13, Lizzie O., age 13 (1910). Lenses dislocated outwards, inner edge of each lens passes across pupil about its centre; fundi n.; irides blue; R. and L. iridodonesis, refraction H. 7, D. She died at the age of fifteen, there had been discharge from an ear and head trouble.

IV. 4, James W., age 4 (1921), an illegitimate child of III. 9, holds things too near him. Corneae of normal size; pupils, equal, central, contract to light; anterior chamber shallower at inner parts; iridodonesis in each eye; lenses, dislocated inwards and tilted backwards at outer parts, occupy rather more than a half of the pupils; retinoscopy showed myopia 20, D. in each eye; optic discs and rest of fundi normal; hair is pale brown; irides blue; high palate.

The mother (II. 4) of the affected members in generation III, aged 48 (1910), had an illegitimate daughter (III. 6) by II. 1. This daughter and her child (IV. 3) both see well and have no sign of dislocation of lenses.

III. 8, John O., and III. 12, his brother Robert, are unaffected. They have both been married for about two years and have no offspring. The former is myopic and there is choroidal atrophy around the left optic disc: right fundus normal. The

latter (III. 12) has a small upper jaw, lower incisors lie in front of the upper incisors when mouth is closed. V. = $\frac{6}{6}$ in each eye; fundi normal. III. 14 and III. 15 died in infancy. III. 20, illegitimate, has the same mother as III. 21 to 26, but a different father. Her lenses are in situ and are clear, no iridodonesis, anterior chambers normal, pupils, circular, contract to light. Her child (IV. 7) has good vision, lenses clear and in situ, no iridodonesis, anterior chambers of normal depth. II. 8, and her children III. 22, 23, 25 and 26 all see well, none are myopic, anterior chambers are of normal depth, no iridodonesis, no edge of lens seen in pupil. The pupils of III. 23 are somewhat ectopic inwards, this does not occur in any of the others. III. 1, III. 3 and IV. 1 all see well, pupils, equal, contract to light, no iridodonesis, anterior chambers of normal depth. III. 27, III. 28. III, 21 and IV. 8 and IV. 9 have good vision, pupils, equal, contract to light, anterior chambers normal, no iridodonesis, retinoscopy shows hypermetropia. III. 29 dark irides, pupils, equal, contract to light, anterior chambers normal, no iridodonesis, good vision, ophthalmoscopic examination, good red reflex, no lens edge seen. III. 22 to 26 all unmarried. II. 7, and II. 13 are miscarriages.

Congenital Dislocation of the Lens. Some published cases for comparison with the present pedigree.

*Fig. B. Bresgen, H. "Zur Heredität der Linsenanomalieen," *Centralblatt f. praktische Augenheilkunde*, Bd. III. S. 104. Leipzig, 1879.

*Fig. C. Pufahl, M. "Ueber Corectopia," *Centralblatt f. praktische Augenheilkunde*, Bd. III. S. 293–300. Leipzig, 1879.

II. 1, aet. 55, R. injured and blind; L. luxation of lens noticed at age of 30; P. up and out, no lens in P. II. 2, aet. 53, R. P. up-out, aphakia; L. P. less excentric, lens luxated downwards.

*Fig. D. Frickhöffer, C. "Ueber Corectopie," *Inaugural Dissertation*. Bonn, 1880.

I. 1, P.'s oval and oblique but central, my. I. 2, my. in both. II. 2, age 21, R. P. ectopic, R. lens free in "Boden am Auge"; L. slightly ectopic, lens seen towards outer edge of pupil. II. 3, ectopic P.'s: R. lens "nach oben, links im Sinne der Pupillenectopie verschoben." II. 4, age 12, ectopic P.'s; R. lens "am Boden des Auges, links gar nicht zu entdecken."

Fig. E. Mules, P. H. "Two instances of Heredity," *Ophthalmic Review*, Vol. II. pp. 48, 49. London, 1883.

II. 2, R. lens displaced down and in, L. lens directly in. II. 10, both lenses displaced below the pupil.

*Fig. F. Rabinowitsch, G. "Sluchai ektopii chrystal s vivichom ego v pered. kameru." *Vestnik Oftalmologii*, Vol. III. p. 128. Kieff, 1886.

Lens dislocated up-out.

* Seen only in abstract.

*Fig. G. Thompson, J. L. "Observations on Displacement of the Crystalline Lens from Congenital and other Causes," *Journal of American Medical Association*, Vol. IX. pp. 674—677. Chicago, 1887.

Two cases in brothers and a third in their cousin.

*Fig. H. Tiffany, F. B. "Ectopia lentis," *Journal of American Medical Association*, Vol. XXV. pp. 796—798. Chicago, 1895.

Defect was bilateral in each case. In one case the displacement in one eye was outward and downward, in another case in one eye it was up and in, in a third case in both eyes it was inwards, in all the other eyes it was up and out. The father was said to have defective eyes.

Fig. I. Peltesohn. "Zwei Fälle angeborener Missbildung am Auge," *Centralblatt f. praktische Augenheilkunde*, Bd. XXI. S. 112—114. Leipzig, 1897.

Bilateral congenital hereditary ectopia lentis in II. 1, aged 39, in his mother I. 1, in her sister I. 2, and in a niece of the latter II. 2; the patient's daughter was similarly affected.

*Fig. J. Parker, E. F. "Five cases of congenital bilateral symmetrical displacement of the lens of the eye in three successive generations of one family," *Journal of American Medical Association*, Vol. XXXI. pp. 708—710. Chicago, 1898.

The dislocation was bilateral up and out. Gives other eight reported instances in which the defect had been found in from two to five generations.

*Fig. K. Ginestous, E. "Luxation congénitale du cristallin," *Bulletin de la Société d'Anatomie et de Physiologie normales et pathologiques de Bordeaux*, Tom. XX. pp. 381—383. Bordeaux, 1899.

In I. 1, displacement was outwards in both; in II. 2 down-out in both; II. 1, up-out.

*Fig. L. Wilder, W. H. "Extraction of congenitally dislocated opaque lenses with presentation of the case," *Ophthalmic Record*, Vol. IX. pp. 174—176. Chicago, 1900.

*Fig. M. Ginestous, E. "La luxation congénitale bilatérale du cristallin," *Recueil d'Ophtalmologie*, Sér. 3, Tom. XXII. p. 282. Paris, 1900.

Fig. N. Paton, L. "Two cases of congenital dislocation and malformation of the lenses occurring in a brother and sister," *Ophthalmic Review*, Vol. XXIII. p. 180. London, 1904.

Fig. O. Charles, J. W. "Ectopia lentis with apparent power of accommodation," *American Journal of Ophthalmology*, Vol. XXII. pp. 131—137. St Louis, 1905.

Fig. P. Adams, P. H. "A family with congenital displacement of lenses," *Transactions of Ophthalmological Society of U. K.*, Vol. XXIX. pp. 274, 275. London, 909.

* Seen only in abstract.

Fig. R. Schmidt-Rimpler. "Ueber Linsen-Luxationen," *Bericht über die 28te Versammlung der Ophthalmologischen Gesellschaft*, 1900, Bd. XXVIII. S. 57—64. Wiesbaden, 1901.

Congenital dislocation through four generations.

II. 3 and 5 were brighter and more intelligent than other members of the family. The author could obtain no history of any other of the relations suffering in the same way. The explanation given of the occurrence of the defect in the mother I. 1 was that "her mother fell down a flight of stairs the night her child was born." II. 1, R. lens displaced down, L. lens completely displaced and not visible. II. 2, R. lens displaced inwards, L. lens up and in. II. 4, R. and L. lenses downwards and L. also a little in. II. 6, R. and L. lenses displaced inwards. II. 7, R. lens upwards, L. up and out. II. 8, R. lens downwards, L. inwards. II. 9, R. and L. lenses upwards. "The interesting point about this series...is in the unusual situation of the displaced lens." All showed iridodonesis; fundus was normal in all cases; no other congenital abnormalities were present.

Fig. Q. Hosford, Stroud. "Congenital dislocation of both lenses with corectopia in six members of a family of seven," *Transactions of Ophthalmological Society of U. K.*, Vol. XXXI. pp. 50, 51. London, 1911.

The parents of the affected siblings and the uncles and aunts are healthy; all members of the family have brown irides. II. 2 had R. lens dislocated up and out, L. lens up. II. 3, 4 and 5 had both lenses dislocated up and out. II. 6 had both lenses out. II. 7 had both lenses dislocated inwards.

Fig. S. Gunn, A. R. "Complete congenital dislocation of the lens: a family history," *The Ophthalmoscope*, Vol. X. pp. 193—195. London, 1912.

Six cases were examined: both lenses were dislocated. In three children the lens could be seen floating in the vitreous.

Fig. T. Foxonet, J. "Glaucome double consécutif à l'ectopie congénitale du cristallin avec luxation dans la chambre antérieure du cristallin gauche. Extraction des deux cristallins dans leur capsule—Guérison." *Annales d'Oculistique*, T. 157, pp. 616—624. Paris, 1920.

*Fig. U. Leoz, G. "Familial congenital subluxation of lens" (4 col. ill.), *Archivos de Oftalmologia hispano-americanos*, Vol. XXII. pp. 177–190. Madrid, 1922. Reference from *The Ophthalmic Year Book*, Vol. XIX. pp. 180, 182. Denver, 1922. Congenital dislocation of lenses with diplocoria occurring in a mother and her two daughters; dislocations were outwards and downwards.

*Fig. V. de Arana, J. "Angeborene Linsenektopie bei zwei Personen der gleichen Familie," *España Oftalmol.*, Jg. VI. No. 7, S. 130–1, 1921 (Spanisch). Reference from *Zentralblatt f. d. gesamte Ophthalmologie*, Bd. VIII. Berlin, 1923 and Bd. VII. Berlin, 1922.

Fig. W. Spiece, W. K. "A case of Ectopia Lentis with Family History," *American Journal of Ophthalmology*, Ser. 3, Vol. I. pp. 681–3. Chicago, 1918.

III. 4. Aged 7, lenses dislocated upwards and outwards.

* Seen only in abstract.

II. 2. Right lens dislocated upwards and outwards, left lens in normal position.

II. 3. Upper outer edge of each lens seen with pupils dilated.

III. 8. Eyes in a similar condition to that of his mother's eyes.

Fig. X. Beauvieux, M. J. C. J. "Étude sur les déplacements congénitaux du cristallin," *Archives d'Ophtalmologie*, T. XXXIII. pp. 16–35. Paris, 1913.

The author's Obs. IV describes ectopia lentis in a male aged 27; the father was alcoholic; mother had myopia of about 18 D.; a brother seen by Lagrange at age of 12 had congenital bilateral dislocation of lens outwards.

Fig. Y. Stanford Morton, A. "Congenital displacement of both lenses occurring in several members of one family," *Royal London Ophthalmic Hospital Reports*, Vol. IX. pp. 435–9. London, 1879.

Ten members of the family were affected in five generations; the displacement was usually inwards or inwards and upwards; the lenses were of normal size and shape but of a slightly milky appearance.

Fig. Z. Clark, C. F. "Coloboma and so-called congenital dislocation of the lens," *Archives of Ophthalmology*, Vol. XLVIII. pp. 475–84. New York, 1919.

I. 1. Blind 30 years before death, supposed to have cataract.

II. 1. Blind 10 years before death.

III. 5. Aged 63, bilateral cataract and coloboma of lens.

III. 8. Aged 55, had cataract, coloboma and dislocation upwards of lens in right eye only; he had two daughters of whom one, aged 25, had bilateral coloboma of lens and the other, aged 33, had coloboma of left lens only.

III. 7. Aged 59, had bilateral dislocation of lens downwards.

III. 10. Aged 53, had cataract; her son, IV. 5, aged 30, had bilateral dislocation of lens upwards.

III. 2. Aged 68, had a daughter, IV. 1, aged 26, with bilateral coloboma of lens.

Fig. AA. Kretschmer. "Angeborene Linsenverschiebung bei Vater und Sohn," *Centralblatt f. praktische Augenheilkunde*, Bd. XXXVIII. S. 8. Leipzig, 1914.

In the son, aged 5, the dislocation was upwards and inwards in each eye. The father had a bilateral but not symmetrical defect; in the right eye the lens was dislocated upwards and outwards, in the left eye the displacement was upwards and inwards.

Fig. BB. Gehrung, J. A. "Congenital dislocation of lenses," *Archives of Ophthalmology*, Vol. XLIII. p, 271. New York, 1914.

A brother and sister showed double dislocation of lenses; their lenses had colobomas. The mother had a coloboma of each lens.

EINE DARSTELLUNG DER STERBETAFEL*.

Von E. I. GUMBEL, Heidelberg.

Man bezeichne mit $l(x)$ die Wahrscheinlichkeit, die einem Neugeborenen zukommt, den $x$ten Geburtstag zu erleben. Dann ist $l(0)=1$ und $l(\omega)=0$, wenn ω das höchste erreichbare Altersjahr bezeichnet. $l(x)$ ist eine monoton abnehmende Funktion von x und stellt den Altersaufbau einer stationären Bevölkerung dar. Zahlreiche Beobachtungen der Funktion $l(x)$ haben ergeben, dass dieselbe sehr stark von der Beobachtungszeit und dem Beobachtungsland, sogar von dem Beruf der beobachteten Bevölkerung abhängt, also keineswegs den Charakter eines Naturgesetzes besitzt. Eine analytische Darstellung ist jedoch für viele praktische Zwecke, besonders für die Versicherungsmathematik vonnöten. Die übliche auf Gompertz und Makeham zurückzuführende Darstellung geht aus von der Sterbensintensität

$$\mu(x)=-\frac{d\,l(x)}{dx\,l(x)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1),$$

und setzt an

$$\mu(x)=A+Bc^x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2),$$

woraus man durch Integration unter Berücksichtigung der ersten Anfangsbedingung erhält

$$l(x)=s^x g^{c^x-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3),$$

wobei

$$\frac{B}{\lg c}=-\lg g \text{ und } A=-\lg s \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

gesetzt ist. Die weitverbreitete Formel (3) hat den Nachteil, dass sie nur für $x>15$ Jahre gilt, weil nur hierfür die Annahme des exponentiellen Wachstums von μ angenähert zutrifft. Für $x<15$ Jahre jedoch fällt das μ sogar. Ferner ist die Bestimmung der Konstanten bzw. die Ausgleichung der Formel ausserordentlich mühevoll. Endlich besitzen die Konstanten, wie man sofort sieht, keine sinnvolle Bedeutung. Denn man kann durch ihre Angabe den Verlauf der $l(x)$ nicht ohne weiteres charakterisieren. Im folgenden wird eine Formel abgeleitet, die von diesen Nachteilen frei ist. Man betrachte

$$E(x)=\frac{1}{l(x)}\int_x^\omega l(x)\,dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

die sogenannte Lebenserwartung eines x-jährigen, so ist

$$-\frac{l'(x)}{l(x)}=\frac{1+E'(x)}{E(x)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5a)$$

* Erst nach Beendigung dieser Arbeit erfuhr ich dass Herr Dr Brownlee (*The Use of Deathrates as a Measure of Hygienic Conditions*, Stationery Office, 1922) bereits den Ansatz (7) gebraucht hat. Er hat aber die mathematische Ergebnisse seiner Hypothese nicht vollständig ausgearbeitet.

also unter Berücksichtigung der ersten Anfangsbedingung

$$l(x) = \frac{E(0)}{E(x)} e^{-\int_0^x \frac{dx}{E(x)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6).$$

Im folgenden wird nun die Annahme gemacht, dass die Lebenserwartung durch den einfachen Ansatz wiedergegeben werden kann:

$$E(x) = E(0)\left(1 - \frac{x}{\omega}\right)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7).$$

Die Bedeutung der 3 hier auftretenden Konstanten ist augenfällig. ω ist das Alter, bei dem die Generation ausgestorben ist. $E(0)$ ist die Lebenserwartung eines Neugeborenen. Auf die Bedeutung von n kommen wir noch zurück.

Man sieht jedoch aus dem Verlauf von $E(x)$ (vergl. Figur 1) sofort, dass die Formel (7) nicht den ganzen Verlauf wiederzugeben vermag. Denn das tatsächliche $E(x)$ steigt von 0 bis etwa 3 Jahre, um dann mit dem Alter zu fallen, während hier stets $E(x)$ mit wachsendem x fällt. Genau derselbe Einwand gilt aber auch für die Gompertz-Makehamsche Formel. Hier fällt das tatsächliche $\mu(x)$ von 0 bis etwa 15 Jahre, um von da an zu steigen, während die Annahme für $\mu(x)$ dahin geht, dass es stets steigt. Während aber die Gompertz-Makehamsche Formel prinzipiell nicht passt bis zum Alter von 15 Jahren, gilt dies hier nur für das Intervall bis zu drei Jahren.

Aus diesem Grund wird man für die Formel nicht das beobachtete $E(0)$ nehmen, vielmehr die Reihe der E vom 3. Jahre ab auftragen und hieraus $E(0)$ extrapolieren. Man muss sich jedoch bewusst bleiben, dass die so berechneten l nur ab $l(3)$ gut passen können. Die frühe Kindersterblichkeit lässt sich also mit unserer Methode nicht erfassen. Hat man also eine bestimmte Sterbetafel, so muss man zunächst nach der bekannten Methode die E's berechnen, die übrigens in den meisten Sterbetafeln bereits angegeben sind. Zunächst werden drei Entartungsfälle betrachtet:

(1) Es sei $n=0$, dann ist $E(x) = E(0) = \text{konstans}$ ..... (7a)

und $$l(x) = e^{-\frac{x}{E(0)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8a).$$

Dies erfüllt zwar die Anfangsbedingung $l(0) = 1$; nicht aber die Bedingung $l(\omega) = 0$. Dies hat praktisch kein Bedenken, solange wie $l(x)$ sehr rasch abnimmt. Übrigens ergibt sich dieser Fall auch als Spezialfall der Gompertz-Makehamschen Formel. Denn $E(x) = \text{konst.}$ bedeutet auch

$$\mu(x) = \text{konst.} = \frac{1}{E(0)} \text{ nach } (5a), \text{ d. h. } c = 1 \text{ oder } B = 0.$$

Dies erlaubt, die Konstanten der Gompertz-Makehamschen Formel mit einer unserer Konstanten zu identifizieren. Denn in beiden Fällen ergibt sich

$$l(x) = s^x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3a),$$

woraus folgt $$A = -\lg s = \frac{1}{E(0)}.$$

(2) Es sei $n=1$, d. h. E eine lineare Funktion des Alters. Dann wird

$$l(x)=\left(1-\frac{x}{\omega}\right)^{\frac{\omega}{E(0)}-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8b),$$

was für den Spezialfall $\omega=2E(0)$ die de Moivresche Hypothese vom linearen Verlauf der $l(x)$ ergibt. Wie man sieht, sind hier beide Grenzbedingungen erfüllt. Wird hier $\omega=\infty$ gesetzt so bekommt man wieder den Fall (1).

(3) Es sei $n=\frac{\omega}{E(0)}$ und gleichzeitig $\omega=\infty$. Dann wird

$$E(x)=E(0)\lim_{\omega=\infty}\left(1-\frac{\frac{x}{E(0)}}{\frac{\omega}{E(0)}}\right)^{\frac{\omega}{E(0)}}=E(0)\,e^{-\frac{x}{E(0)}}\ldots\ldots\ldots\ldots(7c),$$

und daher nach (6) $$l(x)=e^{1+\frac{x}{E(0)}-e^{\frac{x}{E(0)}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8c).$$

$l(x)$ wird also hier sehr rasch abnehmen. Auch dieser Fall erlaubt einen direkten Vergleich mit der Gompertz-Makehamschen Formel, da hierfür

$$l(x)=e^{-\lg g+x\lg s+\lg g c^x}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3b).$$

Beide Ausdrücke stimmen überein, falls

$$g=\frac{1}{e} \text{ und } c=s=e^{\frac{1}{E(0)}}.$$

Diese Bestimmung der Konstanten bedeutet keinen Widerspruch zu der früheren, da im einen Fall $n=0$, im anderen $n=\infty$ gesetzt ist, was natürlich ganz verschiedene Einflüsse haben muss.

Diese drei Entartungsfälle haben gegenüber dem im folgenden behandelten allgemeinen Fall den Vorzug nur eine bezw. zwei Konstanten aufzuweisen. Zu ihrer Bestimmung wird man aus den beobachteten $l(x)$ die $E(x)$ berechnen und ausgleichen. Sind sie konstant, so gilt (8a), linear (8b), exponentiell (8c). Die praktische Bedeutung dieser drei Fälle ist nur gering.

Setzt man im allgemeinen Fall

$$\frac{\omega}{\omega-x}=\xi,$$

so wird $$E(x)=E(0)\,\xi^{-n}.$$

Dies in (6) eingesetzt gibt

$$l(x)=\xi^n e^{-\frac{\omega}{E(0)}\int_1^{\xi}\xi^{n-2}d\xi}, \text{ da } d\xi=\frac{\xi^2}{\omega}dx$$

$$=\xi^n e^{-\frac{\omega}{E(0)(n-1)}(\xi^{n-1}-1)}, \text{ da } n\neq 1.$$

Setzt man $$e^{-\frac{\omega}{E(0)}\frac{1}{n-1}}=D,$$

so wird $$l(x)=\xi^n D^{\xi^{n-1}-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8).$$

Dass dieser Ausdruck der Grenzbedingung $l(\omega) = 0$ genügt, sieht man leicht. Denn für $x = \omega$ wird $\xi = \infty$. Dabei ist aber $D < 1$ solange $n > 1$. Solange dies gilt, nimmt $D^{\xi^{n-1}-1}$ mit wachsendem ξ ab, und zwar stärker als ξ^n zunimmt. Daher ist $n > 1$ die Bedingung für die Anwendbarkeit der Formel (8). n kann also kein echter Bruch sein. Da ω so ziemlich für alle Sterbetafeln angenähert konstant gleich 100 ist, ist $\frac{E(0)}{\omega}$ ein Mass dafür, ob die Sterbetafel günstig ist. Genau das gleiche gilt natürlich für $e^{-\frac{\omega}{E(0)}}$. Die Bedeutung von n ersieht man aus

$$\lg E(x) = \lg E(0) + n \lg \left(1 - \frac{x}{\omega}\right).$$

Trägt man also als Ordinate $\lg E(x)$, als Abscisse $\lg \left(1 - \frac{x}{\omega}\right)$ auf, so muss man, wenn die Formel gilt, in diesem Koordinatensystem eine Gerade erhalten und n ist ihre Neigung. Bei dieser Bestimmung braucht $E(0)$ noch gar nicht bekannt zu sein. Es ergibt sich vielmehr durch Extrapolation als der zu $x = 0$ gehörige Wert der Ordinate. Für $n = 1$ wäre E wie oben angeführt eine lineare Funktion in einem gewöhnlichen Koordinatensystem. Daher ist $n - 1$ ein Mass der Abweichung von diesem einfachsten linearen Charakter.

Im folgenden ist die Gleichung (7) in der ursprunglichen in der logarithmierten Form graphisch für die im *Textbook of the Institute of Actuaries* angegebene Sterbetafel aufgetragen (Figur 1 und 2). Man sieht, dass die E's von 5—85 Jahren sich zwanglos einer Geraden anpassen. Eine Reihe von weiteren Starbetafeln, welche ich untersucht habe, hat dieses Ergebnis und damit die Brauchbarkeit der Formel (7) bestätigt. Da ω konstant gleich 100 gesetzt werden darf, enthält die Gleichung nur zwei Unbekannte, n und $E(0)$. Diese können alsdann durch zwei Beobachtungen, sagen wir $x = 10$ und $x = 80$, bestimmt werden. Man wird jedoch, um sicher zu gehen, $E(0)$ und n mithilfe der Methode der kleinsten Quadrate berechnen. Es ergibt sich für die Tafel des Textbuchs:

$$\omega = 100,$$
$$n = 1{\cdot}5956,$$
$$E(0) = 58{\cdot}32,$$

hieraus
$$D = 0{\cdot}056227.$$

Will man sich jedoch mit dieser einfachen Methode nicht begnügen, so wird man mithilfe der Methode der kleinsten Quadrate $E(0)$, n und ω simultan bestimmen. Man gehe aus von der logarithmierten Gleichung für $E(x)$ und setze n und ω gleich einem Näherungswert plus einem Korrekturglied, also

$$n = 1{\cdot}5 + \nu, \quad \omega = 100 + \delta.$$

Dann wird man nach der Taylor-Entwicklung, wenn man die beobachteten Faktoren auf die linke Seite setzt, die in den Unbekannten $E(0)$, ν, δ lineare Gleichung

$$\lg E(x) - 1{\cdot}5 \lg \left(1 - \frac{x}{100}\right) = \lg E(x) + \nu \lg \left(1 - \frac{x}{100}\right) + \delta \,.\, 0{\cdot}015 \, \frac{x}{100 - x},$$

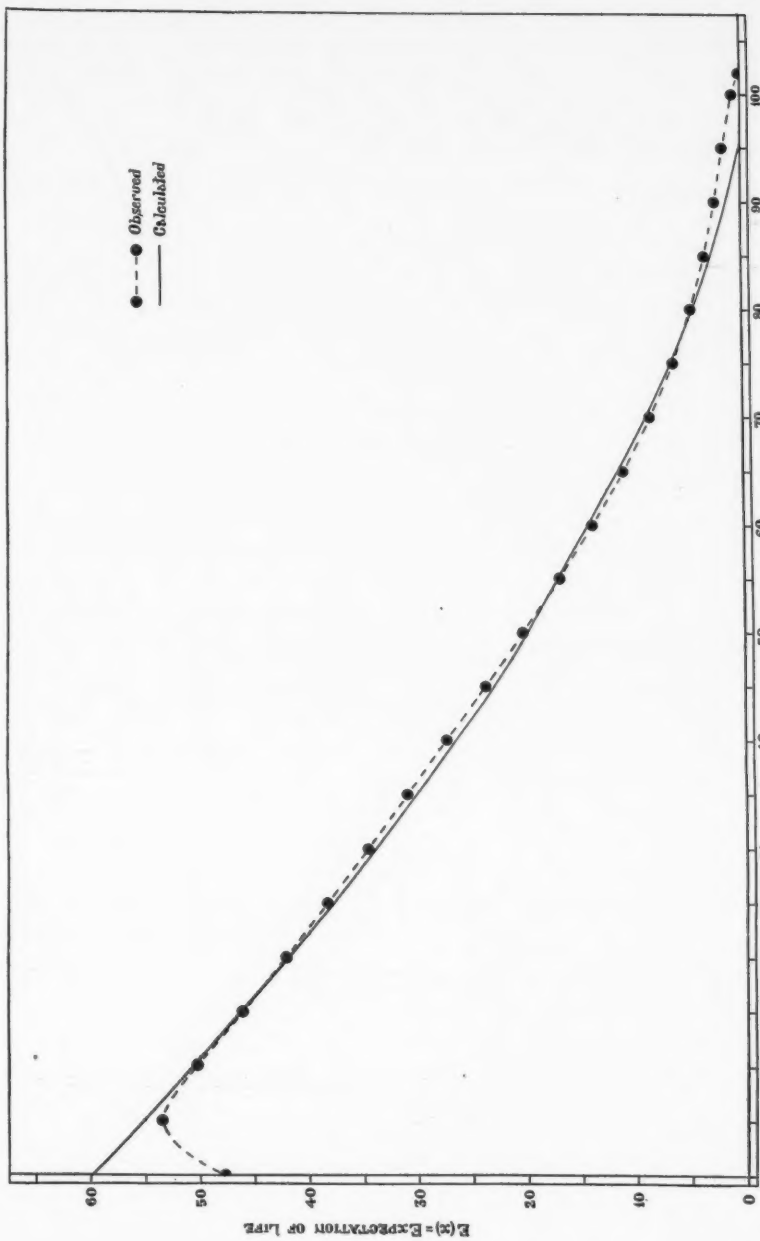

Observed
Calculated
E(x) = Expectation of life
Age in Years
0
10
20
30
40
50
60
70
80
90
100

Fig. 1.

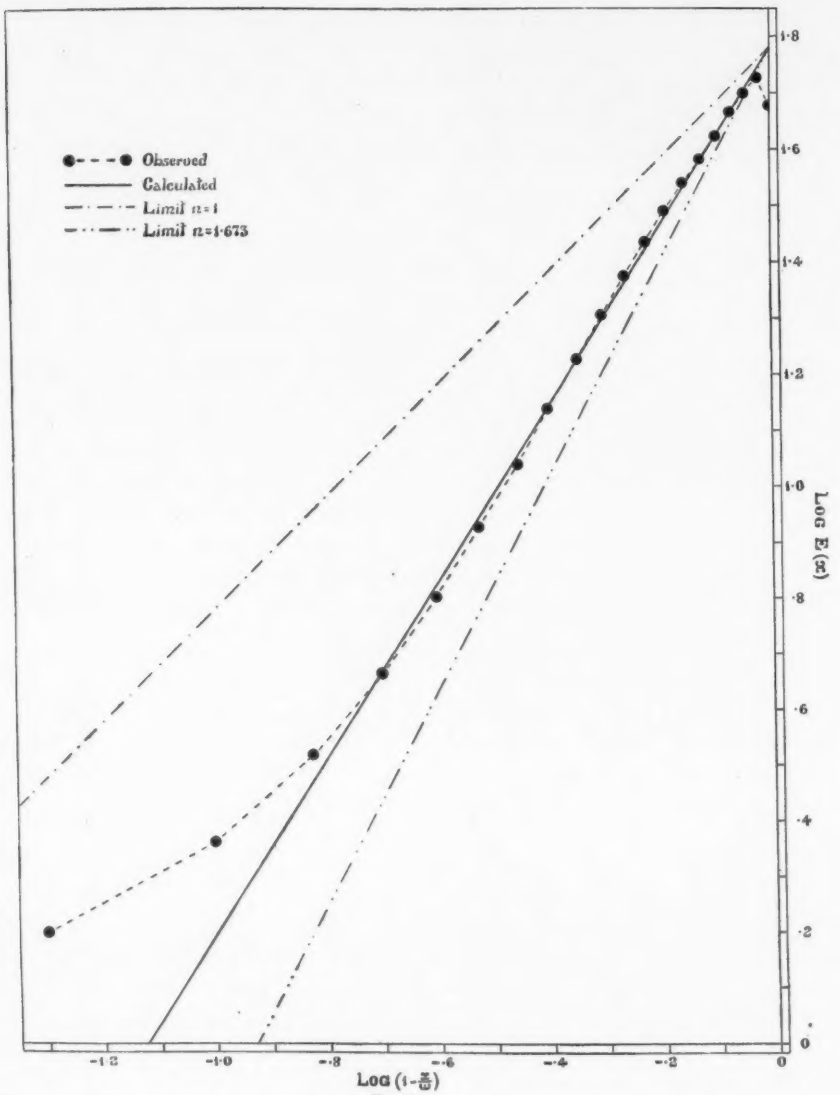

Observed
Calculated
Limit n=1
Limit n=1·673
1·8
1·6
1·4
1·2
1·0
·8
·6
·4
·2
0
Log E(x)
-1·2
-1·0
-·8
-·6
-·4
-·2
0
Log (1-x/ω)

Fig. 2.

erhalten. Es sei die Zahl der Beobachtungen N, so sind $E(0)$, ν, δ zu bestimmen aus

$$N \lg E(0) + \nu \Sigma \lg\left(1 - \frac{x}{100}\right) + \delta\, 0{\cdot}015\, \Sigma \frac{x}{100 - x} = \Sigma \lg E(x) - 1{\cdot}5\, \Sigma \lg\left(1 - \frac{x}{100}\right),$$

$$\lg E(0)\, \Sigma \lg\left(1 - \frac{x}{100}\right) + \nu \Sigma \lg^2\left(1 - \frac{x}{100}\right) + \delta\, 0{\cdot}015\, \Sigma \frac{x}{100 - x} \lg\left(1 - \frac{x}{100}\right) = \Sigma \lg E(x) \lg\left(1 - \frac{x}{100}\right) - 1{\cdot}5\, \Sigma \lg^2\left(1 - \frac{x}{100}\right),$$

$$\lg E(0)\, \Sigma \frac{x}{100 - x} + \nu \Sigma \frac{x}{100 - x} \lg\left(1 - \frac{x}{100}\right) + \delta\, 0{\cdot}015\, \Sigma \left(\frac{x}{100 - x}\right)^2 = \Sigma \lg E(x) \frac{x}{100 - x} - 1{\cdot}5\, \Sigma \lg\left(1 - \frac{x}{100}\right) \frac{x}{100 - x}.$$

Aus diesen drei Gleichungen erhält man nach vorheriger Berechnung der Summen die drei Unbekannten $\lg E(0)$, ν, δ und hieraus $E(0)$, n, und ω. Die so berechneten Werte von n und $E(0)$ werden von den unter der Voraussetzung $\omega = 100$ berechneten natürlich abweichen. Die Anpassung wird man im allgemeinen nur mithilfe des gut passenden Verlaufs der $E(x)$ Kurve vornehmen, da dieser auch versicherungstechnisch der einzig interessante Bereich ist.

Eine obere Grenze für n bekommt man aus der Bedingung, dass $l(x)$ eine monoton abnehmende Funktion von x sein muss. Also dass $l'(x) < 0$. Nun ist

$$l'(x) = \frac{l(x)}{(\omega - x)} (n + \xi^{n-1} (n-1) \lg D).$$

Damit dies negativ, muss $\quad n < \xi^{n-1} \dfrac{\omega}{E(0)}$,

nach der obigen Definition von $\lg D$. Nun ist der rechte Ausdruck am kleinsten für $\xi = 1$, d. h. $x = 0$. Daher gilt diese Ungleichung sicher, wenn sie für $x = 0$ gilt. Wenn man also $l'(0) = 0$ noch zulässt, so gilt

$$n \leqq \frac{\omega}{E(0)}.$$

Dies ist eine zweite Bedingung für die Anwendbarkeit der Formel. Demnach gilt für n im allgemeinen Fall stets die Lagenbeziehung

$$1 < n \leqq \frac{\omega}{E(0)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9).$$

Da $E(0)$ für alle heutigen Sterbetafeln grösser als 40 Jahre ist, gilt $1 < n \leqq 2{\cdot}5$. Dies sind jedoch nur extreme Grenzen. Tatsächlich pflegen die n bei den heutigen Sterbetafeln etwa zwischen 1·5 und 1·8 zu liegen. Entsprechend gilt

$$0 < D \leqq e^{-\frac{\omega}{\omega - E(0)}}.$$

Nun gilt erfahrungsgemäss $40 < E(0) < 60$. Hieraus hat man eine untere Grenze für die obere Grenze von D, nämlich $e^{-2{\cdot}5}$. Hieraus ergibt sich endlich

$$0 < D < 0{\cdot}0821 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10).$$

Das mittlere Alter der über dem Alter x Lebenden $\bar{x}_x$ ist zu definieren als

$$\bar{x}_x = \frac{\int_x^\omega x\, l(x)\, dx}{\int_x^\omega l(x)\, dx} \quad \text{.....(11).}$$

Führt man den Mittelwert der Lebenserwartungen der über x Jahre Alten $\overline{E(x)}$ ein, so ist

$$\overline{E(x)} = \frac{\int_x^\omega l(x)\, E(x)\, dx}{\int_x^\omega l(x)\, dx} \quad \text{.....(12).}$$

Dabei gilt stets die Beziehung*

$$\bar{x}_x = x + \overline{E(x)} \quad \text{.....(13).}$$

Daher wird nach (7) und (8)

$$\bar{x}_x = x + e^{\frac{\omega}{E(0)(n-1)}\left(\frac{\omega}{\omega-x}\right)^{n-1}} \int_x^\omega e^{-\frac{\omega}{E(0)(n-1)}\left(\frac{\omega}{\omega-x}\right)^{n-1}} dx.$$

Dies lässt sich zwar nicht ausintegrieren, wol aber auf eine tabellierte Transzendente reduzieren. Setzt man

$$\frac{\omega}{E(0)(n-1)}\left(\frac{\omega}{\omega-x}\right)^{n-1} = u,$$

so wird

$$dx = \frac{1}{n-1}\left(\frac{\omega^n}{E(0)(n-1)}\right)^{\frac{1}{n-1}} u^{-1-\frac{1}{n-1}}\, du.$$

Daher wird

$$\bar{x}_x = x + e^{\frac{\omega}{E(0)(n-1)}\left(\frac{\omega}{\omega-x}\right)^{n-1}} \left(\frac{\omega^n}{E(0)(n-1)^n}\right)^{\frac{1}{n-1}} \int_{\frac{\omega}{E(0)(n-1)}\left(\frac{\omega}{\omega-x}\right)^{n-1}}^{\infty} e^{-u}\, u^{-1-\frac{1}{n-1}}\, du \quad \text{.....(14).}$$

Dieses sogenannte unvollständige Gammaintegral ist bei Pearson, *Tables of the incomplete Gammafunction* tabelliert. Im Spezialfall $n = 2$ vereinfacht sich dieser Ausdruck zu

$$\bar{x}_x = x + e^{\frac{\omega^2}{E(0)(\omega-x)}} \frac{\omega^2}{E(0)} \int_{\frac{\omega^2}{E(0)(\omega-x)}}^{\infty} e^{-u}\, d\left(-\frac{1}{u}\right),$$

was durch partielle Integration und Einführung des Exponentialintegrals

$$Ei(-x) = \int_\infty^x \frac{dx}{x}\, e^{-x},$$

zu

$$\bar{x}_x = \omega + e^{\frac{\omega^2}{E(0)(\omega-x)}} \frac{\omega^2}{E(0)}\, Ei\left(-\frac{\omega^2}{E(0)(\omega-x)}\right) \quad \text{.....(14d)}$$

* Vergl. "Eine allgemeine Eigenschaft der Sterbetafel." *Zeitschrift für die gesamte Versicherungswissenschaft*, Nr 4, 1924.

wird. Diese Transzendente Ei ist in Laskas Formelsammlung tabelliert. Z. B. ergibt sich für $\omega = 100$ und $E(0) = 50$ das mittlere Alter des Gesamtbevölkerung gleich 28·6. Wie es sein muss, wächst $\bar{x}_x$ mit x, weil das Exponentialintegral hier negativ ist und absolut genommen stärker fällt, als die Exponentialfunktion wächst. Für $x = \omega$ wird $Ei(-\infty) = 0$ sodass $\bar{x}_\omega = \omega$.

In den Entartungsfällen lässt sich die Integration ausführen.

(1) So ist im Fall $n = 0$

$$\bar{x}_x = x + e^{\frac{x}{E(0)}} \int_x^\infty e^{-\frac{x}{E(0)}} dx = x + E(0) \quad \ldots\ldots\ldots\ldots\ldots\ldots (14a).$$

Das mittlere Alter der über x jährigen ist also hier stets grösser als die Lebenserwartung eines x jährigen. Dies gilt, wie mir Herr Professor Pearson mitgeteilt hat, ganz allgemein für die Gompertz-Makehamsche Formel, für die ja der Fall $n = 0$ einen Spezialfall darstellt. Das mittlere Alter der Gesamtbevölkerung ist hiebei gleich $E(0)$.

(2) Im Fall $n = 1$ ist

$$\bar{x}_x = x - \frac{\omega \int_x^\omega \left(1 - \frac{x}{\omega}\right)^{\frac{\omega}{E(0)}} d\left(1 - \frac{x}{\omega}\right)}{\left(1 - \frac{x}{\omega}\right)^{\frac{\omega}{E(0)}}} = x + \frac{E(0)(\omega - x)}{\omega + E(0)}$$

$$= \frac{\omega(x + E(0))}{\omega + E(0)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14b),$$

und im de Moivreschen Fall

$$\bar{x}_x = \tfrac{1}{3}(2x + \omega).$$

Das mittlere Alter der Gesamtbevölkerung ist $\bar{x} = \dfrac{\omega E(0)}{\omega + E(0)}$, bezw. $\dfrac{\omega}{3}$. Vergleicht man das mittlere Alter der über x jährigen mit der Lebenserwartung der x jährigen also $\dfrac{\omega E(0) + \omega x}{\omega + E(0)}$ mit $\dfrac{\omega E(0) - x E(0)}{\omega}$ oder x mit $\dfrac{\omega E(0)^2}{\omega^2 + \omega E(0) + E(0)^2}$, so erhält man: Die Lebenserwartung eines x jährigen ist grösser, gleich oder kleiner als das mittlere Alter der über x jährigen, je nachdem x kleiner, gleich oder grösser ist als $\dfrac{\omega E(0)^2}{\omega^2 + \omega E(0) + E(0)^2}$ also z. B. im de Moivreschen Fall $\dfrac{\omega}{7}$. Der Satz wonach für alle Absterbeordnungen die Lebenserwartung eines Neugeborenen grösser ist als das mittlere Alter der Lebenden lässt sich also nicht auf alle Alter erweitern.

(3) Im Fall $n = \dfrac{\omega}{E(0)}$ und $\omega = \infty$ bekommt man wieder das Exponentialintegral. Denn hier wird

$$\bar{x}_x = x + e^{e^{\frac{x}{E(0)}}} \int_x^\infty e^{-e^{\frac{x}{E(0)}}} dx = x + E(0)\, e^{e^{\frac{x}{E(0)}}} \int_{e^{\frac{x}{E(0)}}}^\infty e^{-v} \frac{dv}{v}$$

$$= x - E(0)\, e^{e^{\frac{x}{E(0)}}} Ei\left(-e^{\frac{x}{E(0)}}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14c),$$

wenn $$e^{\frac{x}{E(0)}} = v$$

gesetzt und daher $$\frac{1}{E(0)}dx = \frac{dv}{v}$$

wird. Z. B. wird für $x = 0$ das mittlere Alter der Gesamtbevölkerung

$$\bar{x} = -E(0) \,.\, e \,.\, Ei(-1) = 0{\cdot}594\, E(0).$$

Es fragt sich, wie sich zwei Absterbeordnungen mit gleichem ω und $E(0)$, aber verschiedenen n unterscheiden. Um den Einfluss einer Veränderung der n bei Konstanten $E(0)$ und ω zu untersuchen bilde man

$$\frac{d\,l(x)}{dn} = l(x)\left(\lg \xi - \frac{\omega}{E(0)(n-1)}\left[\frac{1-\xi^{n-1}}{n-1} + \xi^{n-1}\lg \xi\right]\right).$$

Über das Vorzeichen von $\frac{d\,l(x)}{dn}$ entscheidet also das Vorzeichen des Klammerausdrucks. Danach ist $\frac{d\,l(x)}{dn} \gtreqless 0$, je nachdem

$$\lg \xi + \frac{\omega}{E(0)}\frac{\xi^{n-1}}{(n-1)^2} \gtreqless \frac{\omega}{E(0)}\left\{\frac{1}{(n-1)^2} + \frac{\xi^{n-1}\lg \text{nat}\, \xi}{n-1}\right\} \quad \ldots\ldots\ldots(15).$$

Dies hängt, wie man sieht, von x ab. Solange das Grösserzeichen gilt, wächst l mit n, d. h. grösseren n entsprechen auch grössere l, d. h. ein günstiger Verlauf der Absterbeordnung; umgekehrt bedeutet das Kleinerzeichen einen ungünstigeren Verlauf der Absterbeordnung. Man sieht ohne weiteres, dass $\frac{d\,l(x)}{dn}$ im Verlauf der x von 0 bis ω mindestens einmal das Zeichen wechseln muss, da ja sonst die Bedingung $E(0) = \text{konst.}$ nicht erfüllt wäre. Um den tatsächlichen Verlauf zu illustrieren, sei hier ein schematisches Beispiel für $\frac{\omega}{E(0)} = 2$ und $n = 1{\cdot}5$ angeführt. Dann interessiert, ob

$$\lg \text{nat}\, \xi + 8\sqrt{\xi} \gtreqless 8 + 4\sqrt{\xi}\lg \text{nat}\, \xi.$$

Bis $x = 50$ ist stets die linke, als $x = 55$ stets die rechte Seite grösser. Die Differenz beträgt bei $x = 50$ 0·07, bei $x = 55$ −0·04.

Demnach liegt eine Wurzel der Gleichung zwischen $x = 50$ und $x = 55$, also etwa bis $x = 53$. In dem gewählten Beispiel wird also wenn n wächst, $l(x)$ wachsen oder fallen je nachdem $x \gtrless 53$. Eine Vergrösserung der n bewirkt also eine Verminderung der Sterblichkeit bei den jüngeren und eine Erhöhung bei den älteren Klassen.

Die Figur (3) zeigt den Einfluss der n auf $l(x)$ bei konstantem $E(0)$ und ω; es ist dabei der Einfachheit halber $\omega = 100$, $E(0) = 50$ angenommen. Die Zeichnung zeigt die $l(x)$ Kurve für $n = 0$; 1; 1·5; 2 und ∞. Man sieht sehr deutlich, wie bei zwei Sterbetafeln mit gleichem $E(0)$ und ω eine Vergrösserung des n, solange es endlich bleibt eine Verringerung der Sterblichkeit der Jugendlichen bedeutet. Solange n endlich bleibt bewirkt eine Vergrösserung des n eine Verringerung des mittleren Alters der Gesamtbevölkerung.

Ergibt sich bei der Berechnung von n ein Betrag, grösser als $\frac{\omega}{E(0)}$, so würde dies nichts schaden, solange man sich auf den zur Anpassung verwendeten Bereich der x beschränkt. Darüber hinaus ist dieser Wert jedoch nicht zu verwenden, weil hier $l(x)$ eine kurze Zeit lang wachsen würde. Um dies zu vermeiden, wird man die Rechnung wiederholen, jedoch noch ein kleineres x berücksichtigen, weil eine höhere Kindersterblichkeit das n herabsetzt.

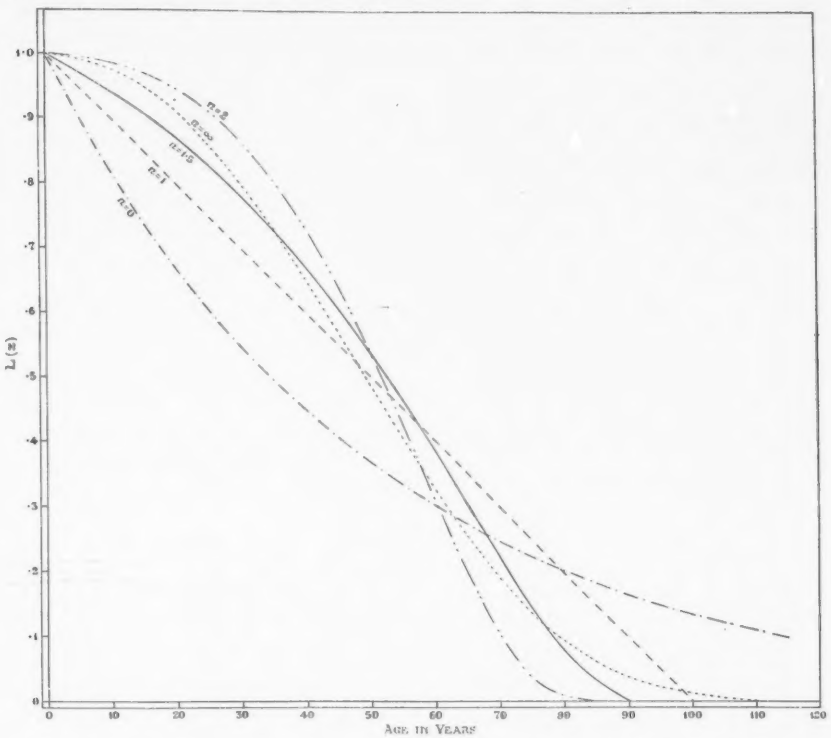


Fig. 3.

Zwei Sterbetafeln mit gleichem $E(0)$ können je nach der Grösse der n verglichen werden. Sind jedoch die beiden $E(0)$ verschieden, so kann diese Vergleichsart nicht durchgeführt werden, da sich die Kurven nicht auf gemeinsame $E(0)$ zurückführen lassen.

Den Einfluss einer Veränderung von ω zu untersuchen, lohnt sich nicht, da diese Konstante im wesentlichen bei allen Tafeln dieselbe ist und Beobachtungen des höchsten Alters infolge der Kleinheit der Zahlen unzuverlässig sind. Der Einfluss des $E(0)$ bei konstantem n und ω ist einfach: wenn $E(0)$ wächst, wachsen sämtliche l. Je grösser $E(0)$, desto günstiger ist der Verlauf der Sterbetafel, was ja selbstverständlich ist.

Aus den Formeln (8) ergibt sich übrigens auch eine Methode zur Konstruktion von Sterbetafeln. Man gehe aus von der Identität

$$\frac{\int_x^\omega \mu(x)\, l(x)\, dx}{\int_x^\omega l(x)\, dx} = \frac{l(x)}{\int_x^\omega l(x)\, dx} = \frac{1}{E(x)}.$$

Hierin kann man $l(x)$ als die Zahl der über dem Alter x Sterbenden und das Integral als die Zahl der über dem Alter x Lebenden auffassen. Dann ist $\frac{1}{E(x)}$ gleich der Sterbeziffer $M(x)$ der über x Jahre Alten. Denn die Sterbeziffer $m(x)$ einer Altersstufe ist gleich dem Verhältnis der in dieser Altersstufe Sterbenden zu den in dieser Altersstufe Lebenden. Bezeichnet man die Zahl der in der wirklichen Bevölkerung zwischen dem Alter x und $x+1$ Lebenden mit $L(x+\frac{1}{2})$ so kann man aus der Kenntniss der einjährigen Sterbeziffern und des Altersaufbaus eine Sterbetafel, welche der Formel (8) angepasst ist, konstruieren. Denn

$$M(x) = \frac{\sum_x^\omega m(x)\, L(x+\frac{1}{2})}{\sum_x^\omega L(x+\frac{1}{2})} = \frac{1}{E(x)} = \left(\frac{\omega}{\omega - x}\right)^n \frac{1}{E(0)} \quad \ldots\ldots\ldots\ldots (16).$$

Hieraus kann man unter der Annahme $\omega = 100$ das $E(0)$ und n mithilfe der Methode der kleinsten Quadrate berechnen. Sie ergeben sich, wenn man diese Gleichung logarithmiert aus den beiden Gleichungen

$$-\Sigma \lg M(x) = N \lg E(0) + n \Sigma \lg \left(1 - \frac{x}{100}\right),$$

$$-\Sigma \lg M(x) \,.\, \lg \left(1 - \frac{x}{100}\right) = \lg E(0) \Sigma \lg \left(1 - \frac{x}{100}\right) + n \Sigma \lg^2 \left(1 - \frac{x}{100}\right),$$

worin N die Zahl der Beobachtungen von $M(x)$ bedeutet. Natürlich wird man dabei die verwendeten x auf die Intervalle $3 < x < 85$ beschränken.

Will man die Annahme $\omega = 100$ nicht von vorneherein machen, so wird man ω, $E(0)$ und n simultan mithilfe derselben Methode berechnen, die oben geschildert ist. Übrigens liesse sich diese Methode natürlich auch ohne Verwendung der hier angegebenen Formeln durchführen. Hat man nämlich mithilfe der $M(x)$ die $E(x)$ berechnet, so ergeben sich die $l(x)$ ganz allgemein aus (6), wobei natürlich das auftretende Integral graphisch oder mechanisch ausgewertet werden müsste. Doch empfiehlt sich dieser umständliche Weg nicht.

Um in ähnlicher Weise mithilfe der beobachteten $m(x)$ eine der Gompertz-Makehamschen Formel angepasste Tafel zu konstruieren hätte man die $m(x)$ in die $\mu(x)$ zu verwandeln. Macht man die Annahme, dass je 3 auf einander folgende Werte von $l(x)$ durch eine in x quadratische Gleichung wiedergegeben werden können, so gilt*

$$m(x) = \mu(x + \tfrac{1}{2}).$$

* Vergl. G. King, *Textbook of the Institute of Actuaries.* London, Layton, 1902, S. 26.

Aus diesen, so als beobachtet geltenden $\mu(x+\frac{1}{2})$ hätte man durch Ausgleichung die 3 Werte A, B, C und hieraus s und g zu berechnen, was allerdings nur nach vorhergehender Kenntniss von Näherungswerten möglich ist. Es mag noch interessieren, welche Auffassung über $E(x)$ sich aus der Gompertz-Makehamschen Formel ergibt. Danach ist

$$E(x) = \frac{1}{s^x g^{c^x}} \int_x^\infty s^x g^{c^x} dx.$$

Setzt man $$c^x \lg g = -u,$$

wobei zu beachten ist, dass $\lg g$ negativ ist, weil B positiv und $c > 1$. Dann wird

$$s^x = \left(\frac{u}{-\lg g}\right)^{\frac{\lg B}{\lg c}}$$

und $$c^x \lg c \, dx = \frac{du}{-\lg g},$$

woraus nach der Definition von u

$$dx = \frac{1}{\lg c} \frac{du}{u}.$$

Daher wird $$E(x) = \frac{1}{\lg c \, (-\lg g)^{\frac{\lg s}{\lg c}} s^x g^{c^x}} \int_{-c^x \lg g}^\infty u^{-1+\frac{\lg s}{\lg c}} e^{-u} du.$$

Das hier auftretende Integral lässt sich mithilfe der von Pearson herausgegebenen *Tables of the incomplete Gammafunction* als Differenz eines vollständigen und eines unvollständigen Gammaintegrals auswerten und man erhält, wenn man die untere Grenze als zweites Argument einführt,

$$E(x) = \frac{\Gamma\left(\frac{\lg s}{\lg c}\right) - \Gamma\left(\frac{\lg s}{\lg c}; -c^x \lg g\right)}{s^x g^{c^x} \lg c \, (-\lg g)^{\frac{\lg s}{\lg c}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7d)$$

was im Fall (3) wieder (7c) ergibt. Dieser, wie man sieht, ziemlich komplizierte Ausdruck für $E(x)$ liegt der Gompertz-Makehamschen Annahme zu zugrunde.

Zusammenfassung. Im obigen wird eine, im allgemeinen für die Jahre 3 bis 85 gut passende einfache Annahme (7) für die Lebenserwartung $E(x)$ gemacht und hieraus die Zahl der Überlebenden des Alters x, $l(x)$, abgeleitet (8). Als Entartungsfälle unserer Annahme ergeben sich die konstante (7a), die lineare (7b) und die exponentiell mit dem Alter abnehmende (7c) Lebenserwartung und entsprechend ein exponentieller (8a), linearer (8b), bezw. rationaler Ausdruck und ein Spezialfall des Gompertz-Makehamschen Ausdrucks für die Überlebenden (8c). Im allgemeinen treten 3 Konstanten auf, das höchste erreichbare Alter ω, die Lebenserwartung eines Neugeborenen $E(0)$ und die dimensionslose Zahl n. Allerdings ist das hier auftretende $E(0)$ nicht einfach das beobachtete sondern ein virtuelles, grösseres, das sich einstellen würde, falls die Ursache des bei kleinen Altern eintretenden Anwachsens von $E(x)$, die Säuglingssterblichkeit nicht existieren würde. Aus der Grenzbedingung $l(\omega) = 0$ und der Forderung, dass $l(x)$ stets abnimmt, ergeben sich die im allgemeinen Fall gültigen Grenzen $1 < n < 2{\cdot}5$ (9). Das n ist umso

grösser, je geringer die Sterblichkeit der jüngeren Klassen (15). Im allgemeinen wird man nur n und ω zu bestimmen haben, doch lassen sich auch alle drei Konstanten simultan berechnen. Das mittlere Alter der über dem Alter x Lebenden führt auf die unvollständige Gammafunktion (14), bezw. das Exponentialintegral (14d) und (14c). In zwei Entartungsfällen lässt es sich jedoch explizit angeben (14a) and (14b). Die Formel erlaubt eine einfache Darstellung der Sterbetafel mithilfe der einjährigen Sterbeziffern und des Altersaufbaus (16). Zum Schluss wird gezeigt, dass die Gompertz-Makehamsche Formel einen ausserordentlich komplizierten Verlauf der $E(x)$ voraussetzt (7d).

Gegenüber der Gompertz-Makehamschen Formel haben die hier vorgeschlagenen Ausdrücke (8), (8a), (8b), (8c) folgende Vorzüge:

(1) Die Bestimmung der auftretenden Konstanten und die Ausgleichung ist ausserordentlich einfach und kann sogar grafisch vorgenommen werden. Man braucht dann nur die in der Sterbetafel angegebenen Werte für E für die gutpassende Strecke von 3 bis 85 Jahren durch eine Gerade wiederzugeben.

(2) Die zugrundeliegende Annahme ist einfacher als die übliche.

(3) Die auftretenden Konstanten ω, $E(0)$ und n haben im Gegensatz zu den Gompertz-Makehamschen eine statistische Bedeutung. Durch ihre Angabe wird der Verlauf von $l(x)$ charakterisiert während dies durch die Angabe der bedeutungslosen Konstanten s, g, c, nicht möglich ist.

(4) Während die Gompertz-Makehamsche Formel im allgemeinen nur zwischen 15 und 65 Jahren gilt dürfte die vorgeschlagene Formel streng für den ganzen Bereich zwischen 3 und 85 Jahren gelten und die Fehler in den beiden andern Bereichen sind geringer als bei Gompertz-Makeham.

NOTE. *To prove that the Expectation of Life at any Age is greater than the Mean Age of all Persons living greater than this Age**.

BY KARL PEARSON, F.R.S.

Let $l_t dt$ be the number living between ages t and $t+dt$ out of l_0 born in a year. Then:

Expectation of Life at age x = mean age at Death of those dying after age x, $= E(x)$, say, or

$$E(x) = \frac{\int_x^w t \frac{dl_t}{dt} dt}{\int_x^w \frac{dl_t}{dt} dt}.$$

Mean age of those living older than x

$$= M(x) = \frac{\int_x^w t\, l_t dt}{\int_x^w l_t dt}.$$

Here w denotes the maximum age of survival.

We have to show that $E(x) > M(x)$.

Now it has been long known that Makeham's curve

$$l_t = k\, s^t g^{c^t}$$

gives a very excellent fit to the curve of survivors. Here k, s, g and c are constants and c is always greater than unity, being of the order 1·09. Consider:

$$\frac{\int_x^w t\, l_t c^t dt}{\int_x^w l_t c^t dt} \text{ and } \frac{\int_x^w t\, l_t dt}{\int_x^w l_t dt}.$$

The former expression must be numerically greater than the latter because it expresses the distance of the mean—from $t=x$ to $t=w$—of a portion of the curve

$$y = l_t c^t,$$

while the latter expresses the distance of the mean of the curve

$$z = l_t$$

for the same range, $t=x$ to $t=w$.

But the first curve is the second curve with each of its ordinates multiplied by the factor c^t. This factor is always greater than unity, *and increases as t increases.* The mean of the first curve is therefore further away from the origin. Accordingly:

$$\frac{\int_x^w t\, l_t c^t dt}{\int_x^w l_t c^t dt} \text{ is } > \frac{\int_x^w t\, l_t dt}{\int_x^w l_t dt},$$

* A paper of Dr Gumbel's suggested to me the above proof based on the Makeham curve.

or multiplying by the constant $\log g \log c$

$$\int_x^w t\, l_t c^t \log g \log c\, dt \times \int_x^w l_t dt$$

is *numerically** $$> \int_x^w l_t c^t \log g \log c\, dt \times \int_x^w t\, l_t dt.$$

But $$\int_x^w t\, l_t \log s\, dt \times \int_x^w l_t dt = \int_x^w l_t \log s\, dt \times \int_x^w t\, l_t dt,$$

since $\log s$ is a constant. Adding this to the above inequality, we have

$$\int_x^w t\, l_t (\log s + \log g \log c c^t)\, dt \times \int_x^w l_t dt > \int_x^w l_t (\log s + \log g \log c c^t) \times \int_x^w t\, l_t dt$$

or $$\frac{\int_x^w t\, l_t (\log s + \log g \log c c^t)\, dt}{\int_x^w l_t (\log s + \log g \log c c^t)\, dt} > \frac{\int_x^w t\, l_t dt}{\int_x^w l_t dt}.$$

But returning to Makeham's formula and taking logarithmic differentials, we have

$$\frac{1}{lt}\frac{dl_t}{dt} = \log s + \log g \log c c^t,$$

and on substitution $$\frac{\int_x^w t \frac{dl_t}{dt} dt}{\int_x^w \frac{dl_t}{dt} dt} > \frac{\int_x^w t\, l_t dt}{\int_x^w l_t dt}.$$

But this is $$E(x) > M(x)$$

or: the expectation of life at any age is greater than the mean age of persons living, of equal or greater age.

* Actually $\log g \times \log c$ and $\log s$ are negative.

ON THE RELATIONSHIP BETWEEN STRENGTH OF GRIP AND CERTAIN MENTAL AND SENSORY CHARACTERS.

By WILLIAM ARTHUR, M.A.

(*Embracing certain Tables prepared by Dr Karl Holzinger.*)

1. *Introductory.* In a paper published recently in *Biometrika** Dr Karl J. Holzinger gave the results of an enquiry into the degree of association of vital capacity, a *dynamic* as distinct from a *static* physical character, with each of the four psychical and sensory characters, reaction time to sight, reaction time to sound, keenness of vision and keenness of hearing, so far as the last can be measured by the pitch of the highest audible musical note. He found that vital capacity has a low but significant positive correlation with shortness of reaction times and with keenness of vision, while it is not significantly correlated with keenness of hearing.

The object of the present note is to investigate the relationship of a second dynamic physical character, strength of grip†, to the same group of mental characters.

2. *Data, Notation and Units.* The following discussion is based on the same data (namely those of Sir Francis Galton's Second Anthropometric Laboratory) as were employed by Koga and Morant in a paper "On the Degree of Association between Reaction Times in the case of Different Senses‡," and by Holzinger in the paper mentioned above. The measurements, which refer to males only, were taken by Galton in his second or South Kensington Museum Laboratory. The number of cases, 3373, is considered to be large enough to give reliable results. In order to avoid unnecessary repetition, the characters occurring, the symbols used to denote them and the units in which they are measured are detailed below.

Character	Symbol	Unit
Age	a	1 year
Strength of Grip	g	1 pound
Vital Capacity	b	1 cubic inch
Reaction Time to Sight	v	0·01 of a second
Reaction Time to Sound	h	0·01 of a second
Visual Acuity (as measured by the distance at which diamond type can be read by the right eye, unaided)	k	1 inch
Keenness of Hearing (as measured by the pitch of the highest audible musical note)	p	1000 vibrations per second

* p. 139 of this volume.

† It was Dr Holzinger's intention to consider strength of grip as a second dynamic physical character, but pressure of time prevented him during his visit to England from proceeding beyond the initial stages of the work. The writer is much indebted to Dr Holzinger for the preparation of Tables I to IV and Table XIX. To him also is due adequate recognition of the importance of using yearly intervals during adolescence and allowing for the hetero-scedasticity in correcting growth curves.

‡ *Biometrika*, Vol. xv. p. 346.

For purposes of comparison the associational constants have in all cases been evaluated separately for grip with the right and grip with the left hand. No confusion will arise from the use of the same symbol to denote strength of grip with either hand, as the hand concerned will be clearly indicated in the text.

3. *Reduction of Strength of Grip to Standard Age.* With the exception of strength of grip all the characters dealt with had been previously "corrected for age" by the authors of the papers cited, the necessity for this being caused by the extreme skewness of the regressions on age, the correlation coefficients being on that account inadequate as measures of the correlation, and the method of partial correlation coefficients consequently breaking down. To reduce the grip of each individual to standard age use was made of the formula $(g_s - \bar{g}_s)/\sigma_s = (g_x - \bar{g}_x)/\sigma_x$, where g_s, g_x denote the grip of any individual at standard age s and at any age x; $\bar{g}_s$, $\bar{g}_x$ the means of grip, σ_s, σ_x the standard deviations of grip at these ages. The process is equivalent to the transferring of each individual to the standard array of grip, and so placing him in that array that his grip-deviate (or deviation from the mean of the array measured in terms of the standard deviation) is unaltered; or it may be regarded as the superimposing of each array of grip on the array chosen as standard, making the means coincide, and then stretching or squeezing it until the standard deviations are the same*. The age to which the other characters had already been reduced, namely 40·5, was naturally chosen as standard for the correction of strength of grip.

Strength of Grip and Age. Before proceeding to apply the above formula to the correction of the two grip entries on each individual record card, it was necessary to obtain a series of smoothed values of the means and the standard deviations of grip for each year of age represented in the data. Tables I and II show the age-grip distributions, the grouping units being three years and five pounds. Table I provides the following constants:

Age	Right Hand Grip	Right Hand Grip and Age
Mean = 24·29 ± 0·13	Mean = 73·01 ± 0·22	$r_{ga} = 0{\cdot}3208 \pm 0{\cdot}0104$
S. D. = 10·95 ± 0·09	S. D. = 18·93 ± 0·16	$\eta_{g.a} = 0{\cdot}7390 \pm 0{\cdot}0053$
		$\eta_{a.g} = 0{\cdot}3888 \pm 0{\cdot}0099$

The extreme skewness of the regression of grip on age and the inadequacy of r as a measure of the correlation are at once evident. The skewness of the regression is demonstrated graphically in Diagram 1.

From Table II we obtain:

Left Hand Grip	Left Hand Grip and Age
Mean = 68·71 ± 0·21	$r_{ga} = 0{\cdot}3089 \pm 0{\cdot}0105$
S. D. = 18·45 ± 0·15	$\eta_{g.a} = 0{\cdot}7233 \pm 0{\cdot}0055$
	$\eta_{a.g} = 0{\cdot}3782 \pm 0{\cdot}0100$

the correlation constants being of the same order as in the case of right hand grip.

The regression curves of grip on age (Diagram 1) are of the same general type as the growth curves of mean vital capacity and other dynamic physical characters.

* [It may also be expressed by saying that the individual on the average maintains his rank with growth. Ed.]

TABLE I.

Right Hand Grip and Age.

Right Hand Grip in Pounds; Central Values.

Central Ages, in Years.

	17	22	27	32	37	42	47	52	57	62	67	72	77	82	87	92	97	102	107	112	117	122	Totals
7·5	9	9	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	21
10·5	2	21	26	30	26	4	1	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	112
13·5	1	5	26	44	59	40	32	28	23	14	12	3	—	1	—	1	1	—	—	—	—	—	290
16·5	—	—	1	4	14	19	23	27	49	67	71	54	45	55	25	20	16	1	3	—	1	—	495
19·5	—	—	1	1	—	1	4	11	16	51	100	92	109	138	107	81	47	24	9	4	2	1	799
22·5	—	—	—	—	1	1	2	3	16	25	55	51	71	77	49	50	31	15	11	5	1	1	465
25·5	—	—	—	—	1	1	—	2	3	21	16	51	45	42	31	35	24	12	4	4	—	1	293
28·5	—	—	—	—	—	—	1	2	6	9	12	25	31	42	32	19	17	6	5	4	—	1	212
31·5	—	—	—	—	—	—	—	1	5	8	7	13	23	28	16	15	7	7	—	2	—	—	132
34·5	—	—	—	—	—	—	—	—	1	8	14	9	19	18	16	12	5	4	1	5	—	—	112
37·5	—	—	—	—	—	—	—	—	—	5	6	8	16	19	8	11	5	5	1	1	—	—	85
40·5	—	—	—	—	—	—	—	1	1	4	5	9	9	14	13	7	4	2	1	3	—	—	73
43·5	—	—	—	—	—	—	—	1	1	3	4	9	8	13	7	6	3	2	1	—	—	—	58
46·5	—	—	—	—	—	—	—	2	4	6	2	7	11	10	7	4	3	3	1	—	1	—	61
49·5	—	—	—	—	—	—	1	1	2	2	5	5	7	8	5	5	1	1	1	—	—	—	44
52·5	—	—	—	1	—	—	—	—	1	6	3	1	4	2	4	3	—	—	—	—	—	—	25
55·5	—	—	—	—	—	—	—	3	2	6	2	2	2	1	1	1	—	1	—	—	—	—	21
58·5	—	—	—	—	—	—	—	—	1	2	5	3	5	4	4	1	—	—	—	—	—	—	25
61·5	—	—	—	—	—	—	—	1	2	2	2	4	2	—	—	—	—	—	—	—	—	—	13
64·5	—	—	—	—	—	—	—	—	3	2	4	4	3	1	1	1	—	—	—	—	—	—	19
67·5	—	—	—	—	—	—	—	—	3	1	—	—	—	—	—	—	1	—	—	—	—	—	5
70·5	—	—	—	—	—	—	—	—	—	1	—	2	2	—	—	—	—	—	—	—	—	—	5
73·5	—	—	—	—	—	—	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	3
76·5	—	—	—	—	—	—	1	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	3
79·5	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	2
Totals	12	35	57	80	101	66	65	86	140	244	328	352	412	473	327	272	165	83	38	28	5	4	3373

TABLE II.

Left Hand Grip and Age.

Left Hand Grip in Pounds; Central Values.

Central Ages, in Years.

	12	17	22	27	32	37	42	47	52	57	62	67	72	77	82	87	92	97	102	107	112	117	122	Totals
7·5	2	12	6	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	21
10·5	—	8	27	29	23	16	5	3	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	112
13·5	—	2	15	30	49	60	38	35	31	13	9	4	1	1	—	1	1	—	—	—	—	—	—	290
16·5	—	—	3	1	9	14	22	32	62	52	62	76	46	37	38	25	10	5	1	—	—	—	—	495
19·5	—	—	1	—	2	1	1	13	30	28	84	129	106	116	104	88	55	23	10	4	4	—	—	799
22·5	—	—	—	—	—	3	2	4	10	24	49	70	67	60	49	52	39	17	10	4	2	3	—	465
25·5	—	—	—	—	—	—	1	3	3	9	31	34	44	47	52	26	22	7	9	2	3	—	—	293
28·5	—	—	—	—	—	—	2	2	7	9	14	18	32	33	40	15	18	8	8	3	2	—	1	212
31·5	—	—	—	—	—	—	1	—	2	6	9	14	20	26	20	11	14	3	5	—	1	—	—	132
34·5	—	—	—	—	—	—	—	1	1	5	10	15	18	19	13	14	7	4	1	2	2	—	—	112
37·5	—	—	—	—	—	—	—	—	4	5	6	5	14	18	14	7	7	3	1	—	1	—	—	85
40·5	—	—	—	—	—	—	2	—	2	2	7	9	13	8	11	8	6	4	1	—	—	—	—	73
43·5	—	—	—	—	—	—	—	—	1	4	4	9	7	7	14	6	4	—	1	1	—	—	—	58
46·5	—	—	—	—	—	—	—	1	5	5	5	4	10	8	7	6	4	1	4	1	—	—	—	61
49·5	—	—	—	—	—	—	—	1	4	1	5	6	10	5	5	2	2	1	2	—	—	—	—	44
52·5	—	—	—	—	—	1	—	—	—	3	5	4	4	3	1	3	1	—	—	—	—	—	—	25
55·5	—	—	—	—	—	—	—	—	4	3	5	3	2	—	3	1	—	—	—	—	—	—	—	21
58·5	—	—	—	—	—	—	—	—	4	2	4	5	2	5	2	1	—	—	—	—	—	—	—	25
61·5	—	—	—	—	—	—	—	1	—	4	—	4	3	1	—	—	—	—	—	—	—	—	—	13
64·5	—	—	—	—	—	—	—	1	1	2	4	5	3	2	1	—	—	—	—	—	—	—	—	19
67·5	—	—	—	—	—	—	—	1	1	1	1	—	—	—	1	—	—	—	—	—	—	—	—	5
70·5	—	—	—	—	—	—	1	—	—	—	—	3	—	1	—	—	—	—	—	—	—	—	—	5
73·5	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	3
76·5	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	3
79·5	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	2	22	52	61	83	96	76	99	173	179	317	417	402	397	375	267	190	76	53	17	15	3	1	3373

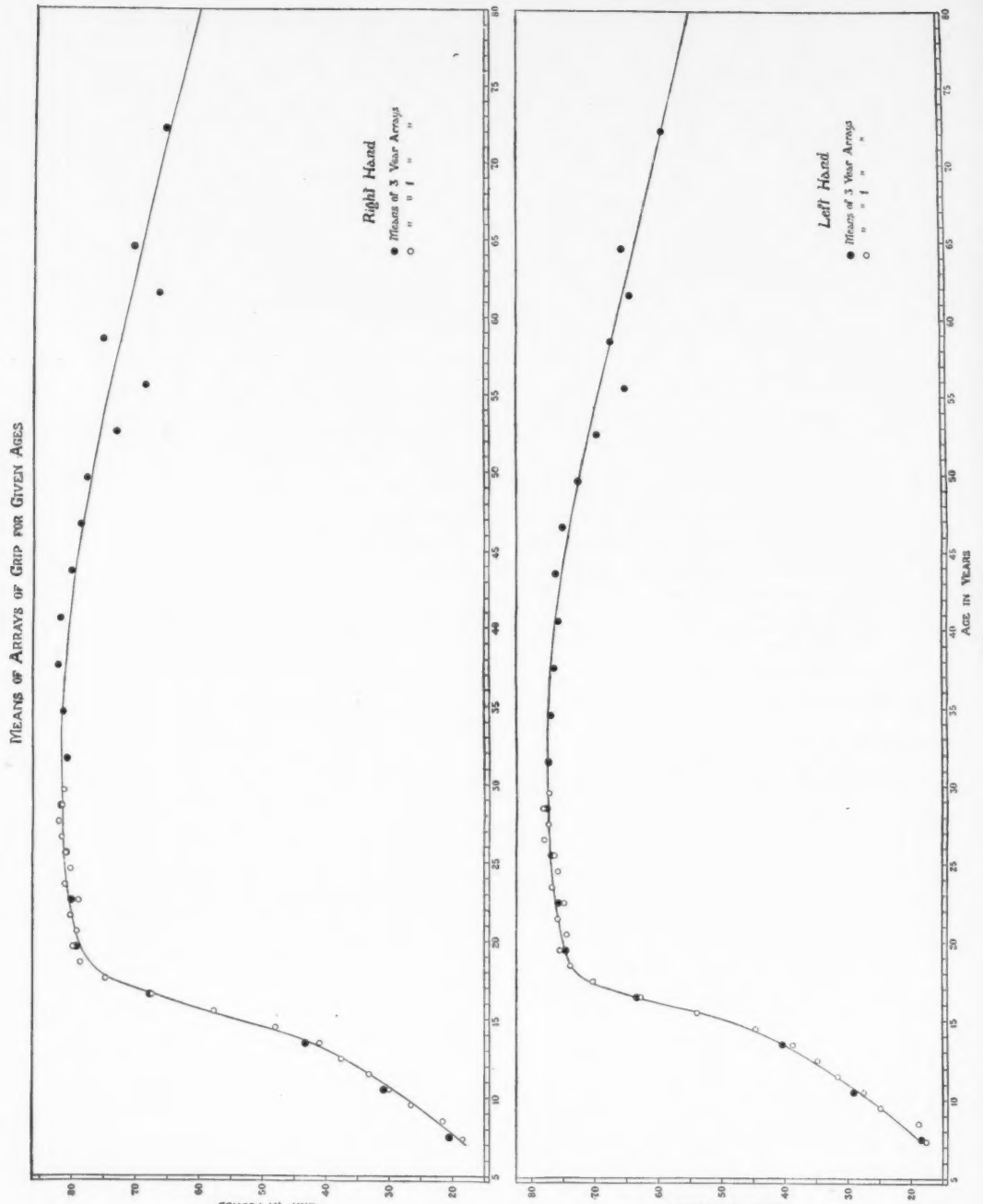

Means of Arrays of Grip for Given Ages
Right Hand
Means of 3 Year Arrays
" " 1 " "
Left Hand
Means of 3 Year Arrays
" " 1 " "
Grip in Pounds
Age in Years

Diagram 1.

Central Ages, in Years.

6·5	
7·5	
8·5	
9·5	
10·5	
11·5	
12·5	
13·5	
14·5	
15·5	
16·5	
17·5	
18·5	
19·5	
20·5	
21·5	
22·5	
23·5	
24·5	
25·5	
26·5	
27·5	
28·5	
29·5	
Totals	

Central Ages, in Years.

6·5	
7·5	
8·5	
9·5	
10·5	
11·5	
12·5	
13·5	
14·5	
15·5	
16·5	
17·5	
18·5	
19·5	
20·5	
21·5	
22·5	
23·5	
24·5	
25·5	
26·5	
27·5	
28·5	
29·5	
Totals	

TABLE III.

Right Hand Grip and Age, up to Age 30.

Right Hand Grip in Pounds; Central Values.

Central Ages, in Years.

	17	22	27	32	37	42	47	52	57	62	67	72	77	82	87	92	97	102	107	112	117	122	Totals
6·5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
7·5	4	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
8·5	4	7	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	14
9·5	1	9	8	7	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	26
10·5	—	8	10	6	3	2	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	30
11·5	1	4	8	17	22	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	56
12·5	—	1	10	17	16	7	6	3	1	2	1	—	—	—	—	—	—	—	—	—	—	—	64
13·5	1	2	13	13	25	16	16	5	6	2	1	1	—	1	—	—	1	—	—	—	—	—	103
14·5	—	2	3	14	18	17	10	20	16	10	10	2	—	—	—	1	—	—	—	—	—	—	123
15·5	—	—	1	2	13	10	12	15	20	17	19	10	5	9	1	—	—	—	—	—	—	—	134
16·5	—	—	—	1	1	8	8	9	16	25	21	20	13	19	6	5	6	—	—	—	—	—	158
17·5	—	—	—	1	—	1	3	3	13	25	31	24	27	27	18	15	10	1	3	—	1	—	203
18·5	—	—	1	—	—	1	2	5	5	15	33	31	44	50	35	26	18	5	1	1	—	1	274
19·5	—	—	—	—	—	—	—	2	4	17	47	31	34	48	42	32	15	10	5	2	—	—	289
20·5	—	—	—	1	—	—	2	4	7	19	20	30	31	40	30	23	14	9	3	1	2	—	236
21·5	—	—	—	—	—	1	2	—	2	10	22	20	26	28	24	16	11	5	6	2	—	—	175
22·5	—	—	—	—	1	—	—	3	9	10	16	19	27	28	13	14	8	5	4	2	1	1	161
23·5	—	—	—	—	—	—	—	1	5	5	17	12	18	21	12	20	12	5	1	1	—	—	129
24·5	—	—	—	—	1	—	—	1	—	8	6	17	17	15	8	10	9	4	—	2	—	—	98
25·5	—	—	—	—	—	1	—	—	1	6	4	18	13	9	9	11	4	3	3	1	—	1	84
26·5	—	—	—	—	—	—	—	1	2	7	6	16	15	18	14	14	11	5	1	1	—	—	111
27·5	—	—	—	—	—	—	—	2	1	5	6	15	10	13	13	10	6	2	3	3	—	1	90
28·5	—	—	—	—	—	—	—	—	1	1	5	8	12	15	12	5	6	—	2	—	—	—	67
29·5	—	—	—	—	—	—	1	—	4	3	1	2	9	14	6	4	5	4	—	1	—	—	54
Totals	12	35	57	79	101	66	63	74	113	187	267	276	301	355	243	206	136	58	32	17	4	4	2686

TABLE IV.

Left Hand Grip and Age, up to Age 30.

Left Hand Grip in Pounds; Central Values.

Central Ages, in Years.

	12	17	22	27	32	37	42	47	52	57	62	67	72	77	82	87	92	97	102	107	112	117	122	Totals
6·5	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
7·5	—	5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
8·5	2	6	5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	14
9·5	—	5	8	8	4	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	26
10·5	—	3	9	9	5	3	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	30
11·5	—	—	10	12	14	13	5	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	56
12·5	—	1	7	12	15	10	8	6	4	1	—	—	—	—	—	—	—	—	—	—	—	—	—	64
13·5	—	1	5	14	17	27	13	11	10	1	1	—	1	1	—	—	1	—	—	—	—	—	—	103
14·5	—	—	3	4	17	23	17	18	17	11	8	4	—	—	—	1	—	—	—	—	—	—	—	123
15·5	—	—	1	1	8	9	13	18	22	16	16	16	8	2	2	2	—	—	—	—	—	—	—	134
16·5	—	—	1	—	1	4	8	9	23	22	18	25	18	11	8	4	4	1	1	—	—	—	—	158
17·5	—	—	1	—	—	1	1	5	17	14	28	35	20	24	28	19	6	4	—	—	—	—	—	203
18·5	—	—	1	—	1	1	—	7	7	10	27	51	36	43	37	23	17	8	1	—	4	—	—	274
19·5	—	—	—	—	—	—	—	3	12	11	31	41	38	41	32	38	28	8	5	1	—	—	—	289
20·5	—	—	—	—	1	—	1	3	11	7	26	37	32	32	35	27	10	7	4	3	—	—	—	236
21·5	—	—	—	—	—	1	2	1	3	9	20	24	26	22	18	19	14	8	4	3	1	—	—	175
22·5	—	—	—	—	—	2	—	2	6	9	15	27	19	22	21	17	10	6	2	—	—	3	—	161
23·5	—	—	—	—	—	—	1	1	6	14	19	22	16	10	16	15	3	4	1	1	—	—	—	129
24·5	—	—	—	—	—	—	1	—	1	5	13	9	16	14	17	10	8	—	2	1	1	—	—	98
25·5	—	—	—	—	—	—	—	1	1	3	8	11	12	15	18	5	3	3	2	1	1	—	—	84
26·5	—	—	—	—	—	—	—	2	1	2	9	13	17	18	17	11	11	4	5	—	1	—	—	111
27·5	—	—	—	—	—	—	1	1	3	3	9	9	14	9	16	8	7	2	4	2	2	—	—	90
28·5	—	—	—	—	—	—	—	—	1	3	2	5	12	15	14	5	5	3	1	1	—	—	—	67
29·5	—	—	—	—	—	—	1	1	3	3	3	4	6	9	9	2	6	3	3	—	—	—	1	54
Totals	2	22	52	61	83	94	72	91	143	136	249	330	297	294	282	207	145	60	38	13	11	3	1	2686

Ages in Years
Diagram 1.

They exhibit a distinct concavity about the period of adolescence; that is, a speeding up of the *rate* of growth at this stage. The rate of growth increases from early childhood onwards, at first slowly then more rapidly, until it reaches a maximum about age 16, and thereafter falls off rapidly as the maximum strength of grip is approached. Efforts to find an algebraic or logarithmic curve to give a satisfactory graduation of the means were unavailing, those curves which gave a good fit for the rest of the system failing entirely to supply the necessary concavity before age 16. Recourse was ultimately had to a spline for the early parts of the curves, the remainder being fitted with cubics by means of least squares. Tables III and IV, showing the age-grip distributions for intervals of one year up to age 30, provided the mean grip for each year up to this age and so furnished a more reliable guide for the splining process. In Diagram 1 single-year means are indicated by clear circles, three-year means by dark.

It is noteworthy that the maximum mean strength of grip occurs in the case of either hand about age 31, much later in life than the maximum mean vital capacity, which was found by Dr Holzinger to occur about age 25, and later than the maxima of other physical characters. It is possible that this may be to some extent due to the element of experience. The acquiring of skill in using muscular power to the best advantage might account for the slight increase (of a little less

TABLE V.

Graduated Means of Arrays of Right Hand and Left Hand Grips for given Ages.

Central Age	Mean Grip Right	Mean Grip Left	Central Age	Mean Grip Right	Mean Grip Left	Central Age	Mean Grip Right	Mean Grip Left
6·5	16·20	15·90	*31·5*	81·93	77·60	*56·5*	73·70	68·46
7·5	19·40	18·70	*32·5*	81·91	77·56	*57·5*	73·15	67·87
8·5	22·75	21·60	*33·5*	81·84	77·49	*58·5*	72·60	67·28
9·5	26·05	24·80	*34·5*	81·75	77·38	*59·5*	72·05	66·69
10·5	29·55	28·25	*35·5*	81·63	77·23	*60·5*	71·48	66·09
11·5	33·30	31·85	*36·5*	81·47	77·06	*61·5*	70·91	65·40
12·5	37·45	35·80	*37·5*	81·29	76·85	*62·5*	70·32	64·89
13·5	42·35	40·25	*38·5*	81·08	76·61	*63·5*	69·77	64·29
14·5	48·70	45·75	*39·5*	80·84	76·34	*64·5*	69·20	63·69
15·5	58·00	53·65	*40·5*	80·58	76·04	*65·5*	68·62	63·10
16·5	67·90	64·85	*41·5*	80·29	75·72	*66·5*	68·05	62·51
17·5	74·30	70·05	*42·5*	79·98	75·36	*67·5*	67·48	61·92
18·5	77·50	72·95	*43·5*	79·64	74·99	*68·5*	66·91	61·35
19·5	78·75	74·50	*44·5*	79·29	74·59	*69·5*	66·34	60·78
20·5	79·55	75·40	*45·5*	78·91	74·17	*70·5*	65·78	60·22
21·5	80·10	75·95	*46·5*	78·51	73·73	*71·5*	65·23	59·67
22·5	80·55	76·25	*47·5*	78·10	73·27	*72·5*	64·68	59·13
23·5	80·90	76·50	*48·5*	77·67	72·79	*73·5*	64·13	58·40
24·5	81·13	76·74	*49·5*	77·22	72·30	*74·5*	63·58	57·80
25·5	81·36	76·97	*50·5*	76·75	71·78	*75·5*	63·03	57·25
26·5	81·55	77·18	*51·5*	76·28	71·26	*76·5*	62·48	56·65
27·5	81·70	77·35	*52·5*	75·78	70·72	*77·5*	61·93	56·05
28·5	81·81	77·47	*53·5*	75·31	70·17	*78·5*	61·38	55·45
29·5	81·89	77·55	*54·5*	74·76	69·61	*79·5*	60·83	54·85
30·5	81·93	77·60	*55·5*	74·24	69·04	*80·5*	60·28	54·25

than a pound in each case) after age 25. The maximum mean grip with the right hand is 81·93 pounds, with the left hand 77·60 pounds. Table V gives the smoothed means derived from the curves fitted.

In obtaining the graduated means of arrays of grip for each year it is legitimate to use means of three-year arrays as a guide to any part of the fitting curve, the three-year mean being of course the mean of the three separate one-year means weighted with the corresponding frequencies, and the position of the average of the three means on the horizontal scale being, except in the case of widely differing frequencies, near the centre of the three-year interval. On the other hand, in finding the graduated standard deviations for each year we must remember that the standard deviation of a three-year array may differ considerably from the mean of the standard deviations of the three constituent arrays. We have in fact, with obvious notation,

$$\sigma^2 = \frac{1}{n}(n_1\sigma_1^2 + n_2\sigma_2^2 + n_3\sigma_3^2) + \frac{1}{n^2}\{n_1n_2(\bar{x}_1 - \bar{x}_2)^2 + n_2n_3(\bar{x}_2 - \bar{x}_3)^2 + n_3n_1(\bar{x}_3 - \bar{x}_1)^2\}.$$

Thus the variance of the three-year array exceeds the weighted mean of the separate variances by a positive quantity which may be substantial if the differences

TABLE VI.

Graduated Standard Deviations of Arrays of Right Hand and Left Hand Grips for given Ages.

Central Age	Standard Deviation Right	Standard Deviation Left	Central Age	Standard Deviation Right	Standard Deviation Left	Central Age	Standard Deviation Right	Standard Deviation Left
6·5	0·97	1·72	*31·5*	12·82	12·74	*56·5*	11·12	10·79
7·5	2·02	2·66	*32·5*	12·79	12·69	*57·5*	11·03	10·69
8·5	3·13	3·69	*33·5*	12·75	12·64	*58·5*	10·94	10·58
9·5	4·32	4·81	*34·5*	12·71	12·59	*59·5*	10·85	10·47
10·5	5·70	6·07	*35·5*	12·67	12·53	*60·5*	10·75	10·36
11·5	7·42	7·42	*36·5*	12·62	12·47	*61·5*	10·66	10·25
12·5	9·78	8·84	*37·5*	12·57	12·41	*62·5*	10·57	10·13
13·5	11·80	10·30	*38·5*	12·52	12·34	*63·5*	10·47	10·00
14·5	12·52	11·80	*39·5*	12·46	12·28	*64·5*	10·38	9·88
15·5	12·78	12·45	*40·5*	12·40	12·21	*65·5*	10·28	9·76
16·5	12·86	12·79	*41·5*	12·34	12·14	*66·5*	10·19	9·64
17·5	12·91	12·96	*42·5*	12·28	12·07	*67·5*	10·09	9·51
18·5	12·94	13·02	*43·5*	12·21	11·99	*68·5*	9·99	9·39
19·5	12·95	13·04	*44·5*	12·13	11·91	*69·5*	9·89	9·26
20·5	12·96	13·05	*45·5*	12·06	11·83	*70·5*	9·79	9·13
21·5	12·96	13·04	*46·5*	11·98	11·75	*71·5*	9·69	9·00
22·5	12·96	13·03	*47·5*	11·90	11·66	*72·5*	9·59	8·87
23·5	12·95	13·00	*48·5*	11·82	11·57	*73·5*	9·49	8·74
24·5	12·95	12·98	*49·5*	11·73	11·48	*74·5*	9·39	8·61
25·5	12·94	12·95	*50·5*	11·64	11·39	*75·5*	9·29	8·48
26·5	12·93	12·93	*51·5*	11·56	11·30	*76·5*	9·19	8·35
27·5	12·91	12·90	*52·5*	11·47	11·20	*77·5*	9·09	8·22
28·5	12·89	12·86	*53·5*	11·39	11·10	*78·5*	8·99	8·09
29·5	12·87	12·82	*54·5*	11·30	11·00	*79·5*	8·89	7·96
30·5	12·85	12·78	*55·5*	11·21	10·90	*80·5*	8·79	7·83

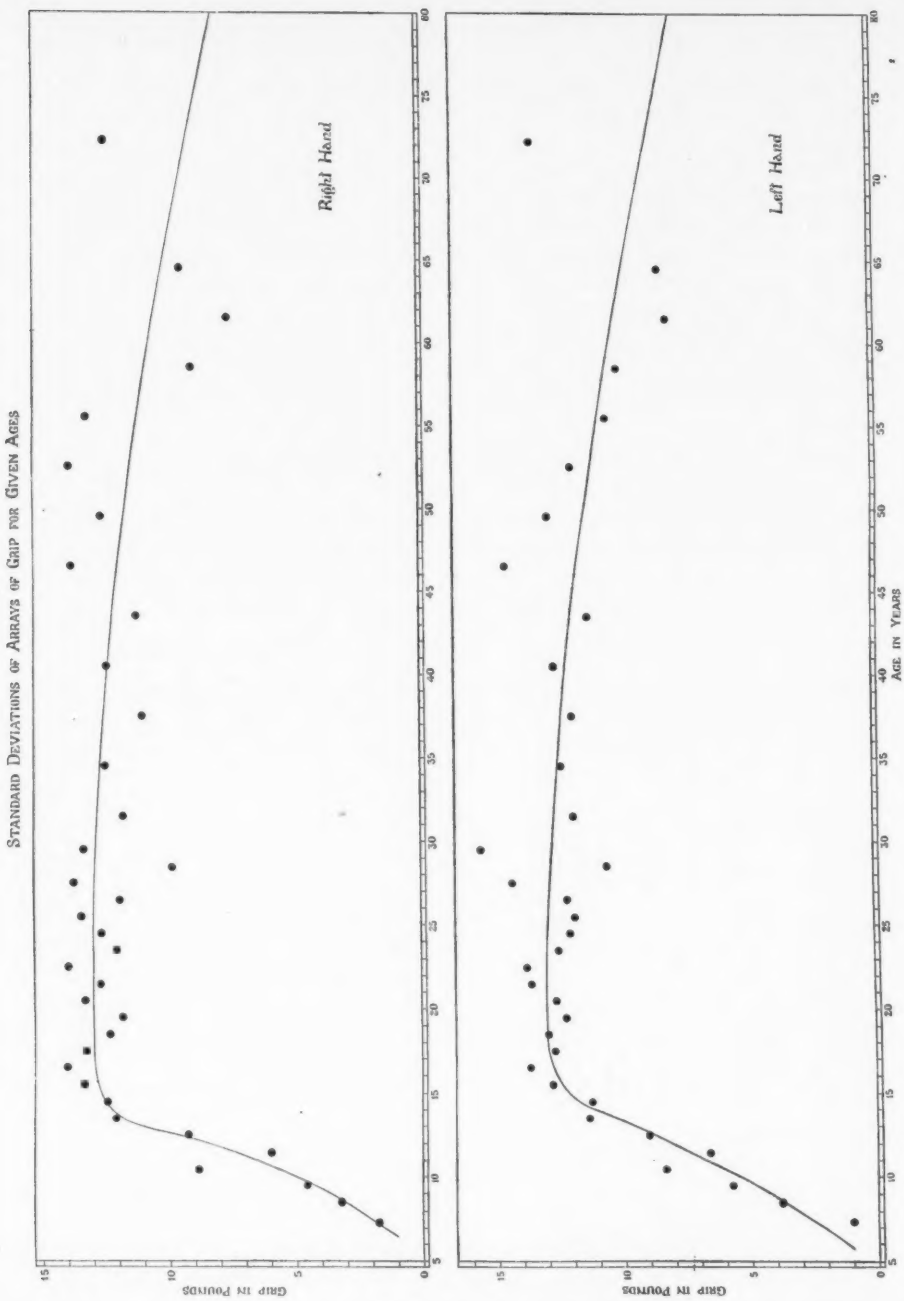

Standard Deviations of Arrays of Grip for Given Ages
Right Hand
Left Hand
Grip in Pounds
Age in Years

Diagram 2.

between the means are not small. Where the differences between successive means are small, or where the regression curve has low gradient, the three-year σ equals, sufficiently closely for our purpose, a mean of the three separate σ's and may be used as a guide for the smoothing curve; but where the mean grip increases (or decreases) rapidly with age, this procedure would introduce a considerable positive error. It is therefore necessary to use the separate σ's for all ages corresponding to steepness in the regression curve, and of course advantageous to use them wherever available. In Diagram 2 standard deviations of one-year arrays, calculated from Tables III and IV, are plotted up to age 30. In this case the entire graduating curves were fitted geometrically. The smoothed values resulting are detailed in Table VI.

4. *Variability of Strength of Grip at different Ages.* When plotted against age the coefficients of variation of arrays of grip exhibit a very marked peak with maximum about age 13·5, followed by distribution about a straight line with very slight downward slope. Diagram 3 shows the curves obtained by plotting the coefficients of variation calculated from the smoothed means and standard deviations. Up to age 20 strength of grip has rapidly changing variability. From a moderately low value at age 6 the variability increases quickly to a maximum about age 13 or 14, then falls equally quickly until age 20 or 21, maintaining thereafter practically the same value until age 80. The rise to the peak and the fall from it correspond respectively to the steepest parts of the curve of standard deviations

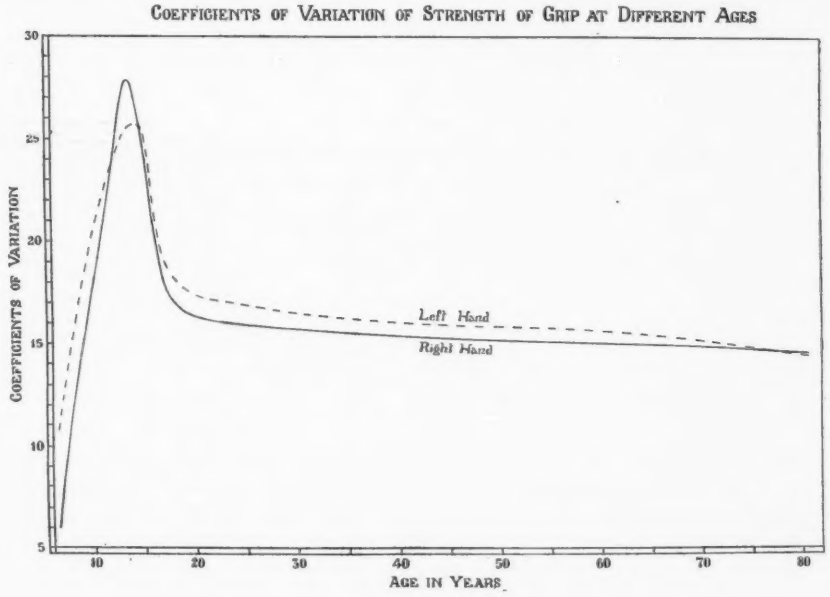


Diagram 3.

and the regression curve. It will be noticed that throughout almost the whole range considered left hand grip is more variable than right. No importance can be attached to the fact that the curve for the left hand drops more rapidly than the other in old age, as the data on which our calculations are based do not contain a sufficient number of elderly people to give results which are thoroughly reliable beyond age 60. Table VII gives the values from which the curves are plotted.

TABLE VII.

Coefficients of Variation of one-year Arrays of Grip.

Age	Coefficient of Variation		Age	Coefficient of Variation	
	Right Hand Grip	Left Hand Grip		Right Hand Grip	Left Hand Grip
6·5	5·99	10·82	*22·5*	16·09	17·09
7·5	10·41	14·22	*23·5*	16·01	16·99
8·5	13·76	17·08	*24·5*	15·96	16·91
9·5	16·58	19·40	*25·5*	15·90	16·82
10·5	19·29	21·49	*30·5*	15·68	16·47
11·5	22·28	23·30	*35·5*	15·52	16·22
12·5	26·11	24·69	*40·5*	15·39	16·06
13·5	27·86	25·59	*45·5*	15·28	15·95
14·5	25·71	25·79	*50·5*	15·17	15·89
15·5	22·03	23·21	*55·5*	15·10	15·79
16·5	18·94	19·72	*60·5*	15·04	15·68
17·5	17·38	18·50	*65·5*	14·98	15·47
18·5	16·70	17·85	*70·5*	14·88	15·16
19·5	16·44	17·50	*75·5*	14·74	14·81
20·5	16·29	17·31	*80·5*	14·58	14·43
21·5	16·18	17·17			

5. *Strength of Grip and Vital Capacity.* For the purpose of determining whether strength of grip is more intimately connected with the other characters considered than is vital capacity we shall at a later stage require the coefficient of correlation of grip and vital capacity, after reduction to standard age 40·5. We shall therefore at this point investigate the association of these two variates. From Table VIII we obtain

Right Hand Grip reduced to Age 40·5	Vital Capacity reduced to Age 40·5	Right Hand Grip and Vital Capacity reduced to Age 40·5
Mean = 80·28 ± 0·14	Mean = 219·92 ± 0·46	$r_{gb} = 0{\cdot}5025 \pm 0{\cdot}0087$
S. D. = 12·39 ± 0·10	S. D. = 39·31 ± 0·32	$\eta_{g.b} = 0{\cdot}5233 \pm 0{\cdot}0084$
		$\eta_{b.g} = 0{\cdot}5197 \pm 0{\cdot}0085$

The correlation, as one would expect, is fairly high, large lung capacity and muscular strength being different elements of bodily fitness. The approach of each of the regressions to linearity is close. Diagram 4 shows the regressions, the corresponding equations being

Probable Right Hand Grip at Age 40·5 ..... $g = 0{\cdot}1593b + 45{\cdot}2359$,

Probable Vital Capacity at Age 40·5 ..... $b = 1{\cdot}5844g + 92{\cdot}7240$.

TABLE VIII.

Right Hand Grip and Vital Capacity, reduced to Age 40·5.

Right Hand Grip in Pounds, reduced to Age 40·5; Central Values.

Vital Capacity in Cubic Inches, reduced to Age 40·5; Central Values.

	32	*37*	*42*	*47*	*52*	*57*	*62*	*67*	*72*	*77*	*82*	*87*	*92*	*97*	*102*	*107*	*112*	*117*	*122*	Totals
67	—	—	—	—	—	1	—	1	—	—	1	—	—	—	—	—	—	—	—	3
82	1	—	—	—	1	—	—	—	1	—	1	—	—	—	—	—	—	—	—	4
97	—	1	—	—	—	—	—	2	3	1	1	—	—	—	—	—	—	—	—	8
112	1	1	1	—	1	1	1	3	—	1	—	1	—	—	—	—	—	—	—	11
127	1	—	—	—	2	7	5	5	7	2	1	2	1	—	—	—	—	—	—	33
142	—	—	—	1	4	3	8	15	14	5	5	—	—	—	—	—	—	—	—	55
157	—	—	—	2	5	13	22	20	16	9	17	2	7	1	—	—	—	—	—	114
172	—	—	1	3	5	14	33	51	40	41	29	15	13	2	4	—	—	—	—	251
187	—	—	—	1	4	17	32	48	73	60	56	25	12	7	1	3	—	—	—	339
202	—	—	1	—	7	16	37	66	83	95	83	64	25	15	5	4	2	—	—	503
217	—	—	—	1	1	7	30	48	87	103	100	77	52	24	9	1	3	—	1	544
232	—	—	—	—	—	6	16	26	62	93	111	84	71	39	15	8	3	—	—	534
247	—	—	—	—	1	1	6	20	35	45	66	79	58	33	24	10	3	1	—	382
262	—	—	—	—	—	1	6	11	24	29	35	43	52	33	13	6	8	2	1	264
277	—	—	—	—	—	—	—	1	12	12	25	31	29	20	9	6	6	—	1	152
292	—	—	—	—	—	—	—	3	3	4	8	16	15	14	18	6	5	1	—	93
307	—	—	—	—	—	—	—	1	1	2	5	6	9	7	8	2	2	—	—	43
322	—	—	—	—	—	—	—	—	1	1	—	1	6	3	4	5	2	—	2	25
337	—	—	—	—	—	—	—	—	—	—	1	2	2	—	1	1	1	2	—	10
352	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
367	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	1	—	—	—	3
382	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	3	2	3	8	31	87	196	321	462	503	546	448	352	198	112	55	35	6	5	3373

TABLE IX.

Left Hand Grip and Vital Capacity, reduced to Age 40·5.

Left Hand Grip in Pounds, reduced to Age 40·5; Central Values.

Vital Capacity in Cubic Inches, reduced to Age 40·5; Central Values.

	27	*32*	*37*	*42*	*47*	*52*	*57*	*62*	*67*	*72*	*77*	*82*	*87*	*92*	*97*	*102*	*107*	*112*	*117*	*122*	Totals
67	—	—	—	—	—	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	3
82	—	1	—	—	—	1	—	—	—	1	1	—	—	—	—	—	—	—	—	—	4
97	—	1	—	—	1	—	1	1	2	—	1	1	—	—	—	—	—	—	—	—	8
112	1	—	1	2	—	1	1	1	1	1	1	—	1	—	—	—	—	—	—	—	11
127	—	—	—	3	1	9	3	5	5	3	3	1	—	—	—	—	—	—	—	—	33
142	—	—	—	2	1	3	12	14	12	3	6	2	—	—	—	—	—	—	—	—	55
157	—	—	1	—	5	15	24	15	12	18	13	7	4	—	—	—	—	—	—	—	114
172	—	1	—	3	3	16	27	45	46	43	33	17	10	4	2	1	—	—	—	—	251
187	—	—	—	1	1	17	30	59	61	59	39	33	22	12	3	2	—	—	—	—	339
202	—	—	—	—	4	14	25	65	89	95	79	63	40	14	8	4	2	1	—	—	503
217	—	—	1	—	4	10	24	36	89	120	97	64	40	44	9	2	3	1	—	—	544
232	—	—	—	—	1	3	16	37	69	90	95	86	74	38	17	5	2	1	—	—	534
247	—	—	—	—	—	2	3	18	35	57	68	55	65	47	17	9	3	1	2	—	382
262	—	—	—	—	—	5	2	10	17	29	49	39	42	45	16	6	2	1	1	—	264
277	—	—	—	—	—	—	1	3	6	16	31	31	25	17	10	8	1	2	1	—	152
292	—	—	—	—	—	—	—	—	3	5	12	15	18	15	13	7	3	2	—	—	93
307	—	—	—	—	—	—	—	—	1	3	6	10	9	4	—	4	5	—	1	—	43
322	—	—	—	—	—	—	1	—	—	2	4	4	—	4	2	3	4	—	—	1	25
337	—	—	—	—	—	—	—	—	—	—	—	1	3	2	1	1	1	1	—	—	10
352	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
367	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	1	—	—	—	—	3
382	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	1	3	3	11	21	96	171	310	448	546	538	430	353	246	100	53	27	10	5	1	3373

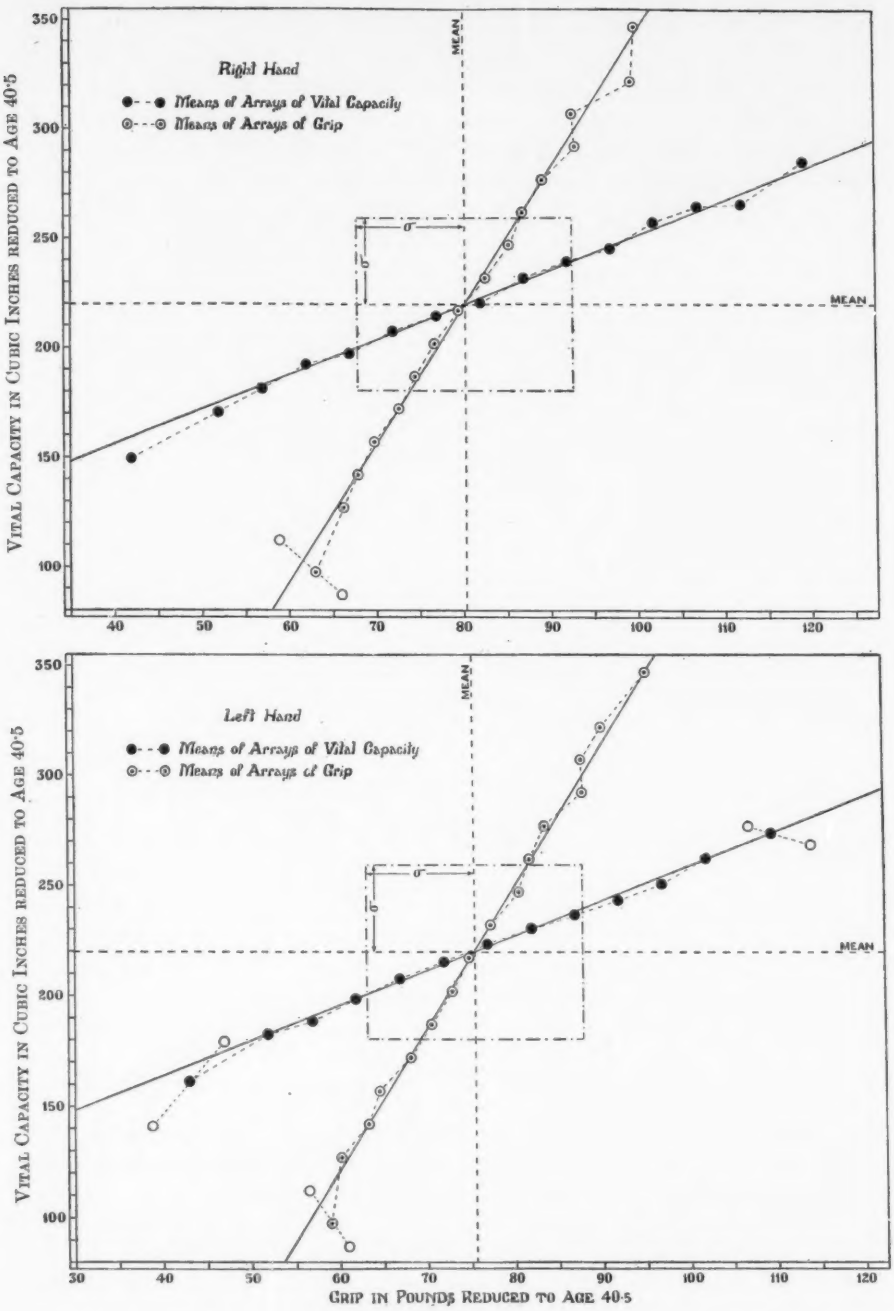

Right Hand
Means of Arrays of Vital Capacity
Means of Arrays of Grip
MEAN
MEAN
σ
σ
Vital Capacity in Cubic Inches reduced to Age 40·5
350
300
250
200
150
100
40
50
60
70
80
90
100
110
120
Left Hand
Means of Arrays of Vital Capacity
Means of Arrays of Grip
MEAN
MEAN
σ
σ
Vital Capacity in Cubic Inches reduced to Age 40·5
350
300
250
200
150
100
30
40
50
60
70
80
90
100
110
120
Grip in Pounds Reduced to Age 40·5

Diagram 4.

Table IX gives

Left Hand Grip reduced to Age 40·5

$$\text{Mean} = 75{\cdot}55 \pm 0{\cdot}14$$
$$\text{S. D.} = 12{\cdot}29 \pm 0{\cdot}10$$

In the case of left hand grip and vital capacity, so closely linear is the regression that if, as we did in the previous case, we combine extreme arrays so that no frequency shall be less than 16, we reach the values

$$r_{gb} = 0{\cdot}4978, \quad \eta_{g.b} = 0{\cdot}4971, \quad \eta_{b.g} = 0{\cdot}4976,$$

each of the correlation ratios being less than the correlation coefficient. Dropping Sheppard's corrections for grouping, which affect η to a greater degree than r, we find

Left Hand Grip and Vital Capacity reduced to Age 40·5

$$r_{gb} = 0{\cdot}4914^{*} \pm 0{\cdot}0088$$
$$\eta_{g.b} = 0{\cdot}4937 \pm 0{\cdot}0088$$
$$\eta_{b.g} = 0{\cdot}4945 \pm 0{\cdot}0088$$

The equations of the regression lines (Diagram 4) are

$$\text{Probable Left Hand Grip at Age 40·5} \ldots\ldots\ldots g = 0{\cdot}1566b + 41{\cdot}1149,$$
$$\text{Probable Vital Capacity at Age 40·5} \ldots\ldots\ldots b = 1{\cdot}5827g + 100{\cdot}3502.$$

6. *Strength of Grip and Reaction Time to Sight.* We proceed now to find the correlation of strength of grip with each of the four chosen mental characters in turn. Table X gives

Reaction Time to Sight reduced to Age 40·5

$$\text{Mean} = 18{\cdot}36 \pm 0{\cdot}05$$
$$\text{S. D.} = 3{\cdot}98 \pm 0{\cdot}03$$

The correlation constants are

Right Hand Grip and Reaction Time to Sight reduced to Age 40·5

$$r_{gv} = -0{\cdot}0913 \pm 0{\cdot}0115$$
$$\eta_{v.g} = 0{\cdot}1077 \pm 0{\cdot}0115$$
$$\eta_{g.v} = 0{\cdot}1116 \pm 0{\cdot}0115$$

There is thus a positive association between strength of grip and quickness of reaction to a sight signal. In order to test the significance of the correlation ratios we must find their mean values for random sampling from uncorrelated material. The mean value of a correlation ratio calculated from a sample of size N divided into K categories is $\bar{\eta} = \sqrt{\left(\frac{K-1}{N}\right)}$†, if there is no correlation in the sampled

* For purposes of comparison with r_{gb} in the case of right hand grip the value $0{\cdot}4978 \pm 0{\cdot}0087$ should be used.

† For proof see Karl Pearson, "On a Correction to be made to the Correlation Ratio η," *Biometrika*, Vol. VIII. p. 254.

TABLE X.

Right Hand Grip and Reaction Time to Sight, reduced to Age 40·5.

Right Hand Grip in Pounds, reduced to Age 40·5; Central Values.

Reaction Time to Sight in hundredths of a Second, reduced to Age 40·5; Central Values.

	32	*37*	*42*	*47*	*52*	*57*	*62*	*67*	*72*	*77*	*82*	*87*	*92*	*97*	*102*	*107*	*112*	*117*	*122*	Totals
5·5	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
7·5	—	—	—	—	1	—	2	—	—	—	—	1	—	—	—	—	—	—	—	4
9·5	—	—	—	1	—	4	3	14	15	13	17	13	11	5	5	2	—	—	—	103
11·5	—	—	—	—	—	—	5	13	19	15	21	17	15	10	2	2	3	1	—	123
13·5	—	—	—	1	1	2	10	16	29	32	43	30	30	11	11	1	3	—	—	220
15·5	1	—	—	3	3	15	35	51	74	88	94	91	65	45	26	14	9	3	—	617
17·5	—	—	1	2	5	13	42	60	88	88	110	66	71	39	19	12	8	2	3	629
19·5	1	1	2	1	8	20	47	76	121	142	146	128	94	57	25	13	9	—	1	892
21·5	—	1	—	—	8	13	20	34	58	49	47	43	32	16	11	5	3	—	1	341
23·5	—	—	—	—	2	13	13	20	18	45	44	28	18	5	9	3	—	—	—	218
25·5	1	—	—	—	1	4	14	17	27	16	12	19	12	4	4	1	—	—	—	132
27·5	—	—	—	—	—	1	3	7	3	4	6	3	3	2	—	—	—	—	—	32
29·5	—	—	—	—	1	1	1	7	8	7	4	7	1	2	—	2	—	—	—	41
31·5	—	—	—	—	—	1	—	2	—	2	—	2	—	2	—	—	—	—	—	9
33·5	—	—	—	—	—	—	—	2	—	1	1	—	—	—	—	—	—	—	—	4
35·5	—	—	—	—	—	—	1	1	1	1	1	—	—	—	—	—	—	—	—	5
37·5	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	2
Totals	3	2	3	8	31	87	196	321	462	503	546	448	352	198	112	55	35	6	5	3373

TABLE XI.

Left Hand Grip and Reaction Time to Sight, reduced to Age 40·5.

Left Hand Grip in Pounds, reduced to Age 40·5; Central Values.

Reaction Time to Sight in hundredths of a Second, reduced to Age 40·5; Central Values.

	27	*32*	*37*	*42*	*47*	*52*	*57*	*62*	*67*	*72*	*77*	*82*	*87*	*92*	*97*	*102*	*107*	*112*	*117*	*122*	Totals
5·5	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
7·5	—	—	—	—	—	—	2	—	1	—	1	—	—	—	—	—	—	—	—	—	4
9·5	—	—	—	1	2	—	7	8	17	9	15	14	10	11	6	2	1	—	—	—	103
11·5	—	—	—	—	—	2	5	11	10	22	19	24	14	7	6	—	—	2	1	—	123
13·5	—	—	—	—	2	3	9	13	25	33	43	29	34	18	5	3	2	1	—	—	220
15·5	—	—	—	4	1	15	25	59	79	95	87	86	65	47	26	20	5	2	1	—	617
17·5	—	—	1	2	3	17	33	54	80	109	106	79	60	49	20	7	6	—	2	1	629
19·5	—	2	—	2	5	20	46	83	117	147	150	106	103	65	25	7	9	4	1	—	892
21·5	—	1	—	1	—	20	21	36	52	45	55	39	32	26	4	5	3	1	—	—	341
23·5	—	—	—	—	3	6	13	21	28	41	38	28	18	12	2	8	—	—	—	—	218
25·5	1	—	1	—	2	4	6	16	24	26	14	15	11	7	4	1	—	—	—	—	132
27·5	—	—	—	1	1	5	2	4	3	4	3	5	1	2	1	—	—	—	—	—	32
29·5	—	—	1	—	1	2	—	2	8	11	4	4	4	2	1	—	1	—	—	—	41
31·5	—	—	—	—	—	1	1	1	1	2	1	1	1	—	—	—	—	—	—	—	9
33·5	—	—	—	—	—	—	—	1	1	—	2	—	—	—	—	—	—	—	—	—	4
35·5	—	—	—	—	—	1	1	—	1	2	—	—	—	—	—	—	—	—	—	—	5
37·5	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	2
Totals	1	3	3	11	21	96	171	310	448	546	538	430	353	246	100	53	27	10	5	1	3373

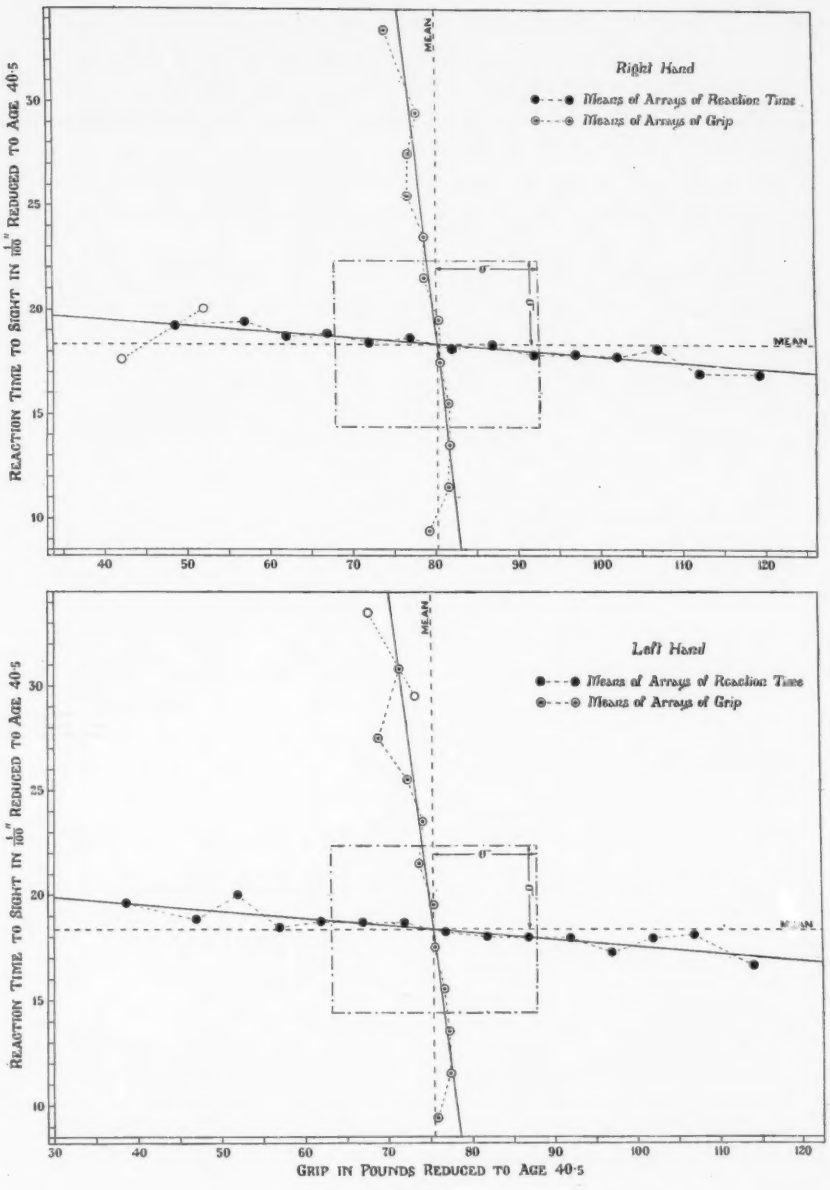

Right Hand
Means of Arrays of Reaction Time
Means of Arrays of Grip
MEAN
MEAN
σ
D
REACTION TIME TO SIGHT IN 1/100" REDUCED TO AGE 40·5
10
15
20
25
30
40
50
60
70
80
90
100
110
120
Left Hand
Means of Arrays of Reaction Time
Means of Arrays of Grip
MEAN
MEAN
σ
D
REACTION TIME TO SIGHT IN 1/100" REDUCED TO AGE 40·5
GRIP IN POUNDS REDUCED TO AGE 40·5

Diagram 5.

population; and the probable error of η when its true value is zero is $0{\cdot}67449/N^{\frac{1}{2}}$, in our case 0·0116. Hence we have

$$\eta_{v.g} - \bar{\eta}_{v.g} = 0{\cdot}1077 - 0{\cdot}0644 = 0{\cdot}0433 = 3{\cdot}73 \times 0{\cdot}0116,$$
$$\eta_{g.v} - \bar{\eta}_{g.v} = 0{\cdot}1116 - 0{\cdot}0571 = 0{\cdot}0545 = 4{\cdot}70 \times 0{\cdot}0116.$$

It is therefore extremely unlikely that either of the correlation ratios found could have arisen as a result of random sampling.

The regressions are closely linear, $\eta_{v.g} - |r_{vg}|$ and $\eta_{g.v} - |r_{vg}|$ being both less than twice the probable error of each of the constants. The prediction equations are

Probable Reaction Time to Sight at Age 40·5…$v = -0{\cdot}02937g + 20{\cdot}7224$,

Probable Right Hand Grip at Age 40·5 ……$g = -0{\cdot}28405v + 85{\cdot}4939$.

The regressions are illustrated in Diagram 5.

The corresponding results for left hand grip, obtained from Table XI, are

Left Hand Grip and Reaction Time to Sight reduced to Age 40·5

$r_{gv} = -0{\cdot}1038 \pm 0{\cdot}0115$
$\eta_{v.g} = 0{\cdot}1194 \pm 0{\cdot}0114$
$\eta_{g.v} = 0{\cdot}1198 \pm 0{\cdot}0114$

The equations of the regression lines (Diagram 5) are

Probable Reaction Time to Sight at Age 40·5…$v = -0{\cdot}03364g + 20{\cdot}9059$,

Probable Left Hand Grip at Age 40·5 ……$g = -0{\cdot}32003v + 81{\cdot}4246$.

The correlation constants are all of the same order as in the case of right hand grip, and similar tests establish the significance of the η's. We infer therefore that there is a slight but significant tendency for muscular fitness and shortness of reaction time to be associated, but remark that the smallness of the correlation ratios renders the prediction of either variate from the other very unreliable.

7. *Strength of Grip and Reaction Time to Sound.* From Table XII we obtain

Reaction Time to Sound reduced to Age 40·5

Mean = 15·52 ± 0·04
S. D. = 3·16 ± 0·03

Right Hand Grip and Reaction Time to Sound reduced to Age 40·5

$r_{gh} = -0{\cdot}1224 \pm 0{\cdot}0114$
$\eta_{h.g} = 0{\cdot}1331 \pm 0{\cdot}0114$
$\eta_{g.h} = 0{\cdot}1359 \pm 0{\cdot}0114$

The negative value of r indicates again the association of short reaction time with muscular fitness. The two correlation ratios are again significantly in excess of the mean values on the supposition of no correlation, namely $\bar{\eta}_{h.g} = 0{\cdot}0644$ and $\bar{\eta}_{g.h} = 0{\cdot}0517$; and the regressions are again closely linear (Diagram 6). The regression equations are

Probable Reaction Time to Sound at Age 40·5…$h = -0{\cdot}03119g + 18{\cdot}0230$,

Probable Right Hand Grip at Age 40·5……$g = -0{\cdot}48015h + 87{\cdot}7289$.

Reaction Time to Sound in hundredths of a

Reaction Time to Sound in hundredths of a

TABLE XII.

Right Hand Grip and Reaction Time to Sound, reduced to Age 40·5.

Right Hand Grip in Pounds, reduced to Age 40·5; Central Values.

Reaction Time to Sound in hundredths of a Second, reduced to Age 40·5; Central Values.

	32	*37*	*42*	*47*	*52*	*57*	*62*	*67*	*72*	*77*	*82*	*87*	*92*	*97*	*102*	*107*	*112*	*117*	*122*	Totals
5·5	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
7·5	—	—	—	—	1	—	—	2	—	—	—	—	—	—	—	—	—	—	—	3
9·5	—	—	—	2	—	—	7	9	13	17	26	18	11	11	4	2	—	—	—	120
11·5	—	—	—	2	2	6	18	29	40	54	63	47	43	32	22	10	7	3	1	379
13·5	—	—	—	2	4	15	36	61	111	119	118	104	75	46	29	15	14	2	—	751
15·5	1	—	3	2	9	36	75	89	132	159	176	160	131	68	32	13	6	1	4	1097
17·5	—	1	—	—	8	15	25	64	74	79	83	55	58	19	12	6	2	—	—	501
19·5	—	1	—	—	3	7	25	36	56	52	59	40	25	16	12	6	2	—	—	340
21·5	—	—	—	—	1	5	3	13	14	9	12	9	3	3	1	1	3	—	—	77
23·5	—	—	—	—	2	1	3	10	13	10	4	8	3	2	—	2	1	—	—	59
25·5	1	—	—	—	—	1	1	3	6	3	4	6	1	1	—	—	—	—	—	27
27·5	1	—	—	—	—	1	1	2	1	1	—	1	1	—	—	—	—	—	—	9
29·5	—	—	—	—	—	—	—	2	1	—	1	—	—	—	—	—	—	—	—	4
31·5	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
33·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
35·5	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	2
37·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39·5	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
41·5	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	3	2	3	8	31	87	196	321	462	503	546	448	35[illegible]	198	112	55	35	6	5	3373

TABLE XIII.

Left Hand Grip and Reaction Time to Sound, reduced to Age 40·5.

Left Hand Grip in Pounds, reduced to Age 40·5; Central Values.

Reaction Time to Sound in hundredths of a Second, reduced to Age 40·5; Central Values.

	27	*32*	*37*	*42*	*47*	*52*	*57*	*62*	*67*	*72*	*77*	*82*	*87*	*92*	*97*	*102*	*107*	*112*	*117*	*122*	Totals
5·5	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
7·5	—	—	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	3
9·5	—	—	—	—	1	1	5	10	11	20	25	14	14	13	3	2	1	—	—	—	120
11·5	—	—	1	1	3	9	11	32	34	56	69	52	46	27	24	7	4	3	—	—	379
13·5	—	—	—	2	1	17	37	61	102	117	125	97	85	62	15	15	9	4	2	—	751
15·5	—	2	2	2	8	29	54	103	158	165	164	147	113	86	35	14	10	3	1	1	1097
17·5	—	—	—	3	5	16	32	45	71	88	76	66	48	30	15	3	2	—	1	—	501
19·5	—	1	—	2	2	7	19	37	41	65	58	39	34	21	6	7	1	—	—	—	340
21·5	—	—	—	—	1	6	6	9	10	14	11	6	5	4	—	4	—	—	1	—	77
23·5	—	—	—	—	—	6	3	8	11	13	5	3	5	2	2	1	—	—	—	—	59
25·5	1	—	—	—	—	1	3	1	5	5	3	6	1	1	—	—	—	—	—	—	27
27·5	—	—	—	1	—	2	—	2	—	1	1	—	2	—	—	—	—	—	—	—	9
29·5	—	—	—	—	—	1	—	—	1	2	—	—	—	—	—	—	—	—	—	—	4
31·5	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
33·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
35·5	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	2
37·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
39·5	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
41·5	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	3	3	11	21	96	171	310	448	546	538	430	353	246	100	53	27	10	5	1	3373

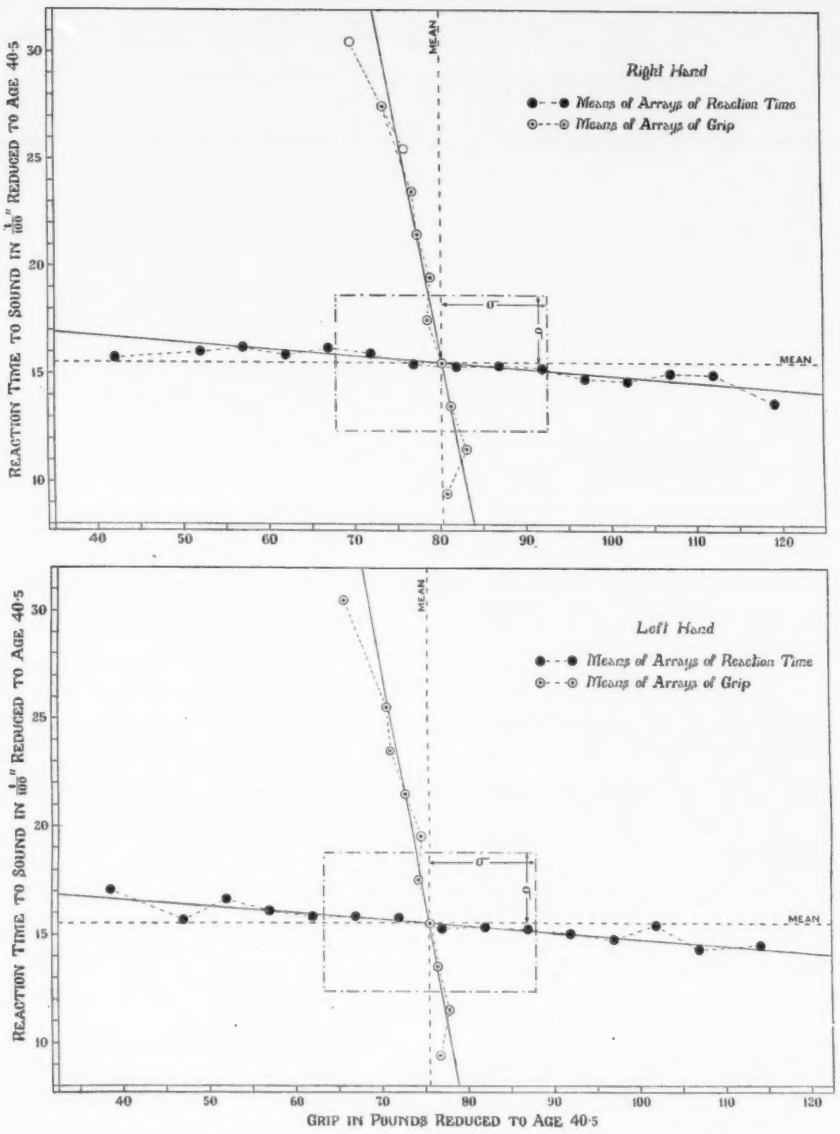

Right Hand
Means of Arrays of Reaction Time
Means of Arrays of Grip
MEAN
σ
σ
MEAN
REACTION TIME TO SOUND IN 1/100" REDUCED TO AGE 40·5
30
25
20
15
10
40
50
60
70
80
90
100
110
120
Left Hand
Means of Arrays of Reaction Time
Means of Arrays of Grip
MEAN
σ
σ
MEAN
REACTION TIME TO SOUND IN 1/100" REDUCED TO AGE 40·5
GRIP IN POUNDS REDUCED TO AGE 40·5

Diagram 6.

Table XIII furnishes for comparison the constants

Left Hand Grip and Reaction Time to Sound reduced to Age 40·5

$$r_{gh} = -0{\cdot}1189 \pm 0{\cdot}0114$$
$$\eta_{h.g} = 0{\cdot}1293 \pm 0{\cdot}0114$$
$$\eta_{g.h} = 0{\cdot}1210 \pm 0{\cdot}0114$$

and the regression equations (Diagram 6),

Probable Reaction Time to Sound at Age 40·5...$h = -0{\cdot}03056g + 17{\cdot}8280$,

Probable Left Hand Grip at Age 40·5 .....$g = -0{\cdot}46283h + 82{\cdot}7299$.

The correlation ratios in the case of grip and reaction time to sight or sound are all of such low order that prediction either way is extremely unreliable. This indeed is suggested at once by a glance at the correlation tables, the scatter in both vertical and horizontal arrays being at once remarked. Further, the divergence of the regressions from linearity is so small that it was not considered either advantageous or justifiable to fit parabolas or cubics to the means. There is however one point which is deserving of notice, namely that each of the four regressions of grip on reaction time shows a tendency to swing to the left for very low reaction times. As will be seen from the appropriate tables, the extreme points plotted are based on a substantial number of cases, 108 and 124. We are therefore possibly justified in drawing the inference that a very quick response to a sight or a sound signal does not connote the highest state of muscular fitness. Exceptional quickness of reaction may be rather an indication of a hyper-nervous condition.

It will be observed that the associational constants in the case of grip and reaction time to sound are all of greater magnitude than the corresponding constants for grip and reaction time to sight. In order to determine whether the difference in degree of association is or is not significant, we may find the probable error of the difference between corresponding correlation coefficients, which, since the regressions are approximately linear, may be taken as adequate measures of association. The probable error of the difference between r_{gv} and r_{gh} is the square root of

$$E^2_{r_{gv}} + E^2_{r_{gh}} - 2R_{r_{gv}r_{gh}}E_{r_{gv}}E_{r_{gh}},$$

where $R_{r_{gv}r_{gh}}$ is the coefficient of correlation between the correlation coefficients and $E_{r_{gv}}$, $E_{r_{gh}}$ are their probable errors. Now

$$^*R_{r_{gv}r_{gh}} = r_{vh} - \tfrac{1}{2}r_{gv}r_{gh}\frac{1 - r^2_{gv} - r^2_{gh} - r^2_{vh} + 2r_{gv}r_{gh}r_{vh}}{(1 - r^2_{gv})(1 - r^2_{gh})},$$

whence, taking $r_{vh} = 0{\cdot}4662$, the value given by Koga and Morant†, we deduce

$$|r_{gv} - r_{gh}| = 0{\cdot}0311 \pm 0{\cdot}0119, \text{ for right hand grip,}$$

and

$$|r_{gv} - r_{gh}| = 0{\cdot}0151 \pm 0{\cdot}0119, \text{ for left hand grip.}$$

* For proof see Karl Pearson and L. N. G. Filon, "Mathematical Contributions to the Theory of Evolution IV, 'On the Probable Errors of Frequency Constants, etc.,'" *Phil. Trans.*, Series A, Vol. 191, p. 263.

† *Biometrika*, Vol. xv. p. 369.

TABLE XIV.

Right Hand Grip and Visual Acuity, reduced to Age 40·5.

Right Hand Grip in Pounds, reduced to Age 40·5; Central Values.

Visual Acuity in inches at which Diamond Type is read, reduced to Age 40·5; Central Values.

	32	*37*	*42*	*47*	*52*	*57*	*62*	*67*	*72*	*77*	*82*	*87*	*92*	*97*	*102*	*107*	*112*	*117*	*122*	Totals
1	—	—	1	1	1	5	4	11	16	12	21	13	11	4	2	1	—	—	—	103
4	1	—	—	1	2	4	15	18	17	22	19	20	10	11	3	1	1	—	—	145
7	—	—	—	1	6	6	25	26	37	31	45	35	19	17	13	3	6	1	—	271
10	—	—	—	—	5	2	17	33	36	38	36	35	19	10	8	1	—	—	—	240
13	—	1	1	1	2	10	30	45	64	68	73	54	52	19	18	5	1	—	—	444
16	1	1	—	2	11	29	42	72	120	107	125	82	66	31	22	14	6	1	1	733
19	—	—	1	2	3	16	25	47	76	94	90	79	68	38	14	14	11	1	3	582
22	—	—	—	—	—	10	23	42	66	81	86	77	67	37	15	8	8	3	—	523
25	1	—	—	—	—	5	12	21	22	39	35	41	25	19	12	7	1	—	—	240
28	—	—	—	—	—	—	3	5	7	9	11	10	12	8	4	1	1	—	1	72
31	—	—	—	—	—	—	—	1	1	1	4	2	2	3	1	—	—	—	—	15
34	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
37	—	—	—	—	1	—	—	—	—	—	1	—	—	1	—	—	—	—	—	3
40	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
43	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
46	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	3	2	3	8	31	87	196	321	462	503	546	448	352	198	112	55	35	6	5	3373

TABLE XV.

Left Hand Grip and Visual Acuity, reduced to Age 40·5.

Left Hand Grip in Pounds, reduced to Age 40·5; Central Values.

Visual Acuity in inches at which Diamond Type is read, reduced to Age 40·5; Central Values.

	27	*32*	*37*	*42*	*47*	*52*	*57*	*62*	*67*	*72*	*77*	*82*	*87*	*92*	*97*	*102*	*107*	*112*	*117*	*122*	Totals
1	—	—	—	—	3	4	6	14	12	18	17	9	13	5	2	—	—	—	—	—	103
4	—	1	—	1	1	6	13	25	18	15	26	15	13	8	2	1	—	—	—	—	145
7	—	—	—	—	4	8	16	30	37	44	36	31	27	15	8	8	6	—	1	—	271
10	—	—	—	—	4	10	13	27	39	37	36	32	15	16	8	2	—	1	—	—	240
13	—	2	1	1	1	15	24	43	52	70	84	49	49	32	9	7	3	1	1	—	444
16	1	—	1	6	4	25	41	69	100	148	108	76	55	56	23	13	4	1	2	—	733
19	—	—	1	2	2	15	25	41	81	90	85	100	58	42	22	9	5	2	1	1	582
22	—	—	—	—	1	9	22	44	60	88	79	74	75	50	12	3	2	4	—	—	523
25	—	—	—	1	1	3	7	11	41	30	48	28	36	15	8	6	5	—	—	—	240
28	—	—	—	—	—	—	3	6	7	5	13	12	9	6	4	4	2	1	—	—	72
31	—	—	—	—	—	—	—	—	1	1	5	3	3	1	1	—	—	—	—	—	15
34	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
37	—	—	—	—	—	1	1	—	—	—	—	—	—	—	1	—	—	—	—	—	3
40	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
43	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
46	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	1	3	3	11	21	96	171	310	448	546	538	430	353	246	100	53	27	10	5	1	3373

We are therefore unable to assert that there is any markedly significant difference in the correlation of the two reaction times with strength of grip. If the true value of the difference is zero, and we assume normality of distribution of the differences, we should expect a result as large as or larger than 0·0311 in one out of thirteen random samples, and a result as large as or larger than 0·0151 in two out of five.

We can apply the same test to find whether the dynamic physical characters, vital capacity and grip, have significantly differing degrees of association with reaction time. The values $r_{bv} = -0{\cdot}1231 \pm 0{\cdot}0114$, $r_{bh} = -0{\cdot}1159 \pm 0{\cdot}0115$ as found by Holzinger* lead to the results,

Right Hand Grip: $|r_{gv} - r_{bv}| = 0{\cdot}0318 \pm 0{\cdot}0115$; $|r_{gh} - r_{bh}| = 0{\cdot}0065 \pm 0{\cdot}0114$,

Left Hand Grip: $|r_{gv} - r_{bv}| = 0{\cdot}0193 \pm 0{\cdot}0115$; $|r_{gh} - r_{bh}| = 0{\cdot}0030 \pm 0{\cdot}0114$,

from which we conclude that grip and vital capacity have the same intensity of correlation with the reaction times†.

8. *Strength of Grip and Keenness of Vision.* From Tables XIV and XV we arrive at the following results:

Visual Acuity reduced to Age 40·5	Right Hand Grip and Visual Acuity reduced to Age 40·5
Mean = 15·93 ± 0·07	$r_{gk} = 0{\cdot}1133 \pm 0{\cdot}0115$
S. D. = 6·35 ± 0·05	$\eta_{k.g} = 0{\cdot}1274 \pm 0{\cdot}0114$
	$\eta_{g\ k} = 0{\cdot}1360 \pm 0{\cdot}0114$

Regression Lines:

Probable Visual Acuity at Age 40·5 ..... $k = 0{\cdot}05806g + 11{\cdot}2642$,

Probable Right Hand Grip at Age 40·5..... $g = 0{\cdot}22121k + 76{\cdot}7546$.

Left Hand Grip and Visual Acuity reduced to Age 40·5

$r_{gk} = 0{\cdot}1194 \pm 0{\cdot}0114$

$\eta_{k.g} = 0{\cdot}1372 \pm 0{\cdot}0114$

$\eta_{g.k} = 0{\cdot}1389 \pm 0{\cdot}0114$

Regression Lines:

Probable Visual Acuity at Age 40·5..... $k = 0{\cdot}06166g + 11{\cdot}2669$,

Probable Left Hand Grip at Age 40·5..... $g = 0{\cdot}23110k + 71{\cdot}8670$.

In both cases $\bar{\eta}_{k.g} = 0{\cdot}0644$ and $\bar{\eta}_{g.k} = 0{\cdot}0544$, so that the four correlation ratios are readily seen to be significant, though of low order. All four regressions, pictured in Diagram 7, are again approximately linear. The correlation coefficients, being positive, indicate the tendency of acuteness of sight and muscular strength to be in conjunction.

* *Biometrika*, Vol. XVI. pp. 145 and 148.

† There is, I think, a slip in *Biometrika*, Vol. XVI. p. 148, lines 20 *et seq.*: For "$r_{12} = 0{\cdot}452$" read "$r_{12} = 0{\cdot}4662$" and for "$0{\cdot}0072 \pm 0{\cdot}0081$" read "$0{\cdot}0072 \pm 0{\cdot}0118$." If the full formula for r_{12} is used, $r_{12} = 0{\cdot}4606$ and the difference becomes "$0{\cdot}0072 \pm 0{\cdot}0119$."

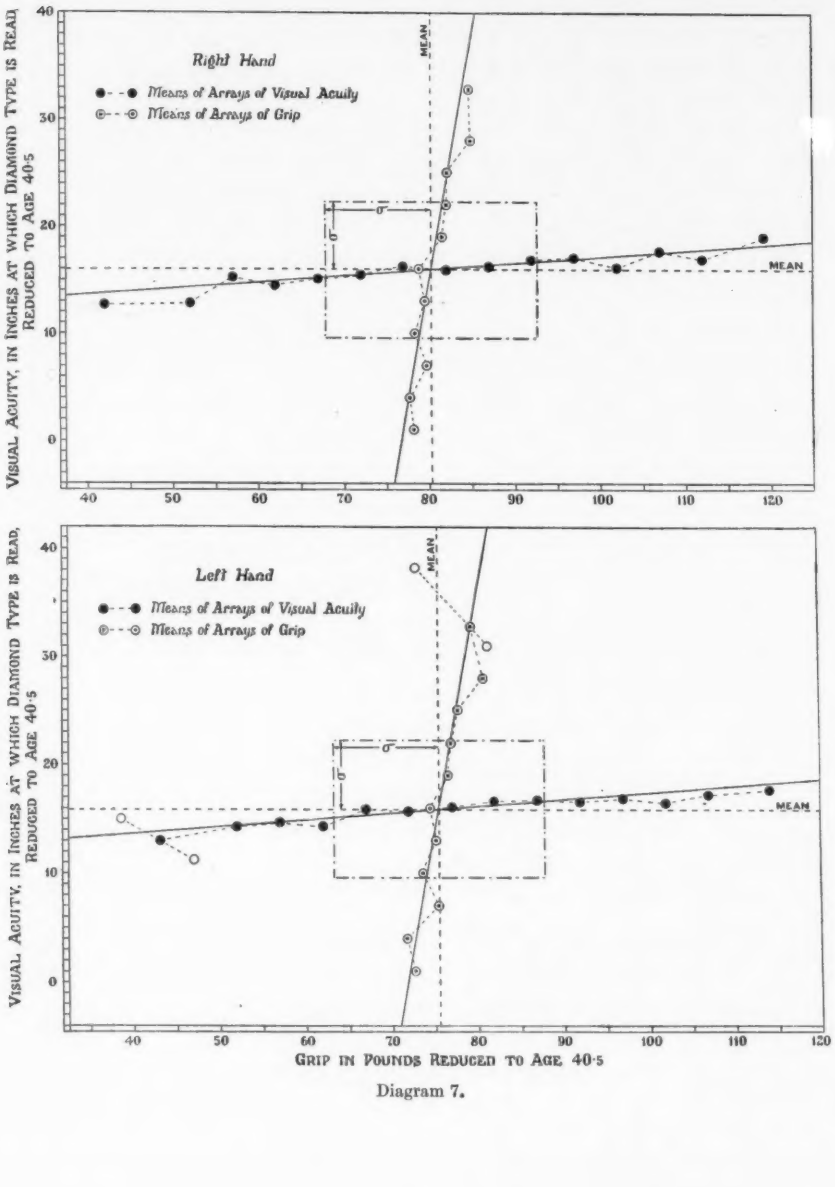

Right Hand
Means of Arrays of Visual Acuity
Means of Arrays of Grip
MEAN
σ
σ
MEAN
Visual Acuity, in Inches at which Diamond Type is Read, Reduced to Age 40·5
Left Hand
Means of Arrays of Visual Acuity
Means of Arrays of Grip
MEAN
σ
σ
MEAN
Visual Acuity, in Inches at which Diamond Type is Read, Reduced to Age 40·5
Grip in Pounds Reduced to Age 40·5

Diagram 7.

TABLE XVI.

Right Hand Grip and Highest Audible Pitch, reduced to Age 40·5.

Right Hand Grip in Pounds, reduced to Age 40·5; Central Values.

Highest Audible Pitch in thousands of vibrations per Second, reduced to Age 40·5; Central Values.

	32	*37*	*42*	*47*	*52*	*57*	*62*	*67*	*72*	*77*	*82*	*87*	*92*	*97*	*102*	*107*	*112*	*117*	*122*	Totals
3·5	—	—	—	—	—	—	1	1	1	2	1	2	2	—	—	—	—	—	—	10
5·5	—	—	—	—	—	—	—	1	—	—	1	1	1	—	—	—	—	—	—	4
7·5	—	—	—	—	—	1	—	3	3	1	1	1	1	1	1	—	—	—	—	13
9·5	1	—	—	—	2	—	2	1	8	6	3	6	2	1	2	—	—	—	—	34
11·5	1	—	1	—	3	4	12	13	28	24	22	15	23	11	5	1	—	—	—	163
13·5	—	1	1	3	7	17	33	45	63	66	68	62	38	28	14	9	8	2	2	467
15·5	1	1	—	1	10	29	55	96	152	158	159	131	96	46	30	12	10	1	1	989
17·5	—	—	1	2	6	27	60	112	143	164	204	171	135	82	40	21	11	2	2	1183
19·5	—	—	—	2	3	6	24	38	48	58	69	41	39	20	16	10	5	1	—	380
21·5	—	—	—	—	—	3	8	9	11	16	15	14	11	5	3	1	1	—	—	97
23·5	—	—	—	—	—	—	1	2	2	4	3	3	3	4	1	—	—	—	—	23
25·5	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	1	—	—	—	4
27·5	—	—	—	—	—	—	—	—	1	2	—	—	—	—	—	—	—	—	—	3
29·5	—	—	—	—	—	—	—	—	1	1	—	—	1	—	—	—	—	—	—	3
Totals	3	2	3	8	31	87	196	321	462	503	546	448	352	198	112	55	35	6	5	3373

TABLE XVII.

Left Hand Grip and Highest Audible Pitch, reduced to Age 40·5.

Left Hand Grip in Pounds, reduced to Age 40·5; Central Values.

Highest Audible Pitch in thousands of vibrations per Second, reduced to Age 40·5; Central Values.

	27	*32*	*37*	*42*	*47*	*52*	*57*	*62*	*67*	*72*	*77*	*82*	*87*	*92*	*97*	*102*	*107*	*112*	*117*	*122*	Totals
3·5	—	—	—	—	—	—	—	2	1	2	1	1	3	—	—	—	—	—	—	—	10
5·5	—	—	—	—	—	—	1	—	—	—	1	—	2	—	—	—	—	—	—	—	4
7·5	—	—	—	—	—	2	1	1	1	3	2	2	1	—	—	—	—	—	—	—	13
9·5	—	—	—	1	1	2	3	1	3	6	6	3	1	5	2	—	—	—	—	—	34
11·5	1	1	1	—	1	10	5	16	19	23	28	16	17	16	5	2	2	—	—	—	163
13·5	—	—	1	2	3	19	32	43	61	79	67	64	40	26	13	7	7	2	1	—	467
15·5	—	2	1	4	7	26	57	90	138	176	154	109	113	64	18	19	6	3	2	—	989
17·5	—	—	—	1	7	24	47	105	157	175	199	171	129	92	47	15	9	2	2	1	1183
19·5	—	—	—	2	2	6	20	44	51	60	62	46	32	31	12	7	3	2	—	—	380
21·5	—	—	—	1	—	6	4	7	10	17	14	13	13	7	2	2	—	1	—	—	97
23·5	—	—	—	—	—	1	—	1	3	4	3	3	2	5	—	1	—	—	—	—	23
25·5	—	—	—	—	—	—	1	—	1	—	—	1	—	—	1	—	—	—	—	—	4
27·5	—	—	—	—	—	—	—	—	3	—	—	—	—	—	—	—	—	—	—	—	3
29·5	—	—	—	—	—	—	—	—	—	1	1	1	—	—	—	—	—	—	—	—	3
Totals	1	3	3	11	21	96	171	310	448	546	538	430	353	246	105	53	27	10	5	1	3373

Using the value of r_{bk} obtained by Holzinger*, namely $0{\cdot}1358 \pm 0{\cdot}0114$, we find for right hand grip $|r_{bk} - r_{gk}| = 0{\cdot}0225 \pm 0{\cdot}0115$, and for left hand grip

$$|r_{bk} - r_{gk}| = 0{\cdot}0164 \pm 0{\cdot}0114;$$

so that grip and vital capacity have the same order of correlation with keenness of vision.

Again, if we use the Koga-Morant value $r_{kv} = -0{\cdot}0644$†, it follows that for right hand grip $|r_{gk}| - |r_{gv}| = 0{\cdot}0220 \pm 0{\cdot}0157$, and for left hand grip

$$|r_{gk}| - |r_{gv}| = 0{\cdot}0156 \pm 0{\cdot}0157;$$

whence we conclude that there is no significant difference in the order of correlation of grip with keenness of sight and with the corresponding reaction time.

9. *Strength of Grip and Keenness of Hearing.* The horizontal and vertical symmetry of Tables XVI and XVII suggests at once that they will furnish low correlations. From the former we find

Highest Audible Pitch reduced to Age 40·5	Right Hand Grip and Highest Audible Pitch reduced to Age 40·5
Mean $= 16{\cdot}31 \pm 0{\cdot}03$	$r_{gp} = 0{\cdot}0568 \pm 0{\cdot}0116$
S. D. $= 2{\cdot}57 \pm 0{\cdot}02$	$\eta_{p.g} = 0{\cdot}0855 \pm 0{\cdot}0115$
	$\eta_{g.p} = 0{\cdot}0762 \pm 0{\cdot}0115$

The mean values of the correlation ratios for zero correlation are $\bar{\eta}_{p.g} = 0{\cdot}0644$, $\bar{\eta}_{g.p} = 0{\cdot}0517$ and the probable error of η when its value is zero is 0·0116. We have then

$$\eta_{p.g} - \bar{\eta}_{p.g} = 0{\cdot}0211 = 1{\cdot}82 \times 0{\cdot}0116; \quad \eta_{g.p} - \bar{\eta}_{g.p} = 0{\cdot}0245 = 2{\cdot}11 \times 0{\cdot}0116.$$

Thus we cannot say that the values found for the η's denote correlation between the variates: they might readily have arisen through random sampling where there was no correlation.

Table XVII gives

Left Hand Grip and Highest Audible Pitch reduced to Age 40·5
$r_{gp} = 0{\cdot}0477 \pm 0{\cdot}0116$
$\eta_{p.g} = 0{\cdot}0802 \pm 0{\cdot}0115$
$\eta_{g.p} = 0{\cdot}0726 \pm 0{\cdot}0116$

and similar tests show that there is not sufficient evidence to establish any degree of correlation between the two variates. In Diagram 8 the closeness to the horizontal and the vertical of the straight lines of best fit indicates how low the observed correlations are.

We shall now consider the question whether reaction time to a given signal depends in different degree on muscular fitness and on the acuteness of the sense employed in receiving the signal. In the case of a sight signal we have, taking $r_{kv} = -0{\cdot}0644 \pm 0{\cdot}0116$, for right hand grip $|r_{vk} - r_{vg}| = 0{\cdot}0269 \pm 0{\cdot}0154$ and for left hand grip $|r_{vk} - r_{vg}| = 0{\cdot}0394 \pm 0{\cdot}0153$. In the case of a sound signal, if we use

* *Biometrika*, Vol. xvi. p. 152.

† *Biometrika*, Vol. xv. p. 367.

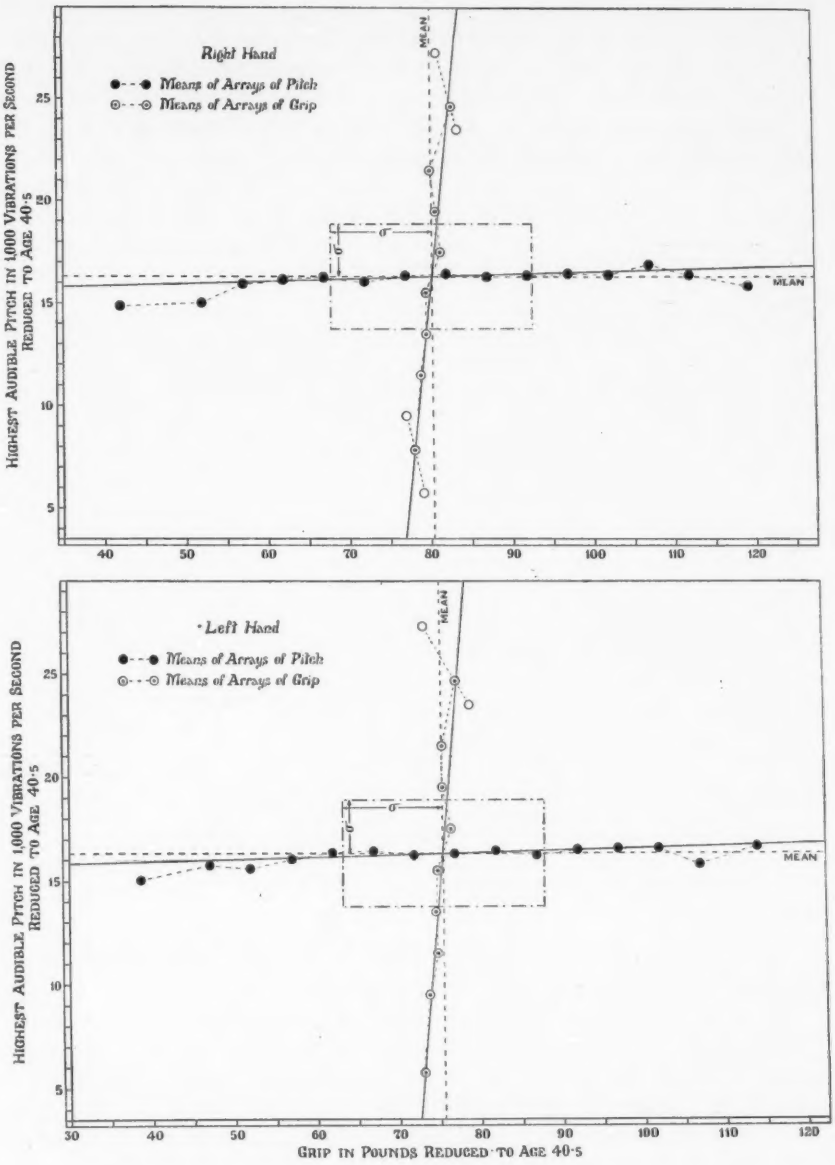

Right Hand
Means of Arrays of Pitch
Means of Arrays of Grip
HIGHEST AUDIBLE PITCH IN 1,000 VIBRATIONS PER SECOND REDUCED TO AGE 40·5
MEAN
σ
Left Hand
Means of Arrays of Pitch
Means of Arrays of Grip
HIGHEST AUDIBLE PITCH IN 1,000 VIBRATIONS PER SECOND REDUCED TO AGE 40·5
GRIP IN POUNDS REDUCED TO AGE 40·5

Diagram 8.

* $r_{hp} = -0{\cdot}0956 \pm 0{\cdot}0115$, we find for right hand grip $|r_{hg} - r_{hp}| = 0{\cdot}0268 \pm 0{\cdot}0158$ and for left hand grip $|r_{hg} - r_{hp}| = 0{\cdot}0233 \pm 0{\cdot}0159$. Thus there appears to be no significance in the observed differences in the correlations of reaction time with strength of grip and with acuteness of the corresponding sense.

Table XVIII is subjoined to give for reference the means, standard deviations and coefficients of variation of the characters under consideration.

TABLE XVIII.

	Character	Mean	Standard Deviation	Coefficient of Variation
Unreduced	Right Hand Grip	73·01 ± 0·22	18·93 ± 0·16	25·93 ± 0·23
	Left Hand Grip	68·71 ± 0·21	18·45 ± 0·15	26·85 ± 0·24
Reduced to Age 40·5	Right Hand Grip	80·28 ± 0·14	12·39 ± 0·10	15·43 ± 0·13
	Left Hand Grip	75·55 ± 0·14	12·29 ± 0·10	16·27 ± 0·14
	Vital Capacity	219·92 ± 0·46	39·31 ± 0·32	17·87 ± 0·15
	Reaction Time to Sight ...	18·36 ± 0·05	3·98 ± 0·03	21·68 ± 0·18
	Reaction Time to Sound ...	15·52 ± 0·04	3·16 ± 0·03	20·36 ± 0·17
	Visual Acuity	15·93 ± 0·07	6·35 ± 0·05	39·86 ± 0·38
	Highest Audible Pitch ...	16·31 ± 0·03	2·57 ± 0·02	15·76 ± 0·13

Certain of the constants in Table XVIII differ sensibly from corresponding values given by Koga and Morant and by Holzinger. This is in large measure probably due to the fact that although the grouping units used by the previous investigators have been employed throughout, different group boundaries have been used. In addition, the values of each of the variates after reduction to standard age have been taken as correct to the nearest unit only, this being the maximum degree of accuracy of the original data. During the process of reduction a few slips were corrected, notably in the case of individuals who, having made a mistake of ten years in stating their date of birth†, were found on adjustment for age to have a superhuman strength of grip.

Since $r_{\bar{v}\bar{h}} = r_{vh}$ the probable error of the difference between $\bar{v}$ and $\bar{h}$ is 0·047, so that the mean reaction time to sight is significantly in excess of the mean reaction time to sound.

10. *Conclusions.* The chief results reached in the foregoing investigation may be summarised as follows.

(1) There is a positive correlation, of low order but significant, between strength of grip with either hand and each of the three characters, quickness of reaction to sight, quickness of reaction to sound and keenness of vision, when all the variates concerned have been reduced to standard age.

(2) These correlations are all substantially of the same order 0·1, the order of the corresponding correlations with vital capacity substituted for grip.

* *Biometrika*, Vol. xv. p. 364.

† Verified from the Laboratory schedules on which age at measurement was entered.

(3) There is not sufficient evidence to establish correlation between strength of grip and keenness of hearing.

It is possible that Strength of Grip, Vital Capacity and other dynamic physical characters may have a significant correlation with Keenness of Hearing as measured by the amplitude rather than the note of a sound. Galton says (*Inquiries into Human Faculty*, p. 377) "The power of hearing shrill notes has nothing to do with sharpness of hearing."

(4) Reaction time to sight or sound is dependent in equal degree, though slightly, on muscular fitness and on sharpness of the sense used in accepting the signal.

(5) Mean reaction time to sight exceeds significantly mean reaction time to sound; the excess being, in seconds, $0{\cdot}0284 \pm 0{\cdot}0005$.

(6) The dynamic physical characters, strength of grip and vital capacity, have after reduction to constant age a positive correlation of order 0·5 and are closely linear.

(7) The variability of strength of grip is greatest "in the 'teens," and grip with the left hand is as a general rule more variable than with the right.

(8) The special form of growth curve which has been indicated in earlier papers is now seen to hold for grip, and it is probably proper to most dynamic physical characters.

The author wishes to acknowledge with gratitude his indebtedness to Professor Karl Pearson, under whose direction in the Galton Laboratory the work was undertaken, and to Miss Ida McLearn to whose excellent draughtsmanship the diagrams bear witness.

APPENDIX.

Diagram 9 is appended to show the growth curve of mean vital capacity with means plotted for each year up to age 30. These means, calculated from Table XIX, give a more trustworthy guide to the shape of the curve during the period of adolescence and demonstrate the concavity clearly.

TABLE XIX.

Vital Capacity and Age, up to Age 30*.

Vital Capacity in Cubic Inches; Central Values.

Age in Years; Central Values.	*42*	*57*	*72*	*87*	*102*	*117*	*132*	*147*	*162*	*177*	*192*	*207*	*222*	*237*	*252*	*267*	*282*	*297*	*312*	*327*	*342*	*357*	*372*	*387*	Totals
6·5	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
7·5	—	2	1	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
8·5	1	2	2	4	4	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	14
9·5	—	1	3	6	6	7	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	26
10·5	—	—	2	4	9	8	4	2	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	30
11·5	—	—	1	9	12	18	10	3	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	56
12·5	—	—	—	5	13	18	10	14	—	2	—	1	1	—	—	—	—	—	—	—	—	—	—	—	64
13·5	—	1	1	8	8	24	27	16	6	6	2	1	2	1	—	—	—	—	—	—	—	—	—	—	103
14·5	—	—	—	1	7	21	15	33	19	11	7	6	—	2	—	—	1	—	—	—	—	—	—	—	123
15·5	—	—	—	—	2	5	14	17	16	24	27	13	6	8	2	—	—	—	—	—	—	—	—	—	134
16·5	—	—	—	1	—	3	3	14	15	29	15	23	17	20	11	4	2	—	—	1	—	—	—	—	158
17·5	—	—	—	1	—	1	—	9	10	26	18	34	32	25	21	13	8	2	2	1	—	—	—	—	203
18·5	—	—	—	1	1	—	2	—	7	15	14	42	58	44	34	27	14	8	3	3	1	—	—	—	274
19·5	—	—	—	—	1	—	3	2	3	15	31	45	49	53	39	19	7	13	5	3	1	—	—	—	289
20·5	—	—	—	—	1	—	1	7	9	13	26	33	34	45	23	16	12	11	3	1	—	—	—	1	236
21·5	—	—	—	—	—	1	—	3	8	7	11	22	29	35	20	18	11	6	2	2	—	—	—	—	175
22·5	—	—	—	—	—	1	—	5	4	5	18	25	24	30	19	12	7	7	3	—	1	—	—	—	161
23·5	—	—	—	—	—	1	—	4	3	7	9	17	20	22	17	15	3	7	2	—	2	—	—	—	129
24·5	—	—	—	—	—	—	1	1	2	9	7	7	15	18	18	8	5	4	1	1	1	—	—	—	98
25·5	—	—	—	—	—	—	1	3	1	7	6	15	13	14	7	8	6	1	1	1	—	—	—	—	84
26·5	—	—	—	—	—	—	—	1	2	6	11	17	15	13	19	15	8	1	3	—	—	—	—	—	111
27·5	—	—	—	—	—	1	—	—	2	3	9	16	11	12	13	5	3	6	3	2	3	1	—	—	90
28·5	—	—	—	—	—	—	—	1	—	3	4	5	14	15	12	6	4	3	—	—	—	—	—	—	67
29·5	—	—	—	—	—	—	1	1	—	2	6	4	10	10	5	9	2	2	2	—	—	—	—	—	54
Totals	1	7	10	42	65	110	95	136	108	192	221	326	350	368	260	175	93	71	30	15	9	1	—	1	2686

* Provided by Dr Holzinger and supplementary to his work, *Biometrika*, Vol. xvi. p. 139.

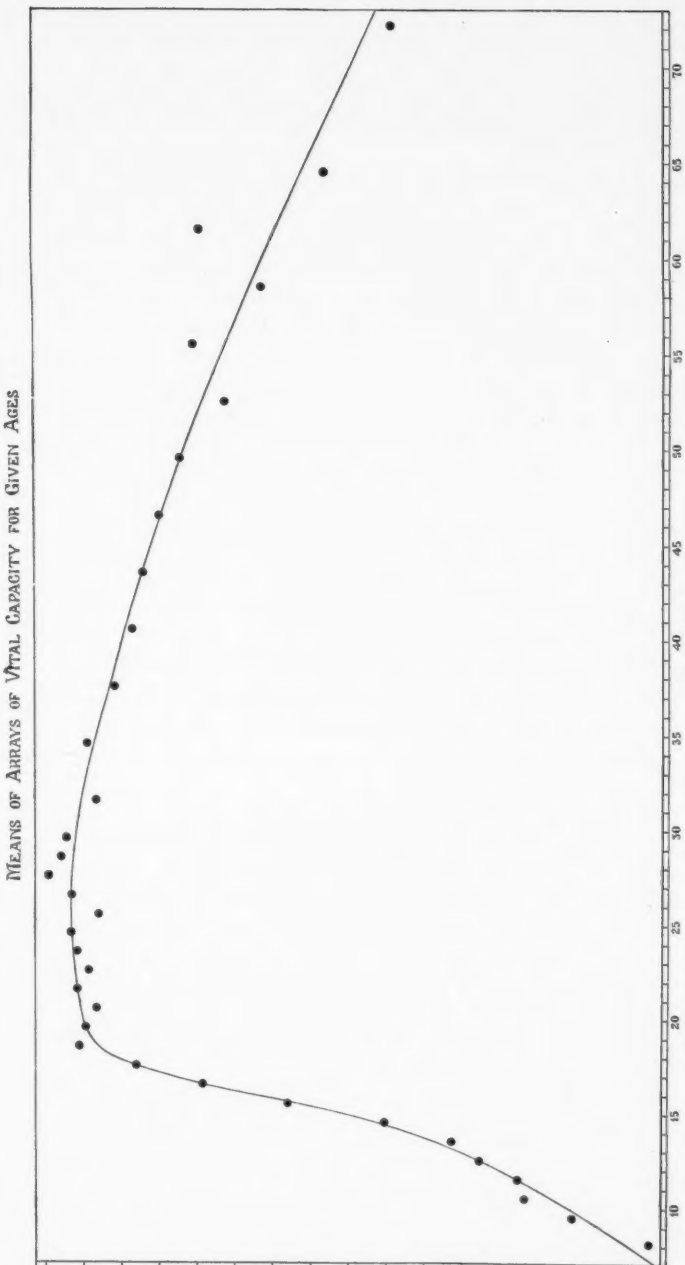


Diagram 9.

This Diagram is prepared from data provided by Dr Holzinger.

ON THE BIOMETRIC CONSTANTS OF THE HUMAN SKULL.

By KARL PEARSON, F.R.S. and ADELAIDE G. DAVIN.

(1) The object of the present paper differs to a large extent from the craniological work which has appeared previously in this Journal. We are not occupying ourselves with the racial characters of a particular group and with the relations of these characters to those of other groups. We shall concern ourselves here in the first place with our crania as representative of *Homo sapiens*, and ask what degrees of variability and of correlation are to be found in and between the constituent parts of the human skull. We are fully aware of the extent to which these correlations change from one local group or race of man to a second. But still there remains a question which to our minds has not yet been fully answered: To what intensity are variation and correlation present in the parts of the human skull? And, accordingly, what field does it present for the play of natural selection? We realise that this field may be larger or smaller according to the local race from which we draw our material. But yet with full recognition of this, it must be admitted that these constants, if in a very inadequate manner, are some of the factors which differentiate the human cranium from that of other primates and that a study of their general relationships is a matter of considerable importance.

The craniologist is always troubled by two fundamental difficulties. The first consists in the sparsity of his material. It is almost impossible to obtain statistically adequate samples. It is only recently that even a dawning of the importance of long series has reached the custodians of osteological museums; their aim had invariably been to obtain a few specimens of the greatest variety of local races, and supplement them whenever possible by a large collection of abnormalities. Such collections are of small use for the study of evolution by biometric methods*. The craniologist is not largely at fault. His material generally lies in small pockets, and there is no systematic method by which the contents of such small pockets may be adequately collected and reported on. In vulgar estimation the craniologist is still something of the body-snatcher. It is not recognised popularly that a study of human remains is of great scientific value and can be made without any irreverence. At the present time when building excavations lead to the discovery of human remains, the aim of the contractor is to get rid of them as quickly as possible without any regard to archaeological inquiry; the aim of the workmen is to carry off and sell as much as they can in their dinner bags, and the

* This criticism applies not only to osteological but to most museum collections whether of mammals, birds or insects. Even fossil shells which might have been collected by the hundred or even thousand will be represented by perhaps a dozen specimens, and thus no material is provided by aid of which the variation within the species in a given epoch—not even the variation of the "specific characters" themselves—can be tested.

aim of the flapper press is to get a photograph and label it "gruesome sight." The one man who may be held to exhibit any reverence is the scientist who desires to study what man was, is and may hereafter be. An Act of Parliament is sadly needed to insist that in all excavations where signs of human remains, bones or artefacts, are met with, there shall be an opportunity for due scientific inquiry, before the operations are proceeded with. There ought to be a standing committee,—say of the Royal Society,—to deal promptly with such discoveries, so as to avoid all unreasonable delay in building operations and yet preserve for adequate examination and study all that is of scientific value*. We trust the reader will pardon a digression only intended to illustrate the difficulty of the craniologist in even collecting small series in this country; in other parts of the world it may be far greater, as for example in China.

The second great difficulty of the craniologist lies in the measurements he proposes to take. He is burdened not only by the past history of his science, but also by the fact that he has got, if possible, to accommodate his system to the needs of the anthropometry of the living. If he neglects the past history of his science and adopts new and possibly better characters for record he cuts himself off from the possibility of forming comparisons with the wide range of measurements on innumerable races already made; and further he has no security that others will follow in his footsteps, so that his measurements may be of service to them. On the other hand the ideally best measurements from the anatomical standpoint are as a rule not those which are (i) most feasible on the living head, or (ii) if feasible, of most importance for racial physiognomy. Thus it comes about that the present—largely standardised—cranial measurements are far from ideal from the anatomical side, and are indeed as most osteological measurements on man distinctly defective, when we turn to comparison with other living forms. Anatomically it would be wise never to take a measurement on the skull which extended beyond the limits of a single constituent bone; all our problems of variation and correlation, besides those of interspecial comparison, would thus be much simpler and anatomically more valuable and intelligible. Such a study wants undertaking in and for itself, and would be exceedingly useful in researches on the primate skull. But it represents an innovation such as it was impossible to venture upon in 1910 with workers trained and interested in the craniometry of the standard type, and while the variability and correlation of the customary measurements were still unknown with any degree of reasonable accuracy. Accordingly it was settled that this investigation should be based on anthropometric rather than on purely osteological characters.

For the purpose in question a very long series was desirable and appeal was made to Professor Sir Flinders Petrie, as Egypt was the only locality from which material such as was needed could be procured. With his customary generosity he responded with a series of nearly 1800 crania dating from the XXVI—XXX Dynasties (B.C. 600 to B.C. 200) from excavations in a single cemetery south of the

* The importance of this matter is not emphasised without the provocative of recent experience !

Gizeh pyramids. Of course after subtraction of broken, abnormal and non-adult crania we had not more than 700 to 900 male and 400 to 600 female specimens to investigate correlation upon. But this material seemed ample for our purpose and far in excess of anything which had hitherto been available for determining biometric cranial constants. The variability of the characters being less than that found for the like cranial characters in smaller series from definite localities, we may conclude that it is reasonably homogeneous*, and certainly adequately homogeneous for our present purpose,—the study of variation and correlation in the human skull.

The measurement and reduction of this large mass of material have been the work of many years and of diverse workers in the Biometric Laboratory. Four successive Crewdson-Benington students:—Kathleen V. Ryley, Evelyn Y. Thomson, B.Sc., H. Dorothy Smith, B.Sc., and Adelaide G. Davin, B.Sc.—undertook the work of measurement. The still more laborious work of tabling and arithmetical reduction was entirely the work of Adelaide G. Davin†, and was carried on amid frequent interruptions by other duties during the years in which she was biological assistant in the Biometric Laboratory.

While it is fourteen years since the material reached this country, it has not been possible to complete and publish it until now; first owing to the heavy work of actually measuring some fifty odd characters in some 1600 and more crania; and secondly on account of the labour of preparing as many schedules, sorting them out, forming over 250 correlation tables and working these out numerically. So much arithmetical work of a like kind has hardly been done single handed before. In the years during which this work was undertaken five were entirely lost by war service. There is therefore reasonable excuse for delay up to the last few years, when the material came into the hands of the Director of the Laboratory, who unfortunately has always to compromise between the demands on his now-limited energies made by teaching duties on the one hand and editorial occupations on the other. Details from the measurements and reductions have been used in earlier papers. Thus by H. Dorothy Smith in 1912 in her "Observations on the Occipital Bone in a series of Egyptian Skulls" and in her "A Study of Pygmy Crania based on Skulls found in Egypt"‡. In the same year also in a paper on "Cranial Type Contours" written by K. Pearson on observations made by Dr R. Crewdson-Benington the type contours based on 100 of the male crania were provided§. In 1913 Kathleen V. Ryley and Dr Julia Bell used the material in their paper on "A Study of the Nasal Bridge...."‖ More recently the material has been used by Miriam L. Tildesley and G. M. Morant in their studies of the Burmese, Tibetan and Nepalese skulls, as a standard for variation in computing

* As homogeneous as the material taken from a single London plague pit for example.

† While the writing of this paper was in progress a certain number (66) of additional tables have been worked out by my colleagues Ethel M. Elderton and Margaret Moul. The values of the coefficients of correlation are included in Table V but not in Table VI.

‡ *Biometrika*, Vol. VIII. pp. 157—266 with 24 plates. Unfortunately the crania by an oversight were there described as of the third dynasty instead of round the third century B.C.

§ *Biometrika*, Vol. VIII. p. 130 *et seq.*

‖ *Biometrika*, Vol. IX. p. 396 *et seq.*

coefficients of Racial Likeness*. Thus the material has not been lying wholly idle for all these years.

(2) *Characters used.* The characters dealt with are those which have been usually measured in the Biometric Laboratory, and agree with some few modifications and additions with those of the International Concordat. We shall therefore not describe here their methods of measurement which have been often considered in papers in this Journal by Fawcett, Macdonell, Thomson and Morant. We give therefore merely an index to the letters by which they are indicated in the tables accompanying this paper.

C = capacity of skull; F = length from ophryon to occipital point; L = maximum length, glabella to occipital point; L' = glabellar horizontal length; B = maximum horizontal breadth on parietal bones; B' = minimum forehead breadth from one temporal crest to the other; H = basiobregmatic height; OH = auricular height as measured on craniophor; LB = length from basion to nasion.

U = horizontal circumference; Q = transverse arc perpendicular to the horizontal plane, passing through "apex" and terminating at the upper surfaces of the ear plugs; Q' = the same measurement when the skull is off the craniophor and terminating at the auricular points; S = the sagittal arc from nasion to opisthion; S_1, S_2, S_3 are the three sub-sagittal arcs, from nasion to bregma, from bregma to lambda and from lambda to opisthion respectively; S_3' = chord from lambda to opisthion.

GH = face height from nasion to pogonion; $G'H$ = upper face height from nasion to alveolar point; GB = face breadth from the lower end of one zygomatic-maxillary suture to the other; J = zygomatic breadth from the most lateral point of one zymotic arch to the corresponding point of the other; NH = nasal height from nasion to the lowest edge of left piriform aperture (the right piriform aperture being used solely if left has been damaged); NB = nasal breadth, i.e. greatest breadth of piriform aperture; $O_1(L)$, $O_1(R)$ greatest breadths of left and right orbits using Fawcett's curvature method to determine margins on nasal side; $O_2(L)$, $O_2(R)$ greatest heights of left and right orbits taken perpendicular to $O_1(L)$ and $O_1(R)$ respectively; G_1 = length of palate, from the point of the spina nasalis to an imaginary line tangential to the inner alveolar rims of the middle incisors; G_1' = similar measurement to G_1 from the base of the spine; G_2 = breadth of palate between the inner alveolar walls of the second molars; GL = profile length from basion to alveolar point; fmb = greatest breadth of foramen magnum; fml = length from basion to opisthion.

$P\angle$ = profile angle, or angle between standard horizontal plane and nasio-alveolar length; $A\angle$, $N\angle$, $B\angle$ are the three angles of the fundamental triangle ANB, which joins alveolar point to nasion and basion at the corresponding points; θ_1 = the angle the basio-nasion line makes with the horizontal plane; θ_2 = the angle the basio-alveolar line makes with the same place.

* *Biometrika*, Vol. XIII. p. 176 *et seq.* (1922), Vol. XIV. p. 207 *et seq.* (1923) and Vol. XVI. p. 13 *et seq.* (1924).

The indices determined individually were: $100\,B/L$ = First Cephalic Index; $100\,B/L'$ = First Cephalic Index (horizontal length); $100\,H/L$ = Second Cephalic Index; $100\,H/L'$ = Second Cephalic Index (horizontal length); $100\,B/H$ = Third Cephalic Index; $100\,GH/GB$ = total face index; $100\,G'H/GB$ = upper face index; $100\,NB/NH$ = nasal index; $100\,O_2/O_1\,(L)$ and $100\,O_2/O_1\,(R)$ = orbital indices left and right; $100\,G_2/G_1$ = palate index; $100\,fmb/fml$ = foraminal index; *Oc. I* = Occipital Index which is obtained from the chord and arc (S_2', S_2) of the occipital bone from lambda to opisthion, it measures the ratio of the radius of curvature of the occipital bone to the occipital chord, and was calculated in each individual case from the formula provided by Pearson, no tables having at that time been computed*. The aim of this index is to provide some crude measure of cerebellar development, and on this account it was originally termed by Pearson the *Cerebellar Index*. Its name was later changed to the Occipital Index as objections were raised to the original terminology.

Since the introduction of the index in 1906 it has been consistently measured by all biometric craniologists and, though the actual number of races for which it has been determined is still small, we have enough data to settle whether it is worth the trouble involved in its measurement and computation. The following table (Table I, pp. 334—35), for which we are largely indebted to the ever ready help of our colleague, G. M. Morant, summarises the available data.

If, as we hold probable, the Occipital Index is a rough measure of the development of the cerebellum, then the above table is not without suggestiveness. The apes would appear to have little motor-coordinating capacity; the Rhodesian man stands fairly close to them. Of palaeolithic man it is not possible to judge closely except in the Předmost skulls, but generally speaking the later palaeolithic men have greater cerebellar capacity than the earlier. Among modern men the negroes stand highest in index and lowest in motor-coordinating capacity, then follow the Melanesian and Polynesian races; the Asiatic races are low. Of European races the Turks, Greeks and Slovenes are lowest, the Teutonic races the highest. The crania of men of distinction appear to have the highest curvature of the occipital portion of the skull, i.e. the least occipital index†. Non-adults have high values of the index approaching those of the apes, and women, taken all round, have a somewhat higher value of this index than men; it is probable, however, that this difference may be fully accounted for by greater male bony growth in the neighbourhood of the inion. While the above values are very incomplete, and not uniformly reliable‡, being only such as have been actually measured or deduced from the measurements of others by workers in the Biometric Laboratory, they still seem to justify the use of this index, and it is hoped that craniologists in the future will determine the arc and chord of the occipital in the sagittal plane, even if they do not compute the index§.

* Tildesley's Tables (*Biometrika*, Vol. XIII., pp. 261, 262) have since been published.

† Many of these are approximative and uncertain.

‡ Very often the numbers are dangerously few; occasionally only the mean values of S_3 and S_3' have been given so that we had to approximate to the mean cerebellar index by corrections.

§ The computation is fairly rapid if Tildesley's table (*Biometrika*, Vol. XIII. p. 261) be used.

(3) *Means and Standardisation.* After this digression touching a character which we believe has not been hitherto adequately discussed, we now turn to the special topic of this paper. We have no intention of discussing the type of cranium of the XXVI—XXX Dynasty Egyptians but for the sake of completeness it is desirable to table here the means, standard deviations and coefficients of variation of this series known in the Biometric Laboratory as "Series E." Corresponding values for the Naqada series and Schmidt's Theban material are given alongside.

One of the difficulties of running a research laboratory is to get team work standardised and to maintain standards during a long range of years. The craniometric work of the Biometric Laboratory was initiated more than thirty-five years ago, at a time when the inherent difficulties of several of the definitions of the *Frankfurter Verständigung* had not been put to the test of mass practical experience. It was absolutely necessary to modify certain definitions, but having done so it was essential to maintain the modified measurements throughout all the work done in the Laboratory. Unfortunately it is very difficult to impress the importance of this on investigators who may be working in the Laboratory for a year or two at most, and who cannot be expected to take a long view of matters and treat their work as simply a part of a scheme extending over a quarter of a century. One research worker has a prejudice against one thing, another desires to measure a novel character in which he is especially interested and accordingly drops other measurements out of a very lengthy list. There is no harm whatever in adding new measurements, *if the old are retained*, but it is heart-rending in a long secular investigation to have to meet the divergences of individual workers naturally more interested in their own particular researches than in any general scheme of which they may never live to see the fruition.

Some illustration of this difficulty must be given here to explain why our measurements are not wholly comparable. For example, if the reader will examine Table III, he will observe that the coefficient of variation of the sagittal arc S is for the first Egyptian series 3·4 to 3·5. On the other hand the variabilities measured by the same coefficient of the component arcs S_1, S_2, S_3 are for males 4·9, 5·8, 5·9 and for females 4·6, 5·2, 6·1. This is partly due to the variety in position of the bregma and the lambda*. Undoubtedly this arises chiefly from the vagaries of the serrations of the sutures, and to a much less extent from the difficulty of accurately determining the exact bregma and lambda in aged crania. But in any case there is a compensatory influence, the total arc relative to its size is less variable than its components in the ratio of about 7 to 11. When a skull is placed on the craniophor and adjusted to the horizontal, the highest point of the skull in the vertical plane through the auricular points is the apex. The apex can be easily determined at any rate on the Ranke type of craniophor. To any one who has observed large numbers of skulls on the craniopher, the position of the bregma relative to the apex is known to be very variable. The direction of the *Frankfurter Verständigung* is to take the vertical height to the skull above the basion, that is to say, to measure the height parallel to the auricular axis and apex plane. This is

* See, however, our p. 345 below.

TABLE I. *The Occipital Index.*

Species or Race, etc.	Number	Male	Number	Female
Old World Monkeys, various ...	—	♂ + ♀ 7	69·3	69·0
New World Monkey, Ateles ...	—	— 1	67·3	
Gibbons*, various. Adult	—	♂ + ♀ 5	67·9	
„ Adolescent	—	♂ + ♀ 6	77·5	73·8
„ Young	—	♂ + ♀ 5	75·2	
Orangs*, (all ages)	—	♂ + ♀ 8	64·2	—
Chimpanzees*, Adult	—	♂ + ♀ 18	62·9	
„ Adolescent	—	♂ + ♀ 6	60·1	62·5
„ Young...	—	♂ + ♀ 8	63·2	
Gorillas*, Adolescent and Young ...	—	♂ + ♀ 10	61·0	—
Rhodesian Man†	—	68·0	—	—
Piltdown Skull	—	59·2 (??)	—	—
La Chapelle Man	—	61·0	—	—
Homo mousteriensis (*Hauseri*) ...	—	58·3 (??)	—	—
Predmost Crania	—	63·4	65·3	—
Gibraltar Skull	—	—	59·5	—
Homo aurignacensis (*Hauseri*) ...	—	58·4	—	—
Engis Skull	—	57·1	—	—
Tilbury Skull	—	56·2	—	—
Cromagnon Skull	—	56·5	—	—
Short Cist Skull	—	60·5	—	—
Neolithic, Trent Skull	—	59·4	—	—
„ Langwith Cave	—	57·2	—	—
Gaboon[1] (1864)	49	68·3	44	69·2
Gaboon[1] (1880)	15	66·3	19	69·5
Congo[1]	48	68·8	27	67·0
Juvenile Negroes[1]	5	66·7	—	—
Angoni[2], Nyassaland	15	60·7	8	62·7
Cameroon[3]	83	65·6	—	—
Modern Abyssinia[4] (Tigre) ...	57	62·0	23	61·8
„ „ (Galla and Somali)	8	62·2	—	—
„ „ (Kohaito) ...	17	59·4	8	59·6
Bushmen[5]	9	59·6	13	58·7
Egypt, Pygmy Crania[6]	—	—	12	63·3
„ "Middle" Dynasties[7] ...	30	62·3	—	—
„ "Early" Dynasties[8]... ...	66	62·0	—	—
„ 26th to 30th Dynasties[9]	885	61·5	592	61·98
„ 18th to 20th Dynasties[10] ...	54	61·4	—	—
„ 18th Dynasty[11]	25	60·6	—	—
„ Naqada, B, T, and R Graves[12]	23	60·5	—	—
„ Naqada, A and Q, Graves[13] .	84	60·2	—	—
„ 1st Dynasty, Abydos[14] ...	32	59·9	—	—
Egyptians: Total unweighted ...	—	61·05	—	—
„ Total weighted	1199	61·4	—	—
Bismarck Archipelago[15]	43	64·7	34	65·3
Santa Crux Islanders[16]	27	60·0	22	59·9
Solomon Islanders[17]...	♂ + ♀ 43	63·0	—	
Moriori[18]	32	62·2	20	64·2
Maori (Weisbach)[19]	15	61·1	—	—
„ (Mollison)[20]	♂ + ♀ 16	64·7	—	
Aborigines (N. S. Wales)[21]	21	60·1	11	63·8
Typical Australian Skull[22]	1	60·2	—	—

* The development of the occipital ridge in the anthropoids obscures this index. All the calvarial sutures seem to close completely very soon after the eruption of the third molars, and the determination of the lambda becomes in the case of adult gorillas impossible, one adult gorilla gave K. Pearson a very doubtful 69·6. "Adolescent" have all teeth, but third molars and canines fully erupted, "Young" are those with no 3rd molars or canines and with 2nd molars not fully erupted. The material does not suffice to separate local varieties. Sir Arthur Keith had in most cases determined age and sex.

† The measurements of the palaeolithic and neolithic crania are based on casts, and in several cases the reconstructions involved in preparing these are open to grave doubt. Professor Macgregor's complete reconstruction of Heidelberg man from the Mauer jaw gives an occipital index of 55·0, which seems to us quite out of the question.

References. [1] *Biometrika*, Vol. VIII. pp. 298—300, Table II. [2] Brit. Mus. (Nat. Hist.). [3] *Archiv f. Anthropologie*, Bd. XL. S. 176. [4] Sergi: *Crania Habessinica*, pp. 306, 395. [5] R. C. of S.s' Museum. [6] *Biometrika*, Vol. VIII. p. 265. [7] Toldt: *Denksch. d. k. Akad. d. Wissensch. Wien, Math. Nat. Kl.* Bd. XCVI. Tafeln S. 51–6. [8] *Idem*. [9] Present Memoir, Table II. [10] Stahr: *Die Rassenfrage in antiken Aegypten* 1907. Tafel. [11] Schultz and Oetteking: *Archiv f. Anthro-*

TABLE I—(*continued*).

Species or Race, etc.	Number	Male	Number	Female
Chinése Northern[23]	39	62·9	9	63·3
„ Southern[24]	19	62·1	—	—
Tibetan, A-Race[25]	37	61·8	—	—
„ B-Race[25]	15	59·2	—	—
„ Non-adult[25]	2	68·0	—	—
Nepalese[26]	47	60·4	[6	59·9 ?]
Hindoos[27]	[9	60·5 ?]	—	—
Burmese, A-Race[28]	44	62·8	39	64·1
„ B-Race[28]	[7	61·5 ?]	17	66·1
„ C-Race[28]	[8	62·8 ?]	18	61·4
„ Non-adult[28]	♂ + ♀ 7	68·4	—	
Mongol Torgods[29]	12	60·7	—	—
Altai Telenghites[29]	60	61·0	15	61·4
Buriats[30]	14	60·4	—	—
Kalmucks[31]	19	60·2	[6	59·6 ?]
Esquimo (cast)[32]	1	67·3	—	—
„ „ [32]	16	59·3	—	—
Turks[32b]	70	63·3	—	—
Greeks[33]	94	62·9	—	—
Serbo-Croatians[34]	80	62·8	—	—
Rumanians[34b]	40	62·7	—	—
Slovenes[35]	60	62·1	—	—
Guanches	79	60·6	44	61·8
Czechs[36]	101	60·5	49	60·5
Swiss (Daniser)[37]	40	60·3	—	—
„ (Alamans *c.* 300 A.D.)[38] ...	♂ + ♀ 176	58·9	—	
Bavarians (Alps)[39]	101	59·0	44	61·1
Tyrolese (Austria)[40]	24	59·6	14	59·7
Walser (Vorarlberg)[41]	43	58·1	25	58·8
Austrians[42]	36	58·3	—	—
Friesians (Terps)[43]	♂ + ♀ 35	59·2	—	
„ (Marken)[43]	♂ + ♀ 13	59·1	—	
Anglo-Saxons[44]	26	58·4	26	59·5
English, 17th Century[45]	40	58·5	49	59·2
„ Murderers[46]	7	58·2	—	—
Robert Burns (from cast)[47]	—	60·0 (?)	—	—
Professor Cullimore[48]	—	59·0	—	—
Robert, the Bruce[49]	—	57·2 (?)	—	—
Marshall Hess[50]	—	57·1	—	—
Friederich Mohs[51]	—	57·0	—	—
Schubert[52]	—	56·5	—	—
Descartes[53]	—	56·1 (?)	—	—
J. S. Bach[53]	—	56·0	—	—
Sir Thomas Browne[54]	—	55·8	—	—
Kant[55]	—	55·4	—	—
Henry Stuart, Lord Darnley[56] ...	—	55·4	—	—
Schiller[53]	—	54·4	—	—
Haydn[53]	—	54·5	—	—

pologie, Bd. XXXVI. Tafeln S. 70—89, Bd. XLIV. S. 78–9 (Sexing). [12] Biometric Lab. Cranial Collection. [13] *Ibid.* [14] *Ibid.* [15] *Archiv f. Anthropologie*, Bd. XLII. S. 196. [16] *Ibid.* Bd. XLVII. S. 127. [17] *Ibid.* Bd. XL. S. 272, Tafeln. [18] *Biometrika*, Vol. XI. S. 90. [19] *Mitth. d. Anthrop. Gesellsch. in Wien*, Bd. XX. S. 32. [20] *Zeitschrift f. Morph. u. Anthrop.* Bd. XI. S. 580. [21] *Mitth. d. Anthrop. Gesellsch. in Wien*, Bd. XLV. S. 79. [22] Cast in Biometric Laboratory. [23] *Zeitschrift f. Morph. u. Anthrop.* Bd. XV. S. 544, Tafel 6 and Haberer: *Schädel...aus Peking*, 1902, S. 86. [24] S_3' for 19 from Schmidt's Katalog, S_3 mean of 74 from various sources; Schmidt does not provide S_3. In all other cases S_3 and S_3' are from the same individual crania. [25] *Biometrika*, Vol. XIV. p. 206. [26] *Ibid.* Vol. XVI. pp. 28—31. [27] Biometric Lab. Cranial Collection. [28] *Biometrika*, Vol. XIII. p. 219. [29] *Zeitschrift f. Morph. u. Anthrop.* Bd. XV. S. 544 u. Tafeln. [30] *Ibid.* S. 544 u. Tafeln. [31] *Ibid.* S. 544 u. Tafeln. [32] Cast of Typical Eskimo skull in Biometric Laboratory; 16 Crania in R. C. of S.s' Museum. [32b] *Mitth. d. Anthrop. Gesellsch. in Wien*, Bd. III. S. 206—213. [33] *Mitth. d. Anthrop. Gesellsch. in Wien*, Bd. XI. S. 72. [34] Weisbach: Suppl. 1884, *Zeitschrift f. Ethnologie*, S. 70–2. [34b] *Denksch. d. k. Akad. d. Wissensch. Wien, Math. Nat. Kl.* Bd. XXX. S. 107—136. [35] *Mitth. d. Anthrop. Gesellsch. in Wien*, Bd. XLII. S. 84, Tabelle I—II. [36] *Archiv f. Anthropologie*, Bd. XXXIX. S. 282. [37] *Zeitschrift f. Morph. u. Anthrop.* Bd. XV. Tafeln S. 544. [38] *Ibid.* Bd. XIV. S. 649. [39] *Beiträge z. Anthrop. u. Urgesch. Bayerns*, Bd. XVIII. S. 1. [40] *Mitth. d. Anthrop. Gesellsch. in Wien*, Bd. XXXIX. S. 17—22. [41] *Zeitschrift f. Ethnologie*, Bd. XLIV. S. 509. [42] *Mitth. d. Anthrop. Gesellsch. in Wien*, Bd. XVIII. S. 79. [43] *Zeitschrift f. Morph. u. Anthrop.* Bd. XVI. S. 396, S. 478. [44] R. C. of S.s' and London Museums Skulls. [45] *Biometrika*, Vol. V. p. 92. [46] Skulls in R. C. of S.s' Museum. [47] Cast in R. C. of S.s' Museum. [48] *Journal of Anatomy*, Vol. LII. p. 282 from plates. [49] From cast in Department of Anatomy, University of Edinburgh, approximate. [50] *Mitth. d. Anthrop. Gesellsch. in Wien*, Bd. XVIII. S. 91. [51] *Ibid.* Bd. XVIII. S. 92. [52] *Ibid.* Bd. XVIII. p. 79. [53] From photographs. The occipital index as obtained from photographs is only approximate, the arc and chord are measured on the *norma lateralis*, the former with a map-wheel. The position of the opisthion, if no *norma basalis* is available, is the chief difficulty, and it has to be placed with careful consideration of the "springings" of the mastoids. [54] *Biometrika*, Vol. XV. p. 59. [55] *Archiv f. Anthropologie*, Bd. XII. S. 405–7. [56] From skull in R. C. of S.s' Collection.

not really such a hard measurement to make with the skull on the craniophor. Macdonell would have nothing to do with it, however much persuasion was applied; he said he must have a fixed point on the skull, he declined to be guided by the apex and persisted in using the more variable bregma. In the same way he would not take the transverse arc Q through the apex, but persisted in taking it through the bregma, and thus introduced a new variation of Q, which I call Q''. The reader who examines Table III will see how Macdonell's transverse arc has increased the variability of Q owing to his taking it through the variable bregma. In the same way his use of H' instead of H in the four height indices has increased and considerably increased seven out of the eight coefficients of variation and six out of the eight standard deviations. The introduction of H' and Q'' weakens the value of Macdonell's results for the purposes of general comparison. Again the adoption of two transverse arcs Q and Q' through the apex does not seem very satisfactory, for it is so likely to lead to one or other being dropped. It is very well to define the transverse arc as in the *Verständigung* as being taken from auricular point to auricular point through the apex. But it is not easy to bend the ordinary steel tape close to the skull round the ridges above the auricular passages and inwards to the auricular points. The use of Q instead of Q' was only adopted after considerable testing and results seem to show that it is more satisfactory and less variable.

When we come to nose, orbit and palate there has been and still is considerable diversity of treatment. Taking the nose first the *Verständigung* is perfectly definite. We have to measure to the lowest point of the border of the piriform aperture from the nasion. In the Laboratory practice the left aperture was used (unless damaged when the right was taken); then the custom of taking both $NH(R)$ and $NH(L)$ crept in, and now it is suggested that a third height should be taken, for the sake of comparison with those who have used a height measured in the median plane. This means the recording of three heights and three separate nasal indices! The difficulty, however, is to determine what the craniologists who have measured in the median plane have really measured. If they define the nasal height to be from nasion to nasal spine, what point of the spine have they measured to? If to the *tip*, then the difficulty at once arises that in ancient skulls the tip is almost invariably rubbed down or broken off, and in aged skulls has been absorbed. If the measurement be taken to the little platform behind the tip, there is no well-defined point on it; there is nothing nearly as definite as the border of the piriform aperture. That nasal indices read with a nasal height measured to piriform aperture and again in the median plane differ very considerably is perfectly true, but to encourage in any way measurement in the median plane is I hold undesirable, for such measurements as have been made in that plane show differences which I hold to be as much due to personal equation in measurement as to racial differentiation. It is very sad, but I doubt if the recorded values of nasal indices made by many observers in different ways are, except in the case of wide racial differences, of any real value for fine analysis.

We next come to the orbital measurements. Here the dacryon provided a definite point indeed for the length measurement, but in its own small way it is as

TABLE II. *Means of Present Series compared with Theban Mummies and the Naqada Crania.*

Character Reference Letter	Males						Females					
	Egyptians XXVI—XXX Dynasties		Egyptians XVIII—XXI Dynasties Thebes, Schmidt		Prehistoric Naqada, Fawcett		Egyptians XXVI—XXX Dynasties		Egyptians XVIII—XXI Dynasties Thebes, Schmidt		Prehistoric Naqada, Fawcett	
C	753	1438·86 ± 2·79	164	1387·63	88	1380·1 ± 7·7	472	1301·08 ± 2·86	74	1250·27	123	1287·9 ± 5·4
F	895	182·84 ± ·13	—	—	121	183·96 ± ·37	590	175·86 ± ·13	—	—	168	177·41 ± ·29
L	895	185·34 ± ·13	169	181·94	139	185·13 ± ·33	589	177·06 ± ·12	77	174·81	185	177·47 ± ·28
L′	872	184·62 ± ·13	—	—	121	184·87 ± ·37	579	176·44 ± ·13	—	—	159	177·89 ± ·29
B	896	138·88 ± ·11	169	136·63	139	134·87 ± ·26	587	135·55 ± ·13	77	134·31	185	131·50 ± ·23
B′	935	94·78 ± ·09	168	93·83	140	91·06 ± ·27	628	92·25 ± ·10	77	91·40	181	88·23 ± ·20
H	884	134·08 ± ·11	169	136·05	134	135·21 ± ·31	582	129·20 ± ·12	77	130·51	169	129·47 ± ·25
OH	887	113·43 ± ·09	169	114·34	140	115·59 ± ·25	588	109·98 ± ·10	76	109·75	174	113·11 ± ·20
LB	896	101·64 ± ·09	169	100·63	109	99·34 ± ·31	595	96·51 ± ·10	76	95·29	141	94·86 ± ·25
U	876	518·66 ± ·31	168	510·76	118	511·02 ± ·81	583	499·69 ± ·33	77	494·75	146	493·74 ± ·64
S	884	371·86 ± ·28	168	372·44	119	373·02 ± ·74	583	359·75 ± ·29	76	357·93	151	363·64 ± ·70
Q	876	307·45 ± ·22	169	—	116	304·22 ± ·63	576	298·25 ± ·25	77	—	151	296·49 ± ·46
Q′	878	307·19 ± ·23	—	306·07	105	303·76 ± ·65	579	296·35 ± ·25	—	296·21	—	—
S_1	928	127·42 ± ·14	—	—	122	127·65 ± ·38	617	123·86 ± ·15	—	—	—	—
S_2	908	128·46 ± ·17	—	—	120	130·10 ± ·46	606	123·90 ± ·18	—	—	—	—
S_3	889	115·23 ± ·15	—	—	108	116·90 ± ·44	593	111·59 ± ·19	—	—	—	—
S_3'	885	97·07 ± ·11	—	—	107	96·60 ± ·31	594	94·66 ± ·13	—	—	—	—
GH	—	—	151	114·31	28	112·02 ± ·76	—	—	72	107·43	27	109·92 ± ·74
G′H	845	70·41 ± ·10	169	68·78	85	67·59 ± ·30	567	66·74 ± ·11	74	65·96	119	65·84 ± ·28
GB	877	95·20 ± ·11	161	95·66	82	95·85 ± ·37	581	91·56 ± ·12	71	91·61	113	92·51 ± ·28
J	785	128·70 ± ·11	166	128·33	53	125·63 ± ·48	500	120·87 ± ·13	71	121·22	63	117·04 ± ·47
NH	898	51 ± ·07	169	50·61	91	48·94 ± ·21	593	48·97 ± ·07	74	48·36	123	46·73 ± ·19
NB	893	24·36 ± ·04	166	25·26	86	25·12 ± ·14	591	23·55 ± ·05	72	24·21	119	24·31 ± ·11
$O_1(L)$	880	40·75 ± ·04	169	38·14	82	43·11 ± ·16	587	39·51 ± ·04	73	37·04	118	41·70 ± ·14
$O_1(R)$	886	41·13 ± ·04			81	42·60 ± ·16	589	39·90 ± ·04			113	41·30 ± ·14
O_1'	—	—	—	—	76	39·10 ± ·17	—	—	—	—	—	—
$O_2(L)$	888	33·83 ± ·04	169	32·96	80	32·67 ± ·17	590	33·70 ± ·05	73	32·25	117	32·16 ± ·13
$O_2(R)$	888	33·73 ± ·04			72	31·87 ± ·18	595	33·49 ± ·05			116	31·87 ± ·14
G_1'	821	46·28 ± ·07	160	46·25	66	47·51 ± ·26	560	44·51 ± ·08	69	44·13	—	—
G_1''	—	—	—	—	75	55·80 ± ·28	—	—	—	—	103	53·89 ± ·26
G_1	817	49·70 ± ·08	—	—	66	52·04 ± ·28	552	47·36 ± ·09	—	—	—	—
G_2	703	38·81 ± ·07	151	40·83	75	40·33 ± ·29	462	37·01 ± ·08	69	38·67	103	38·87 ± ·22
GL	832	95·03 ± ·11	169	95·79	81	94·72 ± ·34	563	90·75 ± ·12	74	91·93	106	91·02 ± ·30
fmb	905	29·96 ± ·05	168	29·80	88	29·54 ± ·15	607	28·79 ± ·05	—	—	—	—
fml	894	35·56 ± ·06	—	35·10	89	35·58 ± ·18	602	34·39 ± ·06	—	—	—	—
P ∠	795	85°·81 ± ·08	47	85°·49	62	84°·41 ± ·25	529	84°·93 ± ·09	28	85°·57	89	84°·49 ± ·26
A ∠	818	74°·08 ± ·08	167	{73°·0}	73	72°·82 ± ·32	550	73°·76 ± ·10	71	{72°·4}	93	72°·62 ± ·22
N ∠	818	63°·92 ± ·08	167	{66°·1}	73	66°·60 ± ·30	550	64°·46 ± ·09	71	{66°·3}	93	66°·40 ± ·31
B ∠	818	41°·85 ± ·06	167	{40°·9}	73	40°·73 ± ·22	550	41°·63 ± ·07	71	{41°·3}	93	40°·98 ± ·19
θ_1	795	30°·03 ± ·07	47	{28°·4}	62	28°·29 ± ·29	529	30°·28 ± ·08	28	{28°·13}	89	29°·09 ± ·25
θ_2	795	11°·89 ± ·08	47	{12°·5}	62	11°·30 ± ·34	529	11°·33 ± ·09	28	{13°·2}	89	11°·82 ± ·26
100 *B*/*L′*	866	75·34 ± ·06	—	—	101	72·70 ± ·19	574	76·89 ± ·07	—	—	146	73·86 ± ·16
100 *B*/*L*	884	75·06 ± ·06	169	75·07	130	72·99 ± ·17	584	76·61 ± ·07	77	76·53	169	74·19 ± ·16
100 *H*/*L′*	863	72·72 ± ·07	—	—	98	73·20 ± ·18	570	73·32 ± ·08	—	—	141	73·11 ± ·16
100 *H*/*L*	871	72·46 ± ·07	169	74·71	131	73·30 ± ·16	570	73·07 ± ·08	77	74·69	166	73·22 ± ·16
100 *H*/*B*	884	{96·54}	169	99·64	131	100·47 ± ·28	582	{95·32}	77	97·26	169	98·66 ± ·25
100 *B*/*H*	873	103·70 ± ·10	169	100·36	131	99·76 ± ·27	570	104·97 ± ·12	77	102·81	163	101·59 ± ·25
100 *GH*/*GB*	—	—	148	112·20	25	117·06 ± ·72	—	—	71	112·75	22	118·87 ± ·97
100 *G′H*/*GB*	823	74·13 ± ·12	155	72·32	76	70·63 ± ·35	547	73·03 ± ·13	71	72·27	106	70·36 ± ·27
100 *NB*/*NH*	881	47·31 ± ·09	166	50·07	77	51·08 ± ·32	580	47·75 ± ·11	72	50·23	113	52·31 ± ·31
100 $O_2/O_1(L)$	876	83·06 ± ·11	—	—	77	74·89 ± ·39	584	85·42 ± ·13	—	—	111	76·91 ± ·29
100 $O_2/O_1(R)$	878	82·20 ± ·12	—	—	76	74·87 ± ·39	585	84·00 ± ·13	—	—	113	77·32 ± ·48
100 O_2/O_1'	—	—	169	86·54	76	82·32 ± ·38	—	—	73	98·81	—	—
100 G_2/G_1	668	78·36 ± ·18	—	—	65	78·02 ± ·57	432	78·41 ± ·22	—	—	—	—
100 G_2/G_1'	—	—	133	88·51	65	85·48 ± ·56	—	—	60	88·34	—	—
100 G_2/G_1''	—	—	—	—	67	71·94 ± ·61	—	—	—	—	97	72·30 ± ·41
100 *fmb*/*fml*	889	84·47 ± ·13	165	85·50	88	83·06 ± ·42	597	83·95 ± ·15	—	—	—	—
Oc. *I.*	884	61·54 ± ·07	—	—	107	60·27 ± ·22	592	61·98 ± ·09	—	—	—	—

TABLE III.

Standard Deviations and Coefficients of Variation of the Present Series

Varying Character Reference Letter	Males					
	Egyptians, XXVI—XXX Dynasties		Prehistoric, Naqada		17th Century, English	
	Standard Deviation	Coefficient of Variation	Standard Deviation	Coefficient of Variation	Standard Deviation	Coefficient of Variation
C	113·51 ± 1·97	7·89 ± ·14	106·68 ± 5·42	7·72 ± ·39	122·37 ± 6·88	8·28 ± ·47
F	5·73 ± ·09	3·14 ± ·05	6·03 ± ·26	3·28 ± ·14	6·17 ± ·25	3·29 ± ·13
L	5·72 ± ·09	3·09 ± ·05	5·99 ± ·26	3·24 ± ·14	6·27 ± ·25	3·31 ± ·13
L′	5·72 ± ·09	3·10 ± ·05	5·75 ± ·23	3·17 ± ·13	5·64 ± ·32	3·00 ± ·17
B	4·76 ± ·08	3·43 ± ·05	4·60 ± ·19	3·29 ± ·13	5·28 ± ·22	3·75 ± ·15
B′	4·05 ± ·06	4·28 ± ·07	4·82 ± ·19	5·29 ± ·21	4·20 ± ·17	4·29 ± ·17
H	5·03 ± ·08	3·75 ± ·06	5·38 ± ·22	3·98 ± ·16	(*H′*) 5·56 ± ·24	(*H′*) 4·21 ± ·18
OH	4·12 ± ·07	3·63 ± ·06	4·46 ± ·18	3·86 ± ·16	4·28 ± ·18	3·73 ± ·15
LB	3·97 ± ·06	3·90 ± ·06	4·85 ± ·22	4·88 ± ·22	4·13 ± ·18	4·07 ± ·18
U	13·77 ± ·22	2·65 ± ·04	13·00 ± ·51	2·54 ± ·11	15·02 ± ·63	2·87 ± ·12
S	12·51 ± ·20	3·36 ± ·05	11·91 ± ·52	3·19 ± ·14	13·69 ± ·57	3·63 ± ·15
Q	9·75 ± ·16	3·17 ± ·05	10·11 ± ·45	3·32 ± ·15	(*Q′*	*Q′*)
Q′	9·89 ± ·16	3·22 ± ·05	—	—	(11·40 ± ·51	3·70 ± ·16)
S_1	6·22 ± ·10	4·88 ± ·08	—	—	5·55 ± ·40†	4·29 ± ·31†
S_2	7·42 ± ·12	5·77 ± ·09	—	—	7·61 ± ·55†	5·91 ± ·43†
S_3	6·81 ± ·11	5·91 ± ·10	—	—	8·48 ± ·64†	7·04 ± ·53†
S_3'	4·81 ± ·08	4·96 ± ·08	—	—	5·94 ± ·45†	6·05 ± ·46†
GH	—	—	5·97 ± ·54	5·33 ± ·48	—	—
G′H	4·15 ± ·07	5·90 ± ·10	4·11 ± ·21	6·08 ± ·31	3·86 ± ·21	5·50 ± ·30
GB	4·67 ± ·08	4·90 ± ·08	4·97 ± ·26	5·18 ± ·27	5·07 ± ·32	5·58 ± ·36
J	4·57 ± ·08	3·55 ± ·06	5·22 ± ·34	4·16 ± ·27	5·57 ± ·40	4·28 ± ·31
NH	2·92 ± ·05	5·65 ± ·09	3·00 ± ·15	6·13 ± ·31	2·60 ± ·14	5·08 ± ·27
NB	1·77 ± ·03	7·27 ± ·12	1·98 ± ·10	7·89 ± ·41	2·16 ± ·12	8·89 ± ·51
$O_1(L)$	1·65 ± ·03	4·06 ± ·07	2·14 ± ·11	4·97 ± ·26	1·81 ± ·11	4·20 ± ·25
$O_1(R)$	1·67 ± ·03	4·06 ± ·07	2·14 ± ·11	5·02 ± ·27	2·02 ± ·12	4·69 ± ·27
$O_2(L)$	1·88 ± ·03	5·56 ± ·09	2·31 ± ·12	7·06 ± ·38	1·88 ± ·11	5·61 ± ·33
$O_2(R)$	1·91 ± ·03	5·67 ± ·09	2·32 ± ·13	7·27 ± ·41	2·22 ± ·13	6·65 ± ·38
G_1'	3·10 ± ·05	6·69 ± ·11	(G_1'')	(G_1'')	2·53 ± ·14	5·67 ± ·33
G_1	3·33 ± ·06	6·70 ± ·11	(3·62 ± ·20	6·49 ± ·36	2·74 ± ·16	5·68 ± ·33
G_2	2·63 ± ·05	6·78 ± ·12	3·75 ± ·21	9·29 ± ·51	2·85 ± ·17	7·75 ± ·46
GL	4·85 ± ·08	5·10 ± ·08	4·58 ± ·44	4·84 ± ·26	4·49 ± ·25	4·68 ± ·26
fmb	2·15 ± ·03	7·18 ± ·11	—	—	1·82 ± ·15†	6·10 ± ·50†
fml	2·47 ± ·04	6·95 ± ·11	—	—	2·86 ± ·23†	8·08 ± ·65†
P ∠	3°·24 ± ·05	3·78 ± ·06	2°·87 ± ·17	3·40 ± ·21	3°·92 ± ·24	4·55 ± ·27
A ∠	3°·46 ± ·06	4·67 ± ·08	4°·16 ± ·23	5·71 ± ·32	3°·41 ± ·20	4·65 ± ·27
N ∠	3°·31 ± ·06	5·18 ± ·09	3°·77 ± ·21	5·66 ± ·32	3°·52 ± ·20	5·40 ± ·31
B ∠	2°·66 ± ·04	6·34 ± ·11	2°·80 ± ·16	6·87 ± ·38	2°·50 ± ·14	6·03 ± ·35
θ_1	2°·89 ± ·05	9·63 ± ·16	3°·34 ± ·20	11·81 ± ·73	2°·53 ± ·16	8·81 ± ·55
θ_2	3°·16 ± ·05	26·56 ± ·48	3°·92 ± ·24	34·69 ± 2·35	3°·34 ± ·21	25·85 ± 1·94
100 *B*/*L′*	2·67 ± ·04	3·55 ± ·02	2·88 ± ·13	3·96 ± ·18	2·97 ± ·17	3·95 ± ·23
100 *B*/*L*	2·68 ± ·06	3·57 ± ·06	2·80 ± ·12	3·83 ± ·16	3·26 ± ·14	4·38 ± ·18
100 *H*/*L′*	2·94 ± ·05	4·04 ± ·07	2·77 ± ·13	3·85 ± ·17	(*H′*) 2·67 ± ·15	(*H′*) 3·80 ± ·22
100 *H*/*L*	2·94 ± ·05	4·06 ± ·07	2·73 ± ·11	3·73 ± ·16	(*H′*) 3·22 ± ·14	(*H′*) 4·61 ± ·20
100 *H*/*B*	4·00 ± ·06	4·14 ± ·07	4·73 ± ·20	4·72 ± ·20	(*H′*) 4·58 ± ·20	(*H′*) 4·86 ± ·21
100 *B*/*H*	4·30 ± ·06	4·14 ± ·07	4·53 ± ·19	4·54 ± ·19	(*H′*) 5·14 ± ·23	(*H′*) 4·83 ± ·21
100 *GH*/*GB*	—	—	5·30 ± ·51	4·53 ± ·43	—	—
100 *G′H*/*GB*	4·96 ± ·08	6·69 ± ·11	4·52 ± ·25	6·41 ± ·35	5·39 ± ·35	7·04 ± ·46
100 *NB*/*NH*	3·82 ± ·06	8·08 ± ·13	4·18 ± ·23	8·18 ± ·44	4·58 ± ·26	9·64 ± ·55
100 $O_2/O_1(L)$	4·95 ± ·08	5·96 ± ·10	5·06 ± ·27	6·76 ± ·37	3·78 ± ·23	4·86 ± ·29
100 $O_2/O_1(R)$	5·05 ± ·08	6·15 ± ·10	5·00 ± ·27	6·68 ± ·36	4·66 ± ·27	6·00 ± ·35
100 G_2/G_1	6·79 ± ·13	8·67 ± ·16	(G_1'') 7·36 ± ·43	(G_1'') 10·23 ± ·60	6·40 ± ·39	8·39 ± ·52
100 *fmb*/*fml*	5·79 ± ·09	6·86 ± ·11	—	—	6·64 ± ·54†	7·89 ± ·65†
Oc. I.	3·30 ± ·05	5·36 ± ·09	—	—	2·15 ± ·16†	3·68 ± ·28†

Macdonell used *H′* height from basion to bregma and *Q″* transverse arc from one auricular point to the other through the bregma. Fawcett used G_1''.

† Supplied from the Moorfields Series.

TABLE III.—(*continued*).

and comparison with the Naqada Values and 17th Century English Values.

Varying Character Reference Letter	Females					
	Egyptians, XXVI—XXX Dynasties		Prehistoric, Naqada		17th Century English	
	Standard Deviation	Coefficient of Variation	Standard Deviation	Coefficient of Variation	Standard Deviation	Coefficient of Variation
C	92·06 ± 2·02	7·08 ± ·16	89·18 ± 3·84	6·92 ± ·30	112·80 ± 6·01	8·68 ± ·47
F	4·77 ± ·09	2·71 ± ·05	5·59 ± ·21	3·15 ± ·12	6·38 ± ·25	3·54 ± ·14
L	4·72 ± ·08	2·66 ± ·05	5·51 ± ·21	3·10 ± ·12	6·22 ± ·25	3·45 ± ·14
L′	4·77 ± ·09	2·70 ± ·05	5·57 ± ·21	3·10 ± ·12	6·33 ± ·40	3·52 ± ·22
B	4·52 ± ·09	3·34 ± ·07	4·55 ± ·16	3·45 ± ·12	4·77 ± ·19	3·54 ± ·14
B′	3·79 ± ·07	4·11 ± ·08	3·94 ± ·14	4·47 ± ·16	4·23 ± ·17	4·55 ± ·18
H	4·37 ± ·09	3·39 ± ·07	4·74 ± ·17	3·66 ± ·13	(*H′*) 4·93 ± ·21	(*H′*) 3·96 ± ·17
OB	3·65 ± ·07	3·32 ± ·07	4·00 ± ·14	3·54 ± ·13	4·50 ± ·18	4·12 ± ·16
LB	3·52 ± ·07	3·65 ± ·07	4·44 ± ·18	4·68 ± ·19	3·91 ± ·17	4·11 ± ·18
U	11·76 ± ·23	2·35 ± ·05	11·47 ± ·46	2·27 ± ·09	14·70 ± ·60	2·92 ± ·12
S	10·49 ± ·21	2·92 ± ·06	12·77 ± ·50	3·51 ± ·14	14·16 ± ·59	3·90 ± ·16
Q	8·72 ± ·17	2·92 ± ·06	8·07 ± ·33	2·72 ± ·11	{ *Q″*	*Q″* }
Q′	8·84 ± ·18	2·98 ± ·06	—	—	{11·67 ± ·50	3·97 ± ·17}
S_1	5·65 ± ·11	4·56 ± ·09	—	—	6·17 ± ·40†	4·91 ± ·32†
S_2	6·43 ± ·12	5·19 ± ·10	—	—	7·29 ± ·48†	5·90 ± ·39†
S_3	6·77 ± ·13	6·06 ± ·12	—	—	9·01 ± ·61†	7·70 ± ·53†
S_3'	4·75 ± ·09	5·02 ± ·10	—	—	6·09 ± ·41†	6·35 ± ·43†
GH	—	—	5·68 ± ·52	5·17 ± ·18	—	—
G′H	3·76 ± ·08	5·64 ± ·11	4·47 ± ·20	6·87 ± ·30	4·71 ± ·28	7·14 ± ·43
GB	4·35 ± ·09	4·75 ± ·09	4·41 ± ·20	4·77 ± ·21	4·59 ± ·29	5·40 ± ·34
J	4·38 ± ·09	3·62 ± ·08	5·58 ± ·34	4·77 ± ·29	4·97 ± ·41	4·13 ± ·34
NH	2·60 ± ·05	5·31 ± ·10	3·18 ± ·14	6·81 ± ·29	2·70 ± ·16	5·55 ± ·32
NB	1·64 ± ·03	6·98 ± ·14	1·80 ± ·08	7·28 ± ·32	1·64 ± ·10	7·06 ± ·42
$O_1(L)$	1·57 ± ·03	3·97 ± ·08	2·21 ± ·10	5·30 ± ·24	1·45 ± ·09	3·53 ± ·22
$O_1(R)$	1·56 ± ·03	3·92 ± ·08	2·22 ± ·10	5·38 ± ·24	1·64 ± ·10	4·00 ± ·24
$O_2(L)$	1·89 ± ·04	5·62 ± ·11	2·12 ± ·09	6·58 ± ·29	1·45 ± ·09	4·31 ± ·26
$O_2(R)$	1·84 ± ·04	5·50 ± ·11	2·18 ± ·10	6·85 ± ·38	1·51 ± ·09	4·47 ± ·27
G_1'	2·70 ± ·05	6·06 ± ·12	{ (G_1'')	(G_1'') }	2·53 ± ·16	6·10 ± ·38
G_1	2·96 ± ·06	6·26 ± ·13	{3·98 ± ·19	7·41 ± ·35	{ 2·95 ± ·19	6·53 ± ·41
G_2	2·53 ± ·06	6·83 ± ·15	3·32 ± ·16	8·55 ± ·40	2·70 ± ·17	7·68 ± ·48
GL	4·08 ± ·08	4·49 ± ·09	4·63 ± ·21	5·09 ± ·24	4·47 ± ·28	4·95 ± ·31
fmb	2·00 ± ·04	6·96 ± ·14	—	—	2·43 ± ·16†	8·39 ± ·57†
fml	2·19 ± ·04	6·37 ± ·12	—	—	2·49 ± ·17†	7·26 ± ·49†
P∠	2°·95 ± ·06	3·48 ± ·07	3°·66 ± ·18	4·33 ± ·22	2°·85 ± ·19	3·27 ± ·22
A∠	3°·41 ± ·07	4·62 ± ·09	3°·14 ± ·16	4·32 ± ·21	3°·31 ± ·21	4·48 ± ·28
N∠	3°·26 ± ·07	5·06 ± ·10	4°·48 ± ·22	6·75 ± ·34	2°·53 ± ·16	3·91 ± ·25
B∠	2°·33 ± ·05	5·60 ± ·11	2°·76 ± ·14	6·73 ± ·33	2°·98 ± ·19	7·20 ± ·46
θ_1	2°·64 ± ·05	8·71 ± ·18	3°·44 ± ·17	11·83 ± ·61	2°·51 ± ·17	8·93 ± ·61
θ_2	2°·90 ± ·06	25·62 ± ·57	3°·65 ± ·18	30·88 ± 1·71	3°·60 ± ·24	7·42 ± 1·99
100 *B*/*L′*	2·57 ± ·05	3·34 ± ·07	2·89 ± ·11	3·92 ± ·12	3·01 ± ·19	4·03 ± ·26
100 *B*/*L*	2·56 ± ·05	3·34 ± ·07	3·12 ± ·11	4·20 ± ·11	2·98 ± ·12	3·99 ± ·17
100 *H*/*L′*	2·86 ± ·06	3·90 ± ·08	2·81 ± ·11	3·85 ± ·15	(*H′*) 2·91 ± ·19	(*H′*) 4·21 ± ·27
100 *H*/*L*	2·85 ± ·06	3·90 ± ·08	2·96 ± ·11	4·04 ± ·15	(*H′*) 2·83 ± ·12	(*H′*) 4·10 ± ·18
100 *H*/*B*	3·69 ± ·07	3·88 ± ·08	4·66 ± ·17	4·73 ± ·18	(*H′*) 3·84 ± ·17	(*H′*) 4·16 ± ·18
100 *B*/*H*	4·07 ± ·08	3·88 ± ·08	4·74 ± ·18	4·66 ± ·18	(*H′*) 4·53 ± ·20	(*H′*) 4·17 ± ·19
100 *GH*/*GB*	—	—	6·73 ± ·68	5·67 ± ·58	—	—
100 *G′H*/*GB*	4·48 ± ·09	6·14 ± ·13	4·15 ± ·19	5·91 ± ·27	6·26 ± ·41	8·04 ± ·52
100 *NB*/*NH*	3·77 ± ·07	7·90 ± ·16	4·86 ± ·22	9·28 ± ·42	3·90 ± ·23	8·16 ± ·49
100 $O_2/O_1(L)$	4·67 ± ·09	5·47 ± ·11	4·57 ± ·21	5·94 ± ·27	4·23 ± ·27	5·18 ± ·33
100 $O_2/O_1(R)$	4·59 ± ·09	5·46 ± ·11	4·78 ± ·21	6·18 ± ·28	4·33 ± ·26	5·25 ± ·32
100 G_2/G_1	6·58 ± ·15	8·40 ± ·19	(G_1'') 5·93 ± ·29	(G_1'') 8·20 ± ·40	6·62 ± ·44	8·52 ± ·57
100 *fmb*/*fml*	5·62 ± ·11	6·69 ± ·13	—	—	6·12 ± ·43†	7·25 ± ·51†
Oc. I.	3·20 ± ·06	5·17 ± ·10	—	—	2·90 ± ·20†	4·90 ± ·33†

Macdonell used *H′* height from basion to bregma, and *Q′* transverse arc from one auricular point to the other through the bregma. Fawcett used G_1''.

† Supplied from the Moorfields Series.

variable in position as the bregma or lambda, and the length measured from the dacryon is no real standard of orbital size. At my suggestion Fawcett adopted the "curvature method," i.e. the pencilling on the nasal wall of a curved line joining the upper and lower orbital margins, maintaining as far as possible their curvature. The tendency of later workers has been to pass to the geodesic between the final points of the orbital margins, and so get lower values for the orbital length and lower values for the orbital index, and this has shown that the curvature method is too subject to personal equation to give adequate accuracy. Yet the orbital index is so important that the determination of a nasal limit for the orbital length requires reconsideration.

Lastly turning to the palate the different methods of determining its length G_1 have rendered useless a good deal of the work done even in the same laboratory. Here unlike the orbital length the uncertainty applies to both ends of the length. The posterior terminal of the palate length was originally taken as the extreme point of the *spina nasalis posterior*, but it was soon discovered that, this spine being often broken off, rubbed down or partly absorbed, the number of skulls for which the palate index could be determined was relatively small. The posterior terminal was then changed to the base of the spine. But this has to be artificially defined by supposing a tangent line drawn to the right and left posterior palatal borders and taking the point in which it meets the median plane as the base of the spine. This is capable of fairly accurate determination. Coming to the anterior terminal of the length various conventions may be adopted, all providing more or less vague points. We may adopt, what after all is fairly definite, the alveolar point, i.e. the mid point of the wall between the middle incisors. Or, we may use the most posterior point on the same wall; or, we may suppose a tangent drawn to the posterior borders of the sockets of the middle incisors and take as our anterior terminal of the palate length the point where this tangent cuts the median plane. Unfortunately this last alternative, although it might be considered a more truthful measure for palate-starting, leads us to a point which "schwebt im Luft," and is accordingly subject to considerable personal equation and liable to optical errors*. In the Biometric Laboratory it is now customary to adopt this imaginary point as anterior terminal and measure two lengths G_1' to the base and G_1 to the tip of the spine. Neither G_1 nor G_1' is what Fawcett measured, she endeavoured to follow the original *Frankfurter Verständigung* and measured from the tip of the spine to the nearest point of the alveolar margin in the median plane, i.e. the second of the above alternatives for the anterior terminal. Her length we call G_1''. It will be clear that these different treatments of palate length render comparisons of palate index difficult. It is conceivable that the alveolar point will ultimately be considered the most definite start for the palate length. The objection that it gives an exaggerated palate length in the case of prognathism seems to apply equally to the other alternatives. Meanwhile the greatest caution should be used in comparing the palate indices determined by different craniologists.

* It would not be hard to devise an instrument of a simple kind to surmount the difficulty, but another difficulty would arise in persuading all craniometricians to use it!

Personally I am inclined to believe that the use of nasal, orbital and palatal indices in the present state of affairs to discriminate degrees of racial likeness may be very misleading. These indices may differ as much owing to the manner in which the lengths have been taken, as owing to innate racial divergences.

(4) *Variation.* The above discussion has seemed to me needful, because in Table III, even in measurements made in the same Laboratory, we must not too hastily conclude that differences in variation are wholly racial, they may in part be due to the workers admittedly or unconsciously not measuring the same character, but in reality a more—or perhaps a less—variable one. Nevertheless a good deal may be learnt from a scrutiny of Table III. In the first place the reader will note that the larger arcs and major lengths have the least variability as determined by the Coefficient of Variation. Accordingly it seemed just possible that this effect might be produced by errors of reading or measurement. Such an error would be practically the same for long or short lengths. As a maximum, suppose it to have a standard deviation of 1 mm. This would alter the Standard Deviation of F in our Series E (Egyptians XXVI—XXX Dynasties) from 5·73 to 5·64 and the Coefficient of Variation from 3·14 to 3·09, and of $G'H$ from 4·15 to 4·03 in Standard Deviation and from 5·90 to 5·72 in Coefficient of Variation. The differences of variation in different characters of the skull do not therefore seem due to reading errors; there must be something determining greater or less variation in the characters themselves.

The explanation of the great variation in the capacity C is easy to give. The variations of C must very largely depend on the variations of L, B and H*, and accordingly if V denote a coefficient of variation, and the subscripts the characters, we have readily if approximately:

$$V_C^2 = V_L^2 + V_B^2 + V_H^2 + 2V_L V_B r_{LB} + 2V_B V_H r_{BH} + 2V_H V_L r_{HL},$$

and if we substitute from Tables III and V the coefficients of variation and of correlation, we find the following results:

Coefficients of Variation of Capacity.

Race and Sex	Computed	Observed
Male Egyptians, XXVI—XXX Dynasties	7·83	7·89
Female " " " "	6·89	7·08
Male and Predynastic Naqada	8·02	7·72
Female " " "	6·88	6·92
Male 17th Century English	7·95	8·28
Female " "	8·29	8·68
Mean	7·64	7·76

The correlations for the Naqada and English crania have been supplied from Fawcett's and Macdonell's memoirs respectively.

* We assume $C = A_1 + A_2 L \times B \times H$, approximately, where A_1 and A_2 are constants.

Considering that the assumption that

$$\text{Capacity} = A_1 + A_2 \times L \times B \times H$$

is only an approximately correct one, the agreement between the computed and observed values is very satisfactory, and it follows that the high value of capacity variation may be legitimately assumed to be the result of the dependence of the capacity on a product of the length, breadth and height of the skull.

Before we classify the variability of the cranial characters we may note that the Coefficient of Variation—the percentage square root mean square deviation on the type values—seems a fitting measure of variation as long as we deal with lengths continuously filled with organic material. But when we come to indices and still more to angles the validity of the Coefficient of Variation as a measure of variation is open to criticism, and we believe it sounder in such cases to use the standard-deviations. Even when the measurement covers spaces in which there exists during growth more than one type of organic material, or an absence of material as in head diameters, and nasal, orbital or foraminal measurements, the Coefficient of Variation is less applicable than to the lengths of cranial arcs, which are more or less of homogeneous material*. But no considerations of the kind arise in the case of indices or arcs. It is accordingly best to deal with their variation as groups apart.

We turn first to the question of sex and we find the following results:

Number of Characters in which there is Male or Female preponderance in Variability.

Class of Character and how determined	XXVI—XXX Dynasty Egyptians	Predynastic Egyptians (Naqada)	17th Century English (London)
Large and Small Lengths ... By Coefficients of Variation ...	5 ♀ / 27 ♂ out of 32	9 ♀ / 16 ♂ out of 25	19 ♀ / 12 ♂ out of 31
Angles ... ... By Standard Deviations ...	0 ♀ / 6 ♂ out of 6	3 ♀ / 3 ♂ out of 6	2 ♀ / 4 ♂ out of 6
Indices ... ... By Standard Deviations ...	0 ♀ / 13 ♂ out of 13	7 ♀ / 5 ♂ out of 12	6 ♀ / 7 ♂ out of 13
Total of Cases with preponderant Variability	5 ♀ / 46 ♂ out of 51	19 ♀ / 24 ♂ out of 43	27 ♀ / 23 ♂ out of 50

It is clear from this table that the degree of preponderant sexual variability may be different from race to race. It is very marked in the XXVI—XXX Dynasty

* The general considerations by which the coefficient of variation was originally reached were of the following nature. Suppose x the value of a length of homogeneous material in an individual, and $\bar{x}$ its mean, σ its standard deviation. Then the deviation $x - \bar{x}$ from type occurs in a mass of material x, and the percentage variability of this character in the individual may be taken as $100\ (x - \bar{x})\ x$, while the square root mean square percentage variability in the population $= V'$ is given by:

$$V' = 100 \sqrt{\frac{1}{N} S \left(\frac{x - \bar{x}}{x}\right)^2}.$$

Egyptians, less so in the Naqada folk and least in the 17th century English. There is not a great difference in the average values in the different variation classes as exhibited below in the classified tables, but on the whole there does appear some preponderance, varying in extent with race, in the male. It is not improbably due to a greater variation in the bony development especially in the regions of muscular attachment in the case of the male.

We shall divide our material as far as the coefficient of variation is concerned into the following grades: *Low variation*, coefficient of variation under 3·5; *Moderately low variation* 3·5 to 4·5; *Moderately high variation* 4·5 to 6·0; *High variation* over 6·0. The classification must be an elastic one based largely on the long Egyptian male series, but modified to some extent by values arising in the other two series. Table IV gives the classified results for each series with the mean value reached in the several grades. Let us examine this classification a little more in detail.

If we place on one side the high variation of cranial capacity of which we have already given the explanation, we are left with relatively short measurements which cross bridge-fashion space unoccupied by bone and proceed from one bone edge to a second. At such points there is greater freedom of expansion than when measurements are made from a suture where bone meets bone. Turning to the second group with moderately high variation we find in the case of upper face height, nasal height, and the profile length at least one free bone edge, and in the orbital heights two. In all these cases there is the spanning of cavities in the measurements. In the occipital arc and chord we have at least one free bone edge as well as the great variance in the position of the lambda; this and the variance in the position of the bregma probably account for S_1 and S_2 appearing in this class. In the total height of the face (GH) and in the profile length (GL) we again have the same factors at work—the existence of a bone edge and the spanning of cavities.

Let $V=100\sigma_x/\bar{x}$ and $v=\sigma_x/\bar{x}$. Then we can write

$$V'=100\left\{\frac{1}{N}S\left(\frac{x-\bar{x}}{\bar{x}}\right)^2\left(1+\frac{x-\bar{x}}{\bar{x}}\right)^{-2}\right\}^{\frac{1}{2}}$$

$$=100\left\{\frac{1}{N}S\left(\frac{x-\bar{x}}{\bar{x}}\right)^2\left(1-\frac{2(x-\bar{x})}{\bar{x}}+3\frac{(x-\bar{x})^2}{\bar{x}^2}+\ldots\right)\right\}^{\frac{1}{2}}$$

$$=100\{v^2-2\sqrt{\beta_1}v^3+3\beta_2 v^4+\ldots\}^{\frac{1}{2}}.$$

Or:

$$V'=V\left\{1-\sqrt{\beta_1}\frac{V}{100}+\tfrac{1}{2}(3\beta_2-\beta_1)\frac{V^2}{(100)^2}+\cdots\right\},$$

where β_1 and β_2 are the usual statistical constants. For approximately normal systems β_1 will be small and β_2 nearly 3, hence we should have

$$V'=V(1+\cdot00045V^2),$$

and accordingly for such large values of V as 5 to 6, V' would hardly differ from V by much more than one per cent. It seemed then best to adopt the simple form V instead of the theoretically better V'. The original assumption from which we started, however, that we are dealing with a deviation $x-\bar{x}$ to be distributed over material x, becomes vague in significance, if that material is heterogeneous in character, or if x covers spaces in which there is an absence of organic material. It is in this direction that we believe search might be made effectively for some of the difference in variation of cranial characters.

TABLE IV. *Classified Variabilities.*

Index Letter to Character	Race and Sex											
	XXVI—XXX Dynasty Egyptians				Prehistoric Egyptians (Naqada)				17th Century English			
	Male		Female		Male		Female		Male		Female	
High	*Coefficients of Variation*											
C	7·89		7·08		7·72		6·92		8·28		8·68	
NB	7·27		6·98		7·89		7·28		8·89		7·06	
fmb	7·18		6·96		—		—		6·10*		8·39*	
fml	6·95	*Mean*	6·37	*Mean*	—	*Mean*	—	*Mean*	8·08*	*Mean*	7·26*	*Mean*
G_2	6·78	7·07	6·83	6·65	9·29	7·85	8·55	7·54	7·75	7·21	7·68	7·39
G_1	6·70		6·26		6·49		7·41		5·68		6·53	
G_1'	6·69		6·06		—		—		5·67		6·10	
Moderately High												
S_3	5·91		6·06		—		—		7·04*		7·70*	
G'H	5·90		5·64		6·08		6·87		5·50		7·14	
S_2	5·77		5·19		—		—		5·91*		5·90*	
$O_2(R)$	5·67		5·50		7·27		6·85		5·61		4·31	
NH	5·65	*Mean*	5·31	*Mean*	6·13	*Mean*	6·81	*Mean*	5·08	*Mean*	5·55	*Mean*
$O_2(L)$	5·56	5·43	5·62	5·21	7·06	5·98	6·58	6·02	5·61	5·54	4·31	5·65
GH	—		—		5·33		5·17		—		—	
GL	5·10		4·49		4·84		5·09		4·68		4.95	
S_3'	4·96		5·02		—		—		6·05*		6·35*	
GB	4·90		4·75		5·18		4·77		5·58		5·40	
S_1	4·88		4·56		—		—		4·29*		4·91*	
Moderately Low												
B'	4·28		4·11		5·29		4·47		4·29		4·55	
$O_1(L)$	4·06		3·97		4·97		5·30		4·20		3·53	
$O_1(R)$	4·06		3·92		5·02		5·38		4·69		4·00	
LB	3·90	*Mean*	3·65	*Mean*	4·88	*Mean*	4·68	*Mean*	4·07	*Mean*	4·11	*Mean*
H	3·75	3·89	3·39	3·71	3·96	4·59	3·66	4·54	4·21†	4·21	3·96†	4·06
OH	3·63		3·32		3·86		3·54		3·73		4·12	
J	3·55		3·62		4·16		4·77		4·28		4·13	
Low												
B	3·43		3·34		3·17		3·45		3·75		3·54	
S	3·36		2·92		3·19		3·51		3·63		3·90	
Q'	3·22		2·98		—		—		—		—	
Q	3·17	*Mean*	2·92	*Mean*	3·32	*Mean*	2·72	*Mean*	3·70†	*Mean*	3·97†	*Mean*
F	3·14	$3·14^5$	2·71	2·82	3·28	3·12	3·15	3·04	3·29	3·36	3·54	3·55
L'	3·10		2·70		3·17		3·10		3·00		3·52	
L	3·09		2·66		3·24		3·10		3·31		3·45	
U	2·65		2·35		2·54		2·27		2·87		2·92	
	Standard Deviations (Angles)											
A ∠	3°·46		3°·41		4°·16		3°·14		3°·41		3°·31	
N ∠	3°·31		3°·26		3°·77		4°·48		3°·52		2°·53	
P ∠	3°·24	*Mean*	2°·95	*Mean*	2°·87	*Mean*	3°·66	*Mean*	3°·92	*Mean*	2°·85	*Mean*
θ_2	3°·16	3°·12	2°·90	2°·80	3°·92	3°·48	3°·65	3°·52	3°·34	3°·20	3°·60	2·96
θ_1	2°·89		2°·64		3°·84		3°·44		2°·53		2°·51	
B ∠	2°·66		2°·33		2°·80		2°·76		2°·50		2°·98	
Higher	*Standard Deviations (Indices)*											
100 G_2/G_1	6·79		6·58		7·36		5·93		6·40		6·62	
100 *fmb/fml*	5·79		5·62		—		—		6·64*		6·12*	
100 *GH/GB*	—		—		5·30		6·73		—		—	
100 $O_2/O_1(R)$	5·05	*Mean*	4·59	*Mean*	5·00	*Mean*	4·78	*Mean*	4·66	*Mean*	4·33	*Mean*
100 *G H/GB*	4·96	4·96	4·48	4·68	4·52	$5·08^5$	4·15	5·05	5·39	5·15	6·26	4·98
100 $O_2/O_1(L)$	4·95		4·67		5·06		4·57		3·78		4·23	
100 *B/H*	4·30		4·07		4·53		4·74		5·14		4·53	
100 *H/B*	4·00		3·69		4·73		4·66		4·58		3·84	
100 *NB/NH*	3·82		3·77		4·18		4·86		4·58		3·90	
Lower												
Oc. I.	3·30		3·20		—		—		2·15*		2·90*	
100 *H/L'*	2·91	*Mean*	2·86	*Mean*	2·77	*Mean*	2·81	*Mean*	2·67	*Mean*	2·91	*Mean*
100 *H/L*	2·94	2·91	2·85	2·81	2·73	$2·79^5$	2·96	$2·94^5$	3·22	2·83	2·83	2·93
100 *B/L*	2·68		2·56		2·80		3·12		3·26		2·98	
100 *B/L'*	2·67		2.57		2·88		2·89		2·97		3·01	

* Supplied from Moorfields Series. † *H* and *Q* measured to bregma.

In the moderately low class, we have the same factors in a more limited extent or affecting greater lengths as in the least forehead breadth, the basal length, the total and auricular heights, and the bizygomatic breadth. It is noteworthy that the variability of the face breadth (GB) measured from a suture point to a suture point is considerably greater on the whole than the bizygomatic variability (S). This confirms a point we have referred to before and shall touch on again, namely, that the variability of the skull as a whole is less than that of its component bones. It might be thought that the orbital length would have fallen into a higher variation class than it does, and this would actually be so, if the measurements of the Prehistoric Egyptians stood alone. The measurement spans a cavity and ends at one terminal in a bony border; possibly an explanation may be found in the bone development being here rather perpendicular to the direction of measurement than in its sense.

Lastly turning to low variation we find included in this grade the principal cranial arcs and the length and breadth of the skull. In the case of the length and breadth the terminals are not sutures or bony borders, but they do span cavities; in the case of the total sagittal arc the terminals are a suture and a bone edge, but the arc is not a short one, and variation at the terminals will have small effect on the total length. It is to be noted that the length of the horizontal arc (U) which does not span cavities and which does not terminate in any suture or face bone border is the least variable of all cranial characters. It seems very noteworthy that the least variable characters of the skull are not the individual bones, but combinations of these which go to make up the broad structure of the skull. There is, we think, little doubt that the variability of measurements on the individual bones of the skull—anatomical as distinguished from anthropometric measurements—must show considerably higher variation than measurements on its structure as a whole. In other words two individuals with closely corresponding anthropometric measurements might have widely different anatomical measurements. The end product might be nearly the same although produced by component cranial bones growing in different proportions. This result which might at first sight appear to connote an important organic law controlling growth is really a quite simple result of the theory of variation and the definition of the coefficient of variation itself. For let X be the sum of n variates $x_1, x_2, \ldots x_n$ the correlations of which are r_{12}, r_{13}, r_{23}, etc., and the standard deviations $\sigma_1, \sigma_2, \ldots \sigma_n$. Then

$$\sigma_X{}^2 = \sigma_1{}^2 + \sigma_2{}^2 + \ldots + \sigma_n{}^2 + 2\sigma_1\sigma_2 r_{12} + 2\sigma_1\sigma_3 r_{13} + \ldots + 2\sigma_{n-1}\sigma_n r_{n-1n}.$$

Now let $\tilde{\sigma}^2$ be the mean squared standard deviation of the components and R a correlation defined by:

$$n\tilde{\sigma}^2 R = \sigma_1\sigma_2 r_{12} + \sigma_1\sigma_3 r_{13} + \ldots + \sigma_{n-1}\sigma_n r_{n-1n},$$

then

$$V_X{}^2 = \frac{(100)^2\sigma_X}{\overline{X}^2} = (100)^2\frac{n\tilde{\sigma}^2}{\overline{X}^2} + \frac{2n\tilde{\sigma}^2 R}{\overline{X}^2}(100)^2.$$

But $\overline{X} = n\tilde{x}$ where $\tilde{x}$ is a mean of the x's. Thus

$$V_X{}^2 = \frac{\tilde{v}^2}{n} + 2\frac{\tilde{v}^2}{n}R.$$

Hence if there be no correlation between the components of X we have $R = 0$ and

$$V_X = \tilde{v}/\sqrt{n}.$$

V_X will decrease rapidly as the number of independently varying parts increases. If the correlation of the components were *perfect*, then and then only would V_X equal $\tilde{v}$. In the case of the parts of the skull, since these are not highly correlated, the second term will be small, and accordingly V_X will be slightly in excess of $\tilde{v}/\sqrt{n}$. Thus in the case of the three sagittal sub-arcs S_1, S_2, S_3, the mean coefficient of variation is 5·52 and this is a reasonably close approach to the actual value of $\tilde{v}$, or 5·51; from the former we deduce:

$$V_X = 3{\cdot}19,$$

whereas its observed value is 3·36, somewhat larger than this as we should expect, since the term $2\tilde{v}^2R/n$, dependent on the correlations, has been neglected. The difference is, however, so small that we may safely predict that the correlations with each other of the sagittal sub-arcs will prove to be small or some even to be negative.

The principle is pretty obvious when we come to examine it algebraically, and without having a biological origin may certainly have biological importance. It may be thus expressed: the percentage variation of a character which is the sum of a number of biologically varying units not highly correlated together, will be very considerably less—roughly as the inverse square root of the number of units—than the mean percentage variation of the units themselves. Thus it comes about that the chief anthropometric characters of head or skull have a markedly less variability than the anatomical characters, by which we understand measurements confined to a single bone.

Passing now to the angles of the fundamental triangle we note that the variability *as measured by the standard deviation* ranges from two and a half to three and a half degrees, the angle at the basion having least variability. When we remember that the basional angle is the least angle of the fundamental triangle we are tempted to apply the coefficient of variation, and should then find it the most variable angle. But the justification for using this coefficient in the case of angles does not appear to be the same as in the case of arcs or lengths; there is not the same conception of a number of roughly equivalent units varying and determining by the sum of their variations the value of the character. Again the line from which we measure the angle may be quite arbitrary as in the case of the profile angle where we might equally well have determined it from the vertical, when its variation as measured by the coefficient of variation would have been nearer 80 °/₀ than 4 °/₀! All that seems feasible in the case of the skull is to recognise a variation of about 3° in the absolute size of its angles.

Finally turning to the indices we see how essentially the variability of an index depends on the variability of its components. Indices formed by the ratio of characters both of which are taken from the low category of variability or one from the low and the other from the moderately low character* fall into the lower

* The occipital index is an exception, but it is not the ratio of two quantities S_3 and S_3' with moderately high variation, but a much more complicated function of them.

category of index variability. Again indices formed by the ratio of characters both taken from the category of high variability stand at the top of the class of high index variability. Thus—owing to the relatively low correlation of cranial characters—the variability of indices depends mainly on the variability of the components of their ratios. There are exceptions which depend upon the absolute size of the index, as in the case of $100\,B/H$ or $100\,H/B$. The reader will recognise this at once, if he notes that if $\bar{I} = 100\bar{X}/\bar{Y}$, then to a first approximation

$$V_I^2 = V_X^2 + V_Y^2 - 2V_X V_Y r_{XY},$$

and accordingly if r_{XY} be small

$$\sigma_I = \frac{\bar{I}}{100}\sqrt{V_X^2 + V_Y^2},$$

and thus the variation, as judged by the standard deviation, depends primarily on V_X and V_Y and secondarily on the magnitude of $\bar{I}$ itself and r_{XY}.

It is quite easy to calculate the variability of indices to a first or indeed to a second approximation from the variability of their components, and accordingly from the standpoint of variability they have not the primary importance of the variability of essential arcs or lengths, and as these might in many if not quite in all cases be reduced to their anatomical components, we realise how a study of arcs, lengths and curvatures of the constituent bones of the skull would enable us by aid of their means, coefficients of variation and correlations to construct those of the anthropometric characters. Light might then ultimately be shed on the nature of variation, as to whether it lies in the size or ultimate number of cells, and how these may be constrained by growth conditions. For while the anthropometric measurements and their variation may be essentially important for the discussion of racial problems, the ultimate source of that variation must lie in the variation of the simplest elements of those characters.

(5) *Correlation.* It is needful to warn the reader at the outset that we are studying the correlation of cranial characters *intraracially.* There may be no association whatever between nasal and palatal indices in the skull, a relatively broad palate being in no way associated with a relatively broad nose; but this in no way hinders a broad nosed race having a broad palate—there may be an *interracial* correlation of nasal and palatal indices which is not organic in the skull but due to racial differentiation. Of course it is quite conceivable that interracial correlations may be of the same sign as intraracial, as when we note that organically a skull with a low orbital index will as a rule have a high nasal index, the correlation being intraracially negative. But even in such a case the intraracial correlation may be no accurate measure of the interracial association. It is therefore very desirable not to confuse the associations found interracially with the organic correlations of the skull. It is these rather that we are here concerned with. A full comparison of intraracial and interracial correlations will only be possible when many long racial series have had their types determined, and not

till then shall we be able to solve with real adequacy the problem of racial relationships. For the present we are considering how within a definite "race"—a group subject for a long period to a stable environment and mating with one another freely—the various anthropometric characters of the skull are correlated.

If we exclude characters such as L', F which are substitutes for L, or Q' for Q, or G_1' for G_1 we have for lengths, angles and indices some 40 characters indicated in the margins of Table V. This would connote 780 coefficients of correlation, or allowing for the two sexes 1560 coefficients. About a third of the tables for these coefficients would require a page of this Journal apiece; the remaining two-thirds might be printed two tables to a page. Thus a thousand pages would be required to print this number of tables! As a matter of fact 312 coefficients, some fifth of the possible coefficients, were calculated, and it was considered that these only would provide tables occupying about 200 royal octavo pages*. The publication of these tables would have had great value in the case of those desiring to test theories of the distribution of frequency in the case of craniometric material, but the post-war cost of printing renders the publication of these tables at present impossible; they must remain in MS. file in the Biometric Laboratory†. While the forming of these 312 tables was a task of considerable magnitude, we are only too conscious of the redundancy of blank cells in Table V. The 312 completed cells seem nowhere when one compares them with the 1248 still uncomputed. We must perforce content ourselves with the excuse that hitherto not a quarter of this number of correlations have been computed for any given race, and these on series scarcely an eighth of the length of the present. We see the big gaps in our results very clearly but we believe that much may still be learnt of the association of the parts of the skull from what has been computed.

Before discussing the results more at length we may consider the association of correlation with sex; taking merely the numerical values without regard to sign we find that for 123 pairs‡ of characters the male is more highly correlated in 69 and the female in 54. Fawcett found for twenty-five pairs of characters in the Naqada prehistoric Egyptians, the male had higher correlations in 18 and the female in only 7 cases. Macdonell for thirty characters in 17th century London crania found a nearer equality, namely in 16 the male and in 14 the female was more highly correlated. Adding all the series together to give 178 cases, the male had superior correlation in 103 and the female in 75 cases. But we doubt if this measures the extent of the difference between the sexes, for the male values might be more widely in excess of the female than *vice versa*. We have, however:

* The publication of the measurements and remarks on the individual skulls would have filled between 25 and 30 of the familiar folding sheets of the cranial memoirs of this Journal, and their publication, however desirable, is now out of the question.

† The similar series for the correlation of the characters of the femur have also had to remain unpublished.

‡ Only 246 tables had been worked out when these statistics were taken. Beside the 312 coefficients of Table V, an additional 8 to 12 are given in the text or footnotes of this paper.

Characters	C	L	B	B'	H	OH	LB	U	S	S_1	S_2	S_3	S_3'	Q	$G'H$
C ♂ ♀	—	+·6661 ±·0137 +·5983 ±·0200	+·6817 ±·0131 +·6798 ±·0168		+·5357 ±·0175 +·4943 ±·0236	+·6128 ±·0154 +·5939 ±·0202	+·2311 ±·0233 +·2297 ±·0296	+·7834 ±·0095 +·7325 ±·0144	+·7626 ±·0103 +·7115 ±·0155					+·7214 ±·0118 +·6914 ±·0163	+·2147 ±·0242 +·1744 ±·0312
L ♂ ♀	+·6661 ±·0137 +·5983 ±·0200	—	+·3971 ±·0191 +·3763 ±·0240		+·3830 ±·0195 +·1896 ±·0272	+·3963 ±·0192 +·3153 ±·0253	+·4515 ±·0181 +·4580 ±·0222	+·8834 ±·0050 +·8652 ±·0071	+·7405 ±·0103 +·6968 ±·0145	+·4822 ±·0173 +·4225 ±·0230	+·3766 ±·0194 +·2972 ±·0253	+·5094 ±·0169 +·4424 ±·0227		+·4449 ±·0184 +·3527 ±·0247	+·2629 ±·0216 +·2926 ±·0264
B ♂ ♀	+·6817 ±·0131 +·6798 ±·0168	+·3971 ±·0191 +·3763 ±·0240	—	+·3303 ±·0201 +·2935 ±·0255	+·3344 ±·0203 +·3389 ±·0250	+·4557 ±·0180 +·4088 ±·0234	+·0861 ±·0225 +·1137 ±·0277	+·6341 ±·0137 +·6607 ±·0158	+·4902 ±·0174 +·4782 ±·0218					+·6817 ±·0122 +·6501 ±·0163	+·1442 ±·0231 +·1892 ±·0280
B' ♂ ♀			+·3303 ±·0201 +·2935 ±·0255	—											
H ♂ ♀	+·5357 ±·0175 +·4943 ±·0236	+·3830 ±·0195 +·1896 ±·0272	+·3344 ±·0203 +·3389 ±·0250		—	+·7281 ±·0107 +·7290 ±·0131		+·3581 ±·0201 +·2559 ±·0266	+·4671 ±·0180 +·4858 ±·0218					+·6592 ±·0129 +·6327 ±·0169	+·1846 ±·0228 +·1855 ±·0279
OH ♂ ♀	+·6128 ±·0154 +·5939 ±·0202	+·3963 ±·0192 +·3153 ±·0253	+·4557 ±·0180 +·4088 ±·0234		+·7281 ±·0107 +·7290 ±·0131	—		+·4719 ±·0179 +·3690 ±·0245	+·6602 ±·0130 +·6344 ±·0170					+·8857 ±·0049 +·8491 ±·0078	+·1277 ±·0231 +·1268 ±·0283
LB ♂ ♀	+·2311 ±·0233 +·2297 ±·0296	+·4515 ±·0181 +·4580 ±·0222	+·0861 ±·0225 +·1137 ±·0277				—		+·1122 ±·0226 +·0601 ±·0281						
U ♂ ♀	+·7834 ±·0095 +·7325 ±·0144	+·8834 ±·0050 +·8652 ±·0071	+·6341 ±·0137 +·6607 ±·0158		+·3581 ±·0201 +·2559 ±·0266	+·4719 ±·0179 +·3690 ±·0245		—	+·7547 ±·0099 +·6797 ±·0153					+·5916 ±·0150 +·5207 ±·0207	
S ♂ ♀	+·7626 ±·0103 +·7115 ±·0155	+·7405 ±·0103 +·6968 ±·0145	+·4902 ±·0174 +·4782 ±·0218		+·4671 ±·0180 +·4858 ±·0218	+·6602 ±·0130 +·6344 ±·0170	+·1122 ±·0226 +·0601 ±·0281	+·7547 ±·0099 +·6797 ±·0153	—	+·6917 ±·0118 +·6790 ±·0151	+·5991 ±·0145 +·3799 ±·0239	+·5273 ±·0164 +·4925 ±·0212		+·6536 ±·0133 +·5930 ±·0185	+·1555 ±·0232 +·1309 ±·0287
S_1 ♂ ♀		+·4822 ±·0173 +·4225 ±·0230							+·6917 ±·0118 +·6790 ±·0151	—	1861 ±·0217 +·1141 ±·0272	+·1545 ±·0222 +·1068 ±·0280			+·1915 ±·0224 +·1675 ±·0278
S_2 ♂ ♀		+·3766 ±·0194 +·2972 ±·0253							+·5991 ±·0145 +·3799 ±·0239	+·1861 ±·0217 +·1141 ±·0272	—	−·1587 ±·0221 −·3422 ±·0247			+·0113 ±·0235 +·0228 ±·0289
S_3 ♂ ♀		+·5094 ±·0169 +·4424 ±·0227							+·5273 ±·0164 +·4925 ±·0212	+·1545 ±·0222 +·1068 ±·0280	−·1587 ±·0221 −·3423 ±·0247	—			+·1014 ±·0233 +·0545 ±·0292
S_3' ♂ ♀													—		
Q ♂ ♀	+·7214 ±·0118 +·6914 ±·0163	+·4449 ±·0184 +·3527 ±·0247	+·6817 ±·0122 +·6501 ±·0163		+·6592 ±·0129 +·6327 ±·0169	+·8857 ±·0049 +·8491 ±·0078		+·5916 ±·0150 +·5207 ±·0207	+·6536 ±·0133 +·5930 ±·0185					—	+·146 ±·023 +·177 ±·028
$G'H$ ♂ ♀	+·2147 ±·0242 +·1744 ±·0312	+·2629 ±·0216 +·2926 ±·0264	+·1442 ±·0231 +·1892 ±·0280		+·1846 ±·0228 +·1855 ±·0279	+·1277 ±·0231 +·1268 ±·0283			+·1555 ±·0232 +·1309 ±·0287	+·1915 ±·0224 +·1675 ±·0278	+·0113 ±·0235 +·0228 ±·0289	+·1014 ±·0235 +·0545 ±·0292		+·1465 ±·0232 +·1778 ±·0281	—
GB ♂ ♀	+·1307 ±·0245 +·1592 ±·0310		+·1086 ±·0228 +·1956 ±·0277												+·229 ±·022 +·302 ±·026
		+·7846	+·3723	+·4179	+·2333	+·2131		+·4835	+·2361					+·3480 ±·0218	+·275 ±·022

Characters.

Q	*G'H*	*GB*	*J*	*NH*	*NB*	O_1 (*L*)	O_2(*L*)	G_1	G_2	*GL*
·7214 ·0118 ·6914 ·0163	+ ·2147 ± ·0242 + ·1744 ± ·0312	+ ·1307 ± ·0245 + ·1592 ± ·0310	+ ·4239 ± ·0212 + ·3876 ± ·0282							
·4449 ·0184 ·3527 ·0247	+ ·2629 ± ·0216 + ·2926 ± ·0264		+ ·3846 ± ·0207 + ·3060 ± ·0276	+ ·2760 ± ·0211 + ·2699 ± ·0263	+ ·2358 ± ·0218 + ·1223 ± ·0281			+ ·3387 ± ·0212 + ·2001 ± ·0283		
·6817 ·0122 ·6501 ·0163	+ ·1442 ± ·0231 + ·1892 ± ·0280	+ ·1086 ± ·0228 + ·1956 ± ·0277	+ ·3732 ± ·0208 + ·3368 ± ·0270	+ ·1132 ± ·0226 + ·1557 ± ·0283	+ ·1118 ± ·0227 + ·0725 ± ·0284					
			+ ·4170 ± ·0199 + ·4883 ± ·0230							
·6592 ·0129 ·6327 ·0169	+ ·1846 ± ·0228 + ·1855 ± ·0279		+ ·2333 ± ·0229 + ·2781 ± ·0281	+ ·1405 ± ·0225 + ·1447 ± ·0277	+ ·0567 ± ·0230 + ·0783 ± ·0284					
+ ·8857 ± ·0049 + ·8491 ± ·0078	+ ·1277 ± ·0231 + ·1268 ± ·0283		+ ·2131 ± ·0230 + ·2169 ± ·0289	+ ·0723 ± ·0227 + ·0395 ± ·0281						
+ ·5916 ± ·0150 + ·5207 ± ·0207			+ ·4835 ± ·0187 + ·4536 ± ·0243							
+ ·6536 ± ·0133 + ·5930 ± ·0185	+ ·1555 ± ·0232 + ·1309 ± ·0287		+ ·2361 ± ·0231 + ·1656 ± ·0299							
	+ ·1915 ± ·0224 + ·1675 ± ·0278									
	+ ·0113 ± ·0235 + ·0228 ± ·0289									
	+ ·1014 ± ·0235 + ·0545 ± ·0292									
—	+ ·1465 ± ·0232 + ·1778 ± ·0281		+ ·3480 ± ·0218 + ·3448 ± ·0268							
+ ·1465 ± ·0232 + ·1778 ± ·0281	—	+ ·2299 ± ·0223 + ·3022 ± ·0262	+ ·2758 ± ·0229 + ·3335 ± ·0274	+ ·7059 ± ·0117 + ·7058 ± ·0142	+ ·0912 ± ·0232 + ·0816 ± ·0284					+ ·2722 ± ·0218 + ·2652 ± ·0267
	+ ·2299 ± ·0223 + ·3022 ± ·0262	—	+ ·3974 ± ·0203 + ·4423 ± ·0244		+ ·3197 ± ·0206 + ·4073 ± ·0235					
+ ·3480 ± ·0218	+ ·2758 ± ·0229	+ ·3974 ± ·0203			+ ·2482 ± ·0227					

G_2	GL	fmb	fml	$P\angle$	$A\angle$	$N\angle$	$B\angle$	100 B/L	100 H/L	100 B/H	100 $G'H/GB$	100 NB/NH	100 $O_2/O_1(L)$	100 $O_2/O_1(R)$	100 G_2/G_1	100 fmb/f
		+ ·2828 ± ·0226 + ·1937 ± ·0299	+ ·2383 ± ·0232 + ·2393 ± ·0294	+ ·1365 ± ·0252 + ·2040 ± ·0313				+ ·0733 ± ·0245 + ·1839 ± ·0301	− ·0028 ± ·0246 − ·0035 ± ·0312	+ ·0554 ± ·0245 + ·1601 ± ·0304		− ·0397 ± ·0249 − ·0915 ± ·0315	− ·0591 ± ·0249 + ·0393 ± ·0316			+ ·07 ± ·02 − ·02 ± ·03
				+ ·1759 ± ·0233 + ·2582 ± ·0276				− ·4902 ± ·0172 − ·4138 ± ·0231	− ·4848 ± ·0175 − ·5272 ± ·0204	+ ·0529 ± ·0229 + ·1773 ± ·0274						
				+ ·1180 ± ·0237 + ·1508 ± ·0289				+ ·5888 ± ·014[illegible] + ·6803 ± ·0150	− ·0059 ± ·0229 + ·0317 ± ·0283	+ ·5089 ± ·0169 + ·5617 ± ·0190						
				+ ·0138 ± ·0239 + ·1029 ± ·0291				+ ·0530 ± ·0229 + ·1806 ± ·0273	+ ·6867 ± ·0121 + ·7365 ± ·0129	− ·6384 ± ·0135 − ·5897 ± ·0184						
				+ ·1979 ± ·0230 + ·2580 ± ·0274												
												− ·0386 ± ·0230 − ·0490 ± ·0284				
	+ ·2722 ± ·0218 + ·2652 ± ·0267			− ·0282 ± ·0239 − ·0196 ± ·0293			+ ·7724 ± ·0095 + ·7796 ± ·0113	− ·0918 ± ·0235 − ·0592 ± ·0289					+ ·1360 ± ·0231 + ·1308 ± ·0282			

	100 G_2/G_1	100 fmb/fml	*Oc. I.*
		+ ·0763	− ·0547
		± ·0245	± ·0246
		− ·0268	− ·1044
		± ·0312	± ·0310
			− ·3236
			± ·0205
			− ·3720
			± ·0243
			− ·0882
			± ·0228
			− ·1033
			± ·0280
			+ ·0577
			± ·0230
			+ ·0881
			± ·0282
			+ ·0113
			± ·0229
			− ·0118
			± ·0283
			− ·2257
			± ·0216
			− ·2218
			± ·0267

$U \begin{cases} \text{ee} \\ \text{oo} \end{cases}$	± ·0095 + ·7325 ± ·0144	± ·0050 + ·8652 ± ·0071	± ·0137 + ·6607 ± ·0158		± ·0201 + ·2559 ± ·0266	± ·0179 + ·3690 ± ·0245		—	+ ·7347 ± ·0099 + ·6797 ± ·0153					+ ·5916 ± ·0150 + ·5207 ± ·0207	
$S \begin{cases} \text{ee} \\ \text{oo} \end{cases}$	+ ·7626 ± ·0103 + ·7115 ± ·0155	+ ·7405 ± ·0103 + ·6968 ± ·0145	+ ·4902 ± ·0174 + ·4782 ± ·0218		+ ·4671 ± ·0180 + ·4858 ± ·0218	+ ·6602 ± ·0130 + ·6344 ± ·0170	+ ·1122 ± ·0226 + ·0601 ± ·0281	+ ·7547 ± ·0099 + ·6797 ± ·0153	—	+ ·6917 ± ·0118 + ·6790 ± ·0151	+ ·5991 ± ·0145 + ·3799 ± ·0239	+ ·5273 ± ·0164 + ·4925 ± ·0212		+ ·6536 ± ·0133 + ·5930 ± ·0185	+ ·15 ± ·02 + ·13 ± ·02
$S_1 \begin{cases} \text{ee} \\ \text{oo} \end{cases}$		+ ·4822 ± ·0173 + ·4225 ± ·0230							+ ·6917 ± ·0118 + ·6790 ± ·0151	—	1861 ± ·0217 + ·1141 ± ·0272	+ ·1545 ± ·0222 + ·1068 ± ·0280			+ ·1 ± ·02 + ·1 ± ·02
$S_2 \begin{cases} \text{ee} \\ \text{oo} \end{cases}$		+ ·3766 ± ·0194 + ·2972 ± ·0253							+ ·5991 ± ·0145 + ·3799 ± ·0239	+ ·1861 ± ·0217 + ·1141 ± ·0272	—	− ·1587 ± ·0221 − ·3422 ± ·0247			+ ·0 ± ·02 + ·0 ± ·02
$S_3 \begin{cases} \text{ee} \\ \text{oo} \end{cases}$		+ ·5094 ± ·0169 + ·4424 ± ·0227							+ ·5273 ± ·0164 + ·4925 ± ·0212	+ ·1545 ± ·0222 + ·1068 ± ·0280	− ·1587 ± ·0221 − ·3423 ± ·0247	—			+ ·1 ± ·02 + ·0 ± ·02
$S_3' \begin{cases} \text{ee} \\ \text{oo} \end{cases}$													—		
$Q \begin{cases} \text{ee} \\ \text{oo} \end{cases}$	+ ·7214 ± ·0118 + ·6914 ± ·0163	+ ·4449 ± ·0184 + ·3527 ± ·0247	+ ·6817 ± ·0122 + ·6501 ± ·0163		+ ·6592 ± ·0129 + ·6327 ± ·0169	+ ·8857 ± ·0049 + ·8491 ± ·0078		+ ·5916 ± ·0150 + ·5207 ± ·0207	+ ·6536 ± ·0133 + ·5930 ± ·0185					—	+ ·1 ± ·0 + ·1 ± ·0
$G'H \begin{cases} \text{ee} \\ \text{oo} \end{cases}$	+ ·2147 ± ·0242 + ·1744 ± ·0312	+ ·2629 ± ·0216 + ·2926 ± ·0264	+ ·1442 ± ·0231 + ·1892 ± ·0280		+ ·1846 ± ·0228 + ·1855 ± ·0279	+ ·1277 ± ·0231 + ·1268 ± ·0283			+ ·1555 ± ·0232 + ·1309 ± ·0287	+ ·1915 ± ·0224 + ·1675 ± ·0278	+ ·0113 ± ·0235 + ·0228 ± ·0289	+ ·1014 ± ·0235 + ·0545 ± ·0292		+ ·1465 ± ·0232 + ·1778 ± ·0281	—
$GB \begin{cases} \text{ee} \\ \text{oo} \end{cases}$	+ ·1307 ± ·0245 + ·1592 ± ·0310		+ ·1086 ± ·0228 + ·1956 ± ·0277												+ · ± ·0 + · ± ·0
$J \begin{cases} \text{ee} \\ \text{oo} \end{cases}$	+ ·4239 ± ·0212 + ·3876 ± ·0282	+ ·3846 ± ·0207 + ·3060 ± ·0276	+ ·3732 ± ·0208 + ·3368 ± ·0270	+ ·4170 ± ·0199 + ·4883 ± ·0230	+ ·2333 ± ·0229 + ·2781 ± ·0281	+ ·2131 ± ·0230 + ·2169 ± ·0289		+ ·4835 ± ·0187 + ·4536 ± ·0243	+ ·2361 ± ·0231 + ·1656 ± ·0299					+ ·3480 ± ·0218 + ·3448 ± ·0268	+ · ± ·0 + · ± ·0
$NH \begin{cases} \text{ee} \\ \text{oo} \end{cases}$		+ ·2760 ± ·0211 + ·2699 ± ·0263	+ ·1132 ± ·0226 + ·1557 ± ·0283		+ ·1405 ± ·0225 + ·1447 ± ·0277	+ ·0723 ± ·0227 + ·0395 ± ·0281									+ · ± ·0 + · ± ·0
$NB \begin{cases} \text{ee} \\ \text{oo} \end{cases}$		+ ·2358 ± ·0218 + ·1223 ± ·0281	+ ·1118 ± ·0227 + ·0725 ± ·0284		+ ·0567 ± ·0230 + ·0783 ± ·0284										+ · ± ·0 + · ± ·0
$O_1(L) \begin{cases} \text{ee} \\ \text{oo} \end{cases}$															
$O_2(L) \begin{cases} \text{ee} \\ \text{oo} \end{cases}$															
$G_1 \begin{cases} \text{ee} \\ \text{oo} \end{cases}$		+ ·3387 ± ·0212 + ·2001 ± ·0283													
$G_2 \begin{cases} \text{ee} \\ \text{oo} \end{cases}$															
$GL \begin{cases} \text{ee} \\ \text{oo} \end{cases}$															+ · ± ·0 + · ± ·0
$jmb \begin{cases} \text{ee} \\ \text{oo} \end{cases}$	+ ·2828 ± ·0226 + ·1937 ± ·0299														

+ ·3916 ± ·0150 + ·5207 ± ·0207			+ ·4835 ± ·0187 + ·4536 ± ·0243							
+ ·6536 ± ·0133 + ·5930 ± ·0185	+ ·1555 ± ·0232 + ·1309 ± ·0287		+ ·2361 ± ·0231 + ·1656 ± ·0299							
	+ ·1915 ± ·0224 + ·1675 ± ·0278									
	+ ·0113 ± ·0235 + ·0228 ± ·0289									
	+ ·1014 ± ·0235 + ·0545 ± ·0292									
—	+ ·1465 ± ·0232 + ·1778 ± ·0281		+ ·3480 ± ·0218 + ·3448 ± ·0268							
+ ·1465 ± ·0232 + ·1778 ± ·0281	—	+ ·2299 ± ·0223 + ·3022 ± ·0262	+ ·2758 ± ·0229 + ·3335 ± ·0274	+ ·7059 ± ·0117 + ·7058 ± ·0142	+ ·0912 ± ·0232 + ·0816 ± ·0284					
	+ ·2299 ± ·0223 + ·3022 ± ·0262	—	+ ·3974 ± ·0203 + ·4423 ± ·0244		+ ·3197 ± ·0206 + ·4073 ± ·0235					
+ ·3480 ± ·0218 + ·3448 ± ·0268	+ ·2758 ± ·0229 + ·3335 ± ·0274	+ ·3974 ± ·0203 + ·4423 ± ·0244	—		+ ·2482 ± ·0227 + ·2330 ± ·0287					
	+ ·7059 ± ·0117 + ·7058 ± ·0142			—	+ ·2387 ± ·0214 + ·2051 ± ·0268					
	+ ·0912 ± ·0232 + ·0816 ± ·0284	+ ·3197 ± ·0206 + ·4073 ± ·0235	+ ·2482 ± ·0227 + ·2330 ± ·0287	+ ·2387 ± ·0214 + ·2051 ± ·0268	—					
						—	+ ·2630 ± ·0212 + ·3905 ± ·0236			
						+ ·2630 ± ·0212 + ·3905 ± ·0236	—			
								—	+ ·1762 ± ·0253 + ·1620 ± ·0316	
								+ ·1762 ± ·0253 + ·1620 ± ·0316	—	
	+ ·2722 ± ·0218 + ·2652 ± ·0267									

		+ ·2722 ± ·0218 + ·2652 ± ·0267			− ·0282 ± ·0239 − ·0196 ± ·0293			+ ·7724 ± ·0095 + ·7796 ± ·0113	− ·0918 ± ·0235 − ·0592 ± ·0289					+ ·1360 ± ·0231 + ·1308 ± ·0282		
													− ·0113 ± ·0242 − ·0502 ± ·0304			
—	+ ·1762 ± ·0253 + ·1620 ± ·0316				− ·3820 ± ·0209 − ·3796 ± ·0259											
+ ·1762 ± ·0253 + ·1620 ± ·0316	—				+ ·0474 ± ·0265 + ·0506 ± ·0331											
		—														
			—	+ ·5241 ± ·0164 + ·5087 ± ·0205												

fmb ♂	+ ·2828 ± ·0226													
♀	+ ·1937 ± ·0299													
fml ♂	+ ·2383 ± ·0232													
♀	+ ·2393 ± ·0294													
L ♂	+ ·1365 ± ·0252	+ ·1759 ± ·0233	+ ·1180 ± ·0237		+ ·0138 ± ·0239	+ ·1979 ± ·0230								
♀	+ ·2040 ± ·0313	+ ·2582 ± ·0276	+ ·1508 ± ·0289		+ ·1029 ± ·0291	+ ·2580 ± ·0274								
A L ♂														
♀														
N L ♂														
♀														
B L ♂														
♀														
100 *B/L* ♂	+ ·0733 ± ·0245	− ·4902 ± ·0172	+ ·5888 ± ·0148		+ ·0530 ± ·0229									
♀	+ ·1839 ± ·0301	− ·4138 ± ·0231	+ ·6803 ± ·0150		+ ·1806 ± ·0273									
100 *H/L* ♂	− ·0028 ± ·0246	− ·4848 ± ·0175	− ·0059 ± ·0229		+ ·6867 ± ·0121									
♀	− ·0035 ± ·0312	− ·5272 ± ·0204	+ ·0317 ± ·0283		+ ·7365 ± ·0129									
100 *B/H* ♂	+ ·0554 ± ·0245	+ ·0529 ± ·0229	+ ·5089 ± ·0169		− ·6384 ± ·0135									
♀	+ ·1601 ± ·0304	+ ·1773 ± ·0274	+ ·5617 ± ·0190		− ·5897 ± ·0184									
100 *G' H/GB* ♂														
♀														
100 *NB/NH* ♂	− ·0397 ± ·0249						− ·0386 ± ·0230							
♀	− ·0915 ± ·0315						− ·0490 ± ·0284							
100 O_2/O_1 (*L*) ♂	− ·0591 ± ·0249													
♀	+ ·0393 ± ·0316													
100 O_2/O_1 (*R*) ♂														
♀														
100 G_2/G_1 ♂														
♀														
100 *fmb/fml* ♂	+ ·0763 ± ·0245													
♀	− ·0268 ± ·0312													
Oc. I. ♂	− ·0547 ± ·0246	− ·3236 ± ·0205	− ·0882 ± ·0228			+ ·0577 ± ·0230	+ ·0113 ± ·0229		− ·2257 ± ·0216					
♀	− ·1044 ± ·0310	− ·3720 ± ·0243	− ·1033 ± ·0280			+ ·0881 ± ·0282	− ·0118 ± ·0283		− ·2218 ± ·0267					

		± ·0267								
		− ·0282 ± ·0239 − ·0196 ± ·0293								− ·3820 ± ·0209 − ·3796 ± ·0259
		+ ·7724 ± ·0095 + ·7796 ± ·0113								
		− ·0918 ± ·0235 − ·0592 ± ·0289								
				− ·0113 ± ·0242 − ·0502 ± ·0304						
		+ ·1360 ± ·0231 + ·1308 ± ·0282								

									—	+ ·5241 ± ·0164 + ·5087 ± ·0205							
									+ ·5241 ± ·0164 + ·5087 ± ·0205	—							
						− ·3820 ± ·0209 − ·3796 ± ·0259	+ ·0474 ± ·0265 + ·0506 ± ·0331				—		− ·6276 ± ·0145 − ·6470 ± ·0171					
												—					
											− ·6276 ± ·0145 − ·6470 ± ·0171		—					
														—			
															+ ·4155 ± ·0190 + ·4391 ± ·0229		
														+ ·4155 ± ·0190 + ·4391 ± ·0229	—		
																—	
											+ ·0437 ± ·0241 + ·0675 ± ·0297		− ·2359 ± ·0225 − ·2495 ± ·0275					
− ·0113 ± ·0242 − ·0502 ± ·0304											− ·1125 ± ·0240 − ·0373 ± ·0296		+ ·2621 ± ·0221 + ·2419 ± ·0274		− ·0102 ± ·0232 + ·0183 ± ·0288	− ·0453 ± ·0232 − ·0065 ± ·0289	+ ·0 ± ·0 + ·0 ± ·0
											− ·1105 ± ·0239 − ·0188 ± ·0297		− ·1532 ± ·0233 − ·2183 ± ·0278		+ ·1179 ± ·0229 + ·1133 ± ·0283	+ ·1176 ± ·0230 + ·0443 ± ·0287	
															+ ·0966 ± ·0230 + ·1007 ± ·0284	+ ·0943 ± ·0231 + ·0537 ± ·0288	
											+ ·3330 ± ·0240 + ·3046 ± ·0308		− ·3218 ± ·0238 − ·3054 ± ·0304		+ ·0689 ± ·0264 − ·0213 ± ·0334	+ ·0640 ± ·0265 + ·0875 ± .0334	
															+ ·0977 ± ·0227 + ·1091 ± ·0280		
											− ·0416 ± ·0242 − ·1153 ± ·0295						

			+ ·0437 ± ·0241 − ·0675 ± ·0297	− ·1125 ± ·0240 − ·0373 ± ·0296	− ·1105 ± ·0239 − ·0188 ± ·0297		+ ·3330 ± ·0240 + ·3046 ± ·0308		− ·0416 ± ·0242 − ·1153 ± ·0295
			− ·2359 ± ·0225 − ·2495 ± ·0275	+ ·2621 ± ·0221 + ·2419 ± ·0274	− ·1532 ± ·0233 − ·2183 ± ·0278		− ·3218 ± ·0238 − ·3054 ± ·0304		
	+ ·4155 ± ·0190 + ·4391 ± ·0229			− ·0102 ± ·0232 + ·0183 ± ·0288	+ ·1179 ± ·0229 + ·1133 ± ·0283	+ ·0966 ± ·0230 + ·1067 ± ·0284	+ ·0689 ± ·0264 − ·0213 ± ·0334	+ ·0977 ± ·0227 + ·1091 ± ·0280	
	—			− ·0453 ± ·0232 − ·0065 ± ·0289	+ ·1176 ± ·0230 + ·0443 ± ·0287	+ ·0943 ± ·0231 + ·0537 ± ·0288	+ ·0640 ± ·0265 + ·0875 ± ·0334		
		—		+ ·0495 ± ·0232 + ·0354 ± ·0289					
			—	− ·4607 ± ·0186 − ·4902 ± ·0221	+ ·2466 ± ·0222 + ·2275 ± ·0275	+ ·2440 ± ·0223 + ·2070 ± ·0279	− ·1325 ± ·0261 − ·1743 ± ·0323	+ ·0013 ± ·0240 + ·0204 ± ·0296	
2	− ·0453 ± ·0232 − ·0065 ± ·0289	+ ·0495 ± ·0232 + ·0354 ± ·0289	− ·4607 ± ·0186 − ·4902 ± ·0221	—	− ·2507 ± ·0215 − ·2164 ± ·0206	− ·2904 ± ·0210 − ·1837 ± ·0274	+ ·0232 ± ·0263 − ·0427 ± ·0329	− ·0420 ± ·0232 − ·0280 ± ·0288	
9	+ ·1176 ± ·0230 + ·0443 ± ·0287		+ ·2466 ± ·0222 + ·2275 ± ·0275	− ·2507 ± ·0215 − ·2164 ± ·0206	—	+ ·8137 ± ·0078 + ·7936 ± ·0105	− ·0342 ± ·0263 − ·0201 ± ·0331	+ ·0586 ± ·0232 + ·0046 ± ·0288	
6	+ ·0943 ± ·0231 + ·0537 ± ·0288		+ ·2440 ± ·0223 + ·2070 ± ·0279	− ·2904 ± ·0210 − ·1837 ± ·0274	+ ·8137 ± ·0078 + ·7936 ± ·0105	—	− ·0270 ± ·0263 + ·0096 ± ·0329	+ ·0346 ± ·0232 + ·0267 ± ·0287	
9	+ ·0640 ± ·0265 + ·0875 ± ·0334		− ·1325 ± ·0261 − ·1743 ± ·0323	+ ·0232 ± ·0263 − ·0427 ± ·0329	− ·0342 ± ·0263 − ·0201 ± ·0331	− ·0270 ± ·0263 + ·0096 ± ·0329	—	− ·0080 ± ·0266 + ·0054 ± ·0333	
7			+ ·0013 ± ·0240 + ·0204 ± ·0296	− ·0420 ± ·0232 − ·0280 ± ·0288	+ ·0586 ± ·0232 + ·0046 ± ·0288	+ ·0346 ± ·0232 + ·0267 ± ·0287	− ·0080 ± ·0266 + ·0054 ± ·0333	—	− ·0317 ± ·0229 − ·0311 ± ·0280
								− ·0317 ± ·0229 − ·0311 ± ·0280	—

Mean Correlation.*

Race	Number of Characters	Male	Female
Naqada Prehistoric Egyptians ...	25	·313	·258
XXVIth—XXXth Dynasty Egyptians	123	·296	·291
17th Century Londoners	30	·376	·343
Total	178	·312	·295

Thus the male mean value in all three series is in excess of the female; the excess, however, is not very great, especially in our longest series. On the whole, however, we think it is necessary to consider that the male skull for anthropometric characters has its parts more highly correlated than the female. It is not very easy to interpret this result in terms of selection. Selection of characters would reduce correlation, but there might be an actual selection of the adjustment of parts tending to increase correlation. Or, again, a slight racial mixture in one sex might emphasise the correlation. One or another of these causes, not necessarily the same, might have operated on the one or other race; it is easier to suppose selection for the women more severe in the case of the prehistoric Egyptians than for the men, but it is, perhaps, less reasonable to suppose it to be true of the 17th century Londoners. On the other hand the variabilities are higher in the prehistoric Egyptians and 17th century Londoners than in the late dynastic Egyptians, which might indicate less purity of race in the former, although it is not evident why this should occur more in the males than the females. On the whole it is best simply to record the fact and await the results of further determinations of correlation in reasonably long cranial series.

We shall now proceed to classify our correlations and shall thus find considerable light thrown on their nature. If the reader turns to Table VI, he will notice some 250 correlations placed in order of magnitude†. In order to interpret them we must make a few preliminary remarks.

(*a*) When one anthropometric measure "covers" a second, we naturally expect a high correlation between them. Thus the nasal height is a part of the upper face height, the frontal, parietal and occipital arcs are all parts of the sagittal arc. But we may give a wider sense to the term "covering" than this. The horizontal circumference may be said to cover the maximum length, the transverse arc the auricular height, and the sagittal arc both the maximum length and the auricular height. Again capacity is clearly a function of the chief cranial diameters and the principal arcs. The "common cause groups" in all these cases are so obvious that we can state *a priori* that there will be high correlations. They are not true organic correlations, but to a large extent result from what we may term mathematical necessity, i.e. from the compound character of the anthropometric measurements.

* Means found without regard to the *sign* of the correlations.

† The actual value of any one of these may be read off by reference to Table V.

TABLE VI. *Classified Table of Cranial Correlations.*

Numerical Values of the Correlation Coefficients.

− 0·7 to − 0·5	− 0·5 to − 0·3	− 0·3 to − 0·1	− 0·1 to + 0·1
− 0·7 to − 0·65	*− 0·5 to − 0·45*	*− 0·3 to − 0·25*	*− 0·10 to − 0·05*
	Nasal & Upper Face Ind. ♀ Max. Length & 1st Ceph. Ind. ♂ Max. Length & 2nd Ceph. Ind. ♂ Nasal & Upper Face Inds. ♂	Nasal & R. Orbital Inds. ♂ Nasal & L. Orbital Inds. ♂ Sagit. Arc & Occip. Ind. ♂	Capacity & Nasal Ind. ♀ Breadth & Occip. Ind. ♂ Capacity & Foram. Ind. ♂ Capacity & Occip. Ind. ♂
− 0·65 to − 0·60 Height & Third Ceph. Ind. ♂	*− 0·45 to − 0·40* Max. Length & 1st Ceph. Ind. ♀	*− 0·25 to − 0·20* Sagit. Arc & Occip. Ind. ♀	*− 0·05 to 0·00* Bizygomatic Breadth & Nasal Ind. ♀ Skull Base & Nasal Inds. ♀ 2nd Ceph. & Nasal Inds. ♂ Nasal & Palatal Inds. ♀ Occip. Ind. & Profile ∠ ♂ Nasal & Foram. Inds. ♂ Capacity & Nasal Ind. ♂ Skull Base & Nasal Inds. ♂ Palatal & L. Orbital Inds. ♂ Occip. & Foram. Inds. ♂ Occip. & Foram. Inds. ♀ Nasal & Foram. Inds. ♀ Capacity & Foram. Ind. ♀ Palatal & R. Orbital Inds. ♂ 1st Ceph. & Palatal Inds. ♀ Palatal & L. Orbital Inds. ♀ Skull Base & Occip. Ind. ♀ Bizygomatic Breadth & Nasal Ind. ♂ 1st Ceph. & Nasal Inds. ♂ 2nd Ceph. & Nasal Inds. ♀ Palatal & Foram. Inds. ♂ Breadth & 2nd Ceph. Ind. ♂ Capacity & 2nd Ceph. Ind. ♀ Capacity & 2nd Ceph. Ind. ♂
− 0·60 to − 0·55 Height & Third Ceph. Ind. ♀	*− 0·40 to − 0·35* Max. Length & Occip. Ind. ♀	*− 0·20 to − 0·15* Nasal & L. Orbital Inds. ♀ Nasal & R. Orbital Inds. ♀ Palate & Upper Face Inds. ♀	
− 0·55 to − 0·50 Max. Length & Second Ceph. Ind. ♀	*− 0·35 to − 0·30* Max. Length & Occip. Ind. ♂	*− 0·15 to − 0·10* Palate & Upper Face Inds. ♂ Capacity & Occip. Ind. ♀ Occip. Ind. & Profile ∠ ♀ Breadth & Occip. Ind. ♀	*0·00 to + 0·05* Foram. & Upper Face Inds. ♂ Foram. & L. Orbital Inds. ♀ Palatal & Foram. Inds. ♀ Palatal & R. Orbital Inds. ♀ Skull Base & Occip. Ind. ♂ Height & Profile ∠ ♂ 1st Ceph. & Nasal Inds. ♀ Upper Face & Foram. Inds. ♀ Nasal & Palatal Inds. ♂ Foram. & R. Orbital Inds. ♀ Breadth & 2nd Ceph. Inds. ♀ 3rd Ceph. & Nasal Inds. ♀ Foram. & Orbital Inds. ♂ Auricular & Nasal Heights ♀ 3rd Ceph. & Nasal Inds. ♂ 2nd Ceph. & L. Orbital Inds. ♀
1st Cephalic Index = 100 *B/L* 2nd „ „ = 100 *H/L* 3rd „ „ = 100 *B/H* Bizygomatic = Max. Breadth between zygomae			*+ 0·05 to + 0·10* Height & 1st Ceph. Ind. ♂ Max. Length & 3rd Ceph. Ind. ♂ 2nd Ceph. & R. Orbital Inds. ♀ Height & Nasal Breadth ♂ Auricular Height & Occipit. Ind. ♂ Foram. & L. Orbital Inds. ♂ Sagit. Arc & Skull Base ♀ Palatal & 2nd Ceph. Inds. ♂ Palatal & 1st Ceph. Inds. ♂ Capacity & 1st Ceph. Ind. ♂ Auricular & Nasal Heights ♂ Breadth & Nasal Breadth ♀ Height & Nasal Breadth ♀ Palatal & 2nd Ceph. Ind. ♀ Breadth & Skull Base ♂ Auricular Height & Occip. Ind. ♀ 2nd Ceph. & R. Orbital Inds. ♂ 1st Ceph. & R. Orbital Inds. ♂ 1st Ceph. & Foram. Inds. ♂

TABLE VI. *Classified Table of Cranial Correlations—(continued).*

Numerical Values of the Correlation Coefficients.

+0·1 to +0·3	+0·3 to +0·5	+0·5 to +0·7	+0·7 to +0·9
+0·10 to +0·15	*+0·30 to +0·35*	*+0·50 to +0·55*	*+0·70 to +0·75*
Height & Profile ∠ ♀	Up. Face Height & Face Breadth ♀	Foram. Length & Breadth ♀	Upper Face & Nasal Heights ♂
1st Ceph. & Orbital Ind. ♀	Max. Length & Bizygomatic ♀	3rd Ceph. Ind. & Breadth ♂	Upper Face & Nasal Heights ♀
1st Ceph. & Foram. Inds. ♀	Max. Length & Auricular Height ♀	Trans. Arc & Horiz. Circum. ♀	Capacity & Sagit. Arc ♀
Breadth & Face Breadth ♂	Facial & Nasal Breadths ♂	Foram. Length & Breadth ♂	Capacity & Trans. Arc ♂
Breadth & Nasal Breadth ♂	Breadth & Min. Forehead Breadth ♂	Capacity & Height ♂	Height & Auricular Height ♂
Sagit. Arc & Skull Base ♂	Up. Face Height & Bizygomatic ♀	*+0·55 to +0·60*	Height & Auricular Height ♀
Breadth & Nasal Height ♂	Breadth & Height ♂	3rd Ceph. Ind. & Breadth ♀	Capacity & Horiz. Circum. ♀
1st Ceph. & L. Orbital Inds. ♀	Breadth & Bizygomatic ♀	1st Ceph. Ind. & Breadth ♂	2nd Ceph. Ind. & Height ♀
Breadth & Skull Base ♀	Breadth & Height ♀	Trans. Arc & Horiz. Circum. ♂	Max. Length & Sagit. Arc. ♂
Breadth & Profile ∠ ♂	Trans. Arc & Bizygomatic ♀	Trans. & Sagit. Arcs ♀	
2nd Ceph. & L. Orbital Inds. ♂	Trans. Arc & Bizygomatic ♂	Capacity & Auricular Height ♀	*+0·75 to +0·80*
1st Ceph. & L. Orbital Inds. ♂		Capacity & Max. Length ♀	Horiz. Circum. & Sagit. Arc ♂
Max. Length & Nasal Breadth ♀	*+0·35 to +0·40*		Capacity & Sagit. Arc ♂
Auricular Height & Up. Face Height ♀	Max. Length & Trans. Arc ♂	*+0·60 to +0·65*	Capacity & Horiz. Circum. ♂
Auricular Height & Up. Face Height ♂	Height & Horiz. Circum. ♂	Capacity & Auricular Height ♂	R. & L. Orbital Inds. ♀
Sagit. Arc & Upper Face Height ♀	Auricular Height & Horiz. Circum. ♀	Height & Trans. Arc ♀	
Capacity & Face Breadth ♂	Breadth & Bizygomatic ♂	Breadth & Horiz. Circum. ♂	*+0·80 to +0·85*
Capacity & Profile ∠ ♂	Max. Length & Breadth ♀	Auricular Height & Sagit. Arc ♀	R. & L. Orbital Inds. ♂
Height & Nasal Height ♂	Max. Length & Height ♂		Auricular Height & Trans. Arc ♀
Breadth & Up. Face Height ♂	Max. Length & Bizygomatic ♂	*+0·65 to +0·70*	
Height & Nasal Height ♀	Capacity & Bizygomatic ♀	Breadth & Trans. Arc ♀	*+0·85 to +0·90*
Trans. Arc & Up. Face Height ♂	Max. Length & Auricular Height ♂	Sagit. & Trans. Arc ♂	Max. Length & Horiz. Circum. ♀
+0·15 to +0·20	Face Breadth & Bizygomatic ♂	Height & Trans. Arc ♂	Max. Length & Horiz. Circum. ♂
Breadth & Profile ∠ ♀	Max. Length & Breadth ♂	Auricular Height & Sagit. Arc ♂	Auricular Height & Trans. Arc ♂
Breadth & Nasal Height ♀		Breadth & Horiz. Circum. ♀	
Sagit. Arc & Up. Face Height ♂	*+0·40 to +0·45*	Capacity & Max. Length ♂	
Capacity & Face Breadth ♀	Facial & Nasal Breadths ♀	Capacity & Breadth ♀	
Palatal Breadth & Length ♀	Breadth & Auricular Height ♀	Sagit. Arc & Horiz. Circum. ♀	
Sagit. Arc & Bizygomatic Breadth ♀	1st & 2nd Ceph. Inds. ♂	Breadth & 1st Ceph. Ind. ♀	
Capacity & Up. Face Height ♀	Min. Forehead & Bizygomatic Breadth ♂	Capacity & Breadth ♂	
Palatal Breadth & Length ♂	Capacity & Bizygomatic ♂	Breadth & Trans. Arc ♂	
Length & Profile ∠ ♂	1st & 2nd Ceph. Inds. ♀	Height & 2nd Ceph. Ind. ♂	
Length & 3rd Ceph. Ind. ♀	Facial & Bizygomatic Breadths ♂	Capacity & Trans. Arc ♀	
Trans. Arc & Up. Face Height ♀	Max. Length & Trans. Arc ♂	Max. Length & Sagit. Arc ♀	
Height & 1st Ceph. Ind. ♀	*+0·45 to +0·50*		
Capacity & 1st Ceph. Ind. ♀	Length & Skull Base ♂		
Height & Up. Face Height ♂	Horiz. Circum. & Bizygomatic ♀		
Height & Up. Face Height ♀	Breadth & Auricular Height ♂		
Breadth & Up. Face Height ♀	Length & Skull Base ♀		
Max. Length & Height ♀	Height & Sagit. Arc ♂		
Capacity & Foram. Breadth ♀	Horiz. Circum. & Auricular Height ♂		
Breadth & Face Breadth ♀	Breadth & Sagit. Arc ♀		
Auricular Height & Profile ∠ ♂	Horiz. Circum. & Bizygomatic ♂		
+0·20 to +0·25	Height & Sagit. Arc ♀		
Capacity & Profile ∠ ♀	Least Forehead & Bizyg. Breadths ♀		
Nasal Breadth & Nasal Height ♀	Breadth & Sagit. Arc ♂		
Up. Face & R. Orbital Inds. ♀	Capacity & Height ♀		
Capacity & Up. Face Height ♂			
Auricular Height & Bizygomatic ♂			
Auricular Height & Bizygomatic ♀			
Capacity & Skull Base ♀			
Up. Face Height & Face Breadth ♂			
Capacity & Skull Base ♂			
Nasal Breadth & Bizygomatic ♀			
Height & Bizygomatic ♂			
Max. Length & Nasal Breadth ♂			
Sagit. Arc & Bizygomatic ♂			
Capacity & Foram. Length ♂			
Capacity & Foram. Length ♀			
Nasal Breadth & Nasal Height ♂			
Up. Face & R. Orbital Inds. ♂			
Nasal Breadth & Bizygomatic ♂			
+0·25 to +0·30			
Height & Horiz. Circum. ♀			
Auricular Height & Profile ∠ ♀			
Length & Profile ∠ ♀			
Length & Up. Face Height ♂			
Length & Nasal Height ♀			
Length & Nasal Height ♂			
Up. Face Height & Bizygomatic ♂			
Height & Bizygomatic ♀			
Capacity & Foram. Breadth ♂			
Length & Up. Face Height ♀			
Least Forehead & Max. Breadths ♀			

(*b*) If the same skull were taken and reproduced with the same proportions of its parts to say a thousand different sizes, there would be considerable correlation between the parts of this series of a thousand crania. A large skull would be large in all its parts and a small skull small in all its parts, and correlation exists at once because we have large and small in the series. This correlation would result if the large and small crania were due to differences in growth. In *absolute* measurements we may expect the occurrence of this *size* correlation; we anticipate that capacity will be correlated with the size of the foramen magnum, or the cranial breadth with the cranial height. To get over this difficulty we introduce indices and consider the correlation of such indices. The correlation of these indices is very instructive. We may have:

(i) The correlation of indices none of the components of which are common factors or factors "covering" others.

(ii) The correlation of indices some of the components of which "cover" others.

(iii) The correlation of indices which have some common component.

(iv) The correlation of an index with an absolute length which is not a component or a length covering a component.

(v) The correlation of an index with an absolute length which does cover a component.

(vi) The correlation of an index with an absolute length which is one of its components.

Bearing these points in mind interpretation of our classified correlations becomes fairly simple. Let us look first at the 18 high positive correlations; 14 of them are the correlations between characters one of which in a general sense "covers" the other. This is not what we can term fairly "organic" correlation; it is so to speak the artificial correlation of a whole with one of its parts. Only two correlations appear to us genuinely organic, the correlation between the shape of the right and left orbits as evidenced by that of the orbital indices for the two sexes. These give a correlation of about 0·8, about the intensity we find for homologous bones of the skeleton, and this is approximately the correlation of homologous parts of the skull*. To the two remaining correlations we will refer shortly.

Let us now turn to the next column, + 0·5 to + 0·7, with 29 coefficients. Of these two coefficients in the lowest group, 0·50 to 0·55, the correlations of foraminal breadth with foraminal length, may be classed as organic; 17 belong to the class in which one character "covers" the other, and of these 17, no less than 14 are in the two higher classes above 0·60. We next have a series of correlations in which while one character does not cover the other, they have in fact a common factor.

* As examples of the correlation of homologous parts we may note: Orbital Breadths, R and L, (σ: + ·8704 ± ·0056, ♀ : + ·8168 ± ·0094), Orbital Heights, R and L, (σ: + ·8664 ± ·0057, ♀ : + ·8896 ± ·0057). If, however, we take parts of the same orbit, $O_1(L)$ and $O_2(L)$, we have

σ: + ·2639 ± ·0212, ♀ : + ·3756 ± ·0240,

a large drop in the intensity of association, or parts of the same organ have far less correlation than homologous parts of different organs.

Thus the transverse arc and horizontal circumference both have a common factor in the general breadth of the skull, the transverse and sagittal arcs in the common auricular height of the skull, and the sagittal arc and horizontal circumference in the length of the skull. Of these six correlations, five occur in the present group and one that of horizontal circumference and sagittal arc for males is one of those we omitted to consider under the group 0·7 to 0·9.

There still remain five correlations and these belong to a most suggestive group. The three cephalic indices: $100\,B/L$, $100\,H/L$ and $100\,B/H$ may be correlated with L, B and H*. Here is the actual table for them:

	$100\,B/L$	$100\,H/L$	$100\,B/H$
L ♂	$-\cdot4902\pm\cdot0172$	$-\cdot4848\pm\cdot0175$	$+\cdot0529\pm\cdot0229$
L ♀	$-\cdot4138\pm\cdot0231$	$-\cdot5272\pm\cdot0204$	$+\cdot1773\pm\cdot0274$
B ♂	$+\cdot5888\pm\cdot0148$	$-\cdot0059\pm\cdot0229$	$+\cdot5089\pm\cdot0169$
B ♀	$+\cdot6803\pm\cdot0150$	$+\cdot0377\pm\cdot0283$	$+\cdot5617\pm\cdot0190$
H ♂	$+\cdot0530\pm\cdot0229$	$+\cdot6867\pm\cdot0121$	$-\cdot6384\pm\cdot0135$
H ♀	$+\cdot1806\pm\cdot0273$	$+\cdot7365\pm\cdot0129$	$-\cdot5897\pm\cdot0184$

Now we see that L, B, or H has a high correlation with the three cephalic indices when they occur as a component, positive when they occur in the numerator, negative when they occur in the denominator. But when L, B, or H does not appear as a component, then the correlation is sensibly zero in the case of males and one female case, in the other two female cases it is significant but small†. We reach therefore the conclusion that there is little or no correlation between an index and an absolute length unless the length is one of the components of the index, or there is a covering relation between the length and one of the components. Now let us look at the correlations between various classes of indices that may occur. We can have the correlation of two indices when:

(*a*) none of the components are identical or "cover" any other;

(*b*) some of the components "cover" each other;

(*c*) some of the components are identical.

We can have the correlation of an index and an absolute quantity when

(*d*) that quantity does not appear as a component and does not cover a component;

(*e*) that quantity covers a component;

(*f*) that quantity is a component.

* H and $100\,H/L$ (♀) form the correlated pair left out of consideration in dealing with our first, the $+0\cdot7$ to $+0\cdot9$ group.

† In dealing with sexed bones we are inclined always to lay more stress on the constants for the male series, because definitely male bones, especially crania, are more easy of identification, but young adult males are not unlikely to be confused with females. Thus female correlations may be emphasise d y some amount of heterogeneity.

Now if the reader will examine the range of correlations from − 0·05 to + 0·05 where we can hardly assert definite significance he will note how many of the index correlations of Class (*a*) fall into this group. No less than 25 out of the 40 correlations falling in the range − 0·05 to + 0·05 are index with index correlations. If we extend the range to include 0·05 to 0·12 we have an additional 13 correlations of index with index, or we may say that the following 38 correlations of indices among themselves for both sexes are insignificant or of no importance: 1st, 2nd and 3rd Cephalic Indices with Nasal Index, 1st and 2nd Cephalic Indices with R. and L. Orbital Indices and with the Palatal Index; the Foraminal with the 1st Cephalic Index, the Palatal Index, the Nasal Index, both R. and L. Orbital Indices and Upper Face Index; the Palatal Index with R. and L. Orbital Indices and Nasal Index; and the Occipital Index with the Foraminal. It would thus seem a principle of great importance that: *There is no correlation between the shapes of individual parts of the skull.*

The exceptions to this principle must now be considered. We have among the present computations four cases to be noted which are when analysed very suggestive:

(i) Nasal and Upper Face Indices. Here we find 100 NB/NH and 100 $G'H/GB$ give fairly high correlations:

$$♂: -\cdot4607 \pm \cdot0186; \quad ♀: -\cdot4902 \pm \cdot0221.$$

Now this is a remarkable confirmation of the rule; these correlations belong to Class (*b*), but *doubly* to that class, for the Upper Face Height covers the Nasal Height and the Face Breadth covers the Nasal Breadth, and these two pairs of covering factors are inverted one factor being in the denominator of one index and the covering factor in the numerator of the other; we have thus a re-enforcing action of the two making for a considerable negative correlation. This correlation is no organic relationship, but due to the "covering" nature of the components of the two indices.

(ii) Nasal and Orbital Indices. We have 100 NB/NH with 100 O_2/O_1 (R) giving:

$$♂: -\cdot2904 \pm \cdot0210; \quad ♀: -\cdot1837 \pm \cdot0274,$$

and 100 NB/NH with 100 O_2/O_1 (L) giving:

$$♂: -\cdot2507 \pm \cdot0215; \quad ♀: -\cdot2164 \pm \cdot0206.$$

The existence of these negative correlations is amply accounted for by the facts that Nasal Breadth in part covers Orbital Length and Nasal Height in part covers Orbital Height. Thus we are again thrown back on "covering" action*.

(iii) Palatal and Upper Face Indices. Here the values of the correlation of 100 G_2/G_1 with 100 $G'H/GB$ are

$$♂: -\cdot1325 \pm \cdot0261; \quad ♀: -\cdot1743 \pm \cdot0323.$$

The values are again significantly negative and the explanation is to be found in the fact that the facial breadth covers the palatal breadth. We have not, however,

* The association of a high Nasal Index with a low Orbital Index—or more generally of a mongoloid nose with almond eyes—is seen in its relation to such negative correlation.

the double covering as in the case of the Nasal with the Upper Face or Orbital Indices and accordingly the negative correlation is lower.

(iv) Upper Face and Orbital Indices. Here we have the values of the correlations of 100 $G'H/GB$ with 100 O_2/O_1:

Left orbit: ♂: + ·2466, ♀: + ·2275, and

Right orbit: ♂: + ·2440, ♀: + ·2070.

Here the Face Breadth GB completely covers the Orbital Length O_1 and the Upper Face Height GH' partially covers O_2. Thus "covering" again suffices to account for the correlation, which need not be attributed to any mysterious organic relationship.

It will be noted that in the above cases we are dealing not with two components of the indices being the same, but solely with two components "covering" each other in whole or part. The typical cases when one component is common to two indices were exhaustively treated by the late Dr Macdonell* for the case of the three cephalic indices and he showed how the existence of the common component of the indices is the dominating factor in such index correlations. We give here the correlations of the three cephalic indices and in a footnote† those calculated from the Coefficients of Variation formulae, and we see that their values confirm the results already reached.

Correlation of Cephalic Indices (Direct Tabulation).

Sex	First, 100 B/L	Second, 100 H/L	Third, 100 B/H
First, 100 B/L ♂	—	+ ·4155 ± ·0190	+ ·4361 ± ·0186
First, 100 B/L ♀	—	+ ·4391 ± ·0229	+ ·4180 ± ·0234
Second, 100 H/L ♂	+ ·4155 ± ·0190	—	− ·6316 ± ·0138
Second, 100 H/L ♀	+ ·4391 ± ·0229	—	− ·6250 ± ·0173
Third, 100 B/H ♂	+ ·4361 ± ·0186	− ·6316 ± ·0138	—
Third, 100 B/H ♀	+ ·4180 ± ·0234	− ·6250 ± ·0173	—

* *Biometrika*, Vol. II. p. 239; see also Fawcett, Vol. I. p. 456.

† The table below shows the extent to which the deduction of correlations from the coefficients of variation is reliable. For close agreement second order terms are desirable.

Correlation of Cephalic Indices from Coefficients of Variation.

Sex	First, 100 B/L	Second, 100 H/L	Third, 100 B/H
First, 100 B/L ♂	—	+ ·3591 ± ·0199	+ ·5196 ± ·0167
First, 100 B/L ♀	—	+ ·4507 ± ·0225	+ ·4370 ± ·0229
Second, 100 H/L ♂	+ ·3591 ± ·0199	—	− ·5665 ± ·0155
Second, 100 H/L ♀	+ ·4507 ± ·0225	—	− ·6119 ± ·0177
Third, 100 B/H ♂	+ ·5196 ± ·0167	− ·5665 ± ·0155	—
Third, 100 B/H ♀	+ ·4370 ± ·0229	− ·6119 ± ·0177	—

We have seen that when cranial indices have no common component they have for practical purposes no correlation; that when one component "covers" another they have some correlation increasing with the extent of the "covering," and that when they have common components they have considerable correlation, positive or negative, according as the common components are in like or unlike positions. It seems idle to speak of any such correlations as "organic." There are either no, or at best very small "organic" correlations between the shapes of different portions of the skull. The observed correlations are due to the existence of common or covering factors in the indices. And notwithstanding the criticisms which have been raised against the term "spurious correlation," it seems to be the appropriate word to describe these correlations which are due, or largely due, to "covering" factors.

We now turn to those remaining correlations which are of no significance, or are so small as to be of no practical value*. Among such values we note those of cranial capacity and the chief indices—in other words the shape of the parts of the skull has no relation of value to the size of the brain as a whole. Thus we have capacity with Foraminal Index (♂: +·0763, ♀: −·0268), with Occipital Index (♂: −·0547, ♀: −·1044), with Nasal Index (♂: −·0397, ♀: −·0915), with Orbital Index (L)† (♂: −·0591 ± ·0249, ♀: +·0393 ± ·0316), with 2nd Cephalic Index, or 100 H/L (♂: −·0028, ♀: −·0035), with 3rd Cephalic Index, or 100 B/H (♂: +·0554, ♀: +·1601) and with 1st Cephalic Index, or 100 B/L (♂: +·0733, ♀: +·1839). Here the females in the 1st and 3rd Cephalic Indices give results divergent from the males, but we prefer for reasons already stated (p. 353, ftn. †) to consider the males as giving the better result. It is clear in any case that almost any measure of size on the skull, even the dimensions of the foramen, would be of more value than the cephalic indices, which the late Dr Beddoe proposed to use in estimating cranial capacity.

Into this group of no prognostic value fall also a number of correlations of indices with cranial lengths not "covering" or only covering in a minor degree one or other of their components. Such are the Nasal Index with Skull Base (♂: −·0386, ♀: −·0490) and with Bizygomatic Breadth‡ (♂: −·0113, ♀: −·0502); the Occipital Index with Skull Base (♂: +·0577, ♀: +·0881), with Breadth (♂: −·0882, ♀: −·1037), with Auricular Height (♂: +·0577, ♀: +·0881) and with Profile Angle (♂: −·04116, ♀: −·1153). Into this group also fall with the exception of two female correlations already referred to the correlations of the three cephalic indices with the non-covering major lengths.

The remaining correlations in this group are chiefly of non-covering absolute measurements, but they deserve some consideration. The following characters have

* By "no practical value" we understand that they would be of no service in reconstructing a skull from a known fragment, i.e. they have no prognostic value.

† This correlation is practically insignificant, and, as far as we can venture to extend it interracially, does not suggest any relation between size of brain and eye-shape.

‡ It may be suggested that the bizygomatic breadth covers the nasal breadth, but apart from the fact that the position of the bizygomatic breadth is well up the piriform aperture, the value of this breadth is essentially dependent on the curvature of the zygomatic arches.

little association with each other: Auricular Height with Nasal Height (♂: +·0723, ♀: +·0395); it might be supposed that Auricular Height would cover Nasal Height, but the relationship is far less than in the case of Cranial Height and Nasal Height (♂: +·1405, ♀: +·1447). Another exception of the same kind is that of Skull Base and Sagittal Arc (♂: +·1122, ♀: +·0601), where we should expect a considerable influence of the covering factor as we find it between Maximum Length of Skull and both Sagittal Arc and Skull Base. Indeed Breadth of Skull and Skull Base, which also falls into this group (♂: +·0861, ♀: +·1137), shows as much associa[illegible]n. Another noteworthy exception is that of Nasal Breadth, its correlation with Cranial Height (♂: +·0567, ♀: +·0783) is of no importance as we should anticipate on our hypothesis; its correlation with Cranial Breadth (♂: +·1118, ♀: +·0725) is, notwithstanding the "covering," of no real importance, while the correlations of both Nasal Breadth (♂: +·2358, ♀: +·1223) and Nasal Height (♂: +·2760, ♀: +·2699) with the non-covering Maximum Length are of a higher order of importance and not without considerable interest. Considerable length of skull is associated in a definite but mild degree with a large nose. The remaining correlation falling into the group −0·1 to +0·1 is that of Cranial Height with the Profile Angle (♂: +·0138, ♀: +·1029). It is not clear why the Profile Angle should be related to the Cranial Length, Breadth or Height, but if we had *a priori* to select one of these for a higher association it would be the Cranial Height. It is not so, however—the Profile Angle is associated with Auricular Height (♂: +·1979, ♀: +·2580) which might be presumed to have a "covering" factor, but not so highly with Cranial Height (♂: +·0138, ♀: +·1029) as with Length (♂: +·1759, ♀: +·2582), Breadth (♂: +·1180, ♀: +·1508), or indeed Capacity (♂: +·1365, ♀: +·2040). Clearly a long-headed person, and to a less extent a large-headed person, are less likely to be prognathic. It must, we think, be admitted that we have here real organic correlations dominating any "covering" factor influence*.

We have now to consider the small positive† correlations omitting those which have been incidentally discussed in earlier sections. In the first place we have to note a number of cases in which the absolute magnitudes concerned have a "covering" factor. Thus we have a whole series involving Capacity, namely: Capacity with Face Breadth (♂: +·1307, ♀: +·1592), with Upper Face Height (♂: +·2147, ♀: +·1744), with Foraminal Breadth (♂: +·2828, ♀: 1937), with Foraminal Length (♂: +·2383, ♀: +·2393), with Skull Base (♂: +·2311, ♀: +·2297).

* The detection of the "covering" factor requires sometimes a little thought. Thus there is a fair-sized correlation between Profile Angle ($P\angle$) and Palatal Index ($\cdot 100\ G_2/G_1$), i.e.

♂: +·3330 ± ·0240, ♀: ·3046 ± ·0308;

the explanation is that a short palate draws the alveolar point inwards and the vertical from the nasion may fall even anterior to that through the alveolar point. Again there is a high negative correlation between the Profile Angle and the Nasional Angle ($N\angle$) (♂: −·6276 ± ·0145, ♀: −·6470 ± ·0171). This is due to the fact that on the average a large Profile Length (GL) means a large Nasional Angle, but this throws the alveolar point forward and gives a prognathic face or small Profile Angle. The reader will find it an interesting study to seek interpretations of the correlations (especially in regard to their signs) of Profile and Nasional Angles with the facial Characters and Indices.

† Table VI shows that the whole of the negative correlations have already been dealt with and are chiefly illustrations of the influence of "covering factors."

Although these are of a higher order than the relation of Capacity to the Cephalic Indices, they have little prognostic value; they merely assert that Capacity covers these minor lengths. We next have a series of linear measurements involving "covering" factors: Breadth of Skull and Face Breadth (♂: +·1080, ♀: +·1956), Auricular Height and Upper Face Height (♂: +·1277, ♀: +·1268), Height and Upper Face Height (♂: +·1846, ♀: +·1855), Nasal Breadth and Bizygomatic Breadth (♂: +·2482, ♀: +·2330), Sagittal arc and Upper Face Height (♂: +·1555, ♀: +·1309), a correlation probably due to the "covering" action of the lower part of the parietal and upper part of the occipital arcs (S_2 and S_3), Transverse Arc and Upper Face Height (♂: +·1465, ♀: +·1778), and possibly we ought to include here although on the border line to a higher value Maximum Breadth and Least Forehead Breadth (♂: +·3303, ♀: +·2935).

But in this same group occur a number of pairs of which the correlation can hardly be attributed to "covering" factors. These are Breadth and Upper Face Height (♂: +·1442, ♀: +·1892), Sagittal Arc and Bizygomatic Breadth (♂: +·2361, ♀: +·1635), Auricular Height and Bizygomatic Breadth (♂: +·2131, ♀: +·2169), Height and Bizygomatic Breadth (♂: +·2333, ♀: +·2781), Maximum Length and Upper Face Height (♂: +·2629, ♀: +·2926), Bizygomatic Breadth and Upper Face Height (♂: +·2758, ♀: +·3335), Height and Horizontal Circumference (♂: +·3581, ♀: +·2550). To these we must add certain paired measurements: Palatal Breadth and Palatal Length (♂: +·1762, ♀: +·1620), Nasal Breadth and Nasal Height (♂: +·2387, ♀: +·2057), Upper Face Height and Upper Face Breadth (♂: +·2299, ♀: +·3022), Length and Breadth of R. Orbit (♂: +·2348, ♀: +·3665), Length and Breadth of L. Orbit* (♂: +·2630 ± ·0212, ♀: +·3905 ± ·0212). To none of these can we give any other interpretation than that of organic correlation. Before we proceed to consider this further let us analyse the remaining group, that of positive correlations from 0·3 to 0·5. These moderately large correlations consist of three groups: First of index correlations, where common factors exist. These we have already discussed. Secondly, of the correlations of absolute lengths with large "covering" factors. Thirdly, of the correlations of absolute but considerable lengths with no "covering" factors.

In the second group we find the correlations of Facial Breadth with Nasal Breadth (♂: +·3197, ♀: +·4073), Cranial Breadth with Bizygomatic Breadth (♂: +·3732, ♀: +·3368), Transverse Arc with Bizygomatic Breadth (♂: +·3480, ♀: +·3448), Capacity with Bizygomatic Arc (♂: +·4239, ♀: +·3876), Facial Breadth with Bizygomatic Arc (♂: +·3974, ♀: +·4423), Minimum Forehead with Bizygomatic Breadth (♂: +·4170, ♀: +·4883), Maximum Length with Skull Base (♂: +·4515, ♀: +·4580), Horizontal Circumference and Bizygomatic Breadth (♂: +·4835, ♀: +·4536), Cranial Height and Sagittal Arc (♂: +·4671, ♀: +·4858), and Capacity and Cranial Height (♂: +·5357, ♀: +·4943). All these substantial correlations may be due—at any rate largely, if not wholly—to "covering" factors. Such factors will not however account for the nearly equally large third group which contains the following pairs: Maximum Length and Bizygomatic Breadth

* Not entered in Table V to avoid still more unduly increasing its dimensions.

(♂: +·3846, ♀: +·3060), Maximum Length and Auricular Height (♂: +·3963, ♀: +·3153), Cranial Breadth and Cranial Height (♂: +·3344, ♀: +·3389), Maximum Length and Transverse Arc (♂: +·4449, ♀: +·3527), Auricular Height and Horizontal Circumference (♂: +·4719, ♀: +·3690), Maximum Length and Cranial Breadth (♂: +·3971, ♀: +·3763), Maximum Length and Cranial Height (♂: +·3830, ♀: +·1896), Cranial Breadth and Auricular Height (♂: +·4537, ♀: +·4088), Cranial Breadth and Sagittal Arc (♂: +·4902, ♀: +·4782). It is clear that these eighteen correlation coefficients cannot be in any way directly associated with "covering" factors, they have not common parts. Taking them in association with similar factors in the preceding class (+·1 to +·3), we must admit that between, say, +·2 and +·5 there exist some forty out of two hundred and fifty correlations, which cannot be due to "covering" factors, and can only be described as "organic." As we have not selected our pairs to correlate with any regard to the existence or non-existence of such factors, it is fairly safe to say that about 16 °/₀ of cranial correlations will be found to be of this "organic" character.

It is quite another problem, however, to interpret wherein the organic association lies. It may be supposed to result from the existence of certain control determinants which guide the development of the component parts of the skull. A chief difficulty of such a hypothesis, besides its more or less transcendental character, lies in the fact that we have observed that there are no correlations—in the absence of "covering" factors—between the *shapes* of different parts of the skull as judged by indices, unless the indices are indices of homologous parts. This certainly does not look like a tactical development dictated "according to plan" by a staff-corps of control determinants. Is it possible therefore that all we have to do with in those organic correlations is a phenomenon of growth? The "plan" consists in a large head or a small head and the bones develop from their centres of ossification until they join. One bone may grow slightly more quickly than another, and hence we have the variability of the system and in particular the universal, if not over emphatic asymmetry of the cranium. But on the whole homologous bones grow under like conditions at the like rates, and thus arise the high correlations of homologous parts; they are not absolutely equal and therefore their correlations are not perfect. In the same way the factor of growth exists but is less influential in non-homologous bones: the "plan" is a skull of definite type of a more or less definite size and the bones grow each more or less independently until they meet and unite. The reader may object that in such a suggestion no explanation is provided for individual resemblance within the family or more widespread resemblance within the race. To this one may reply that heredity of family or race provides the brain shape, brain size, and brain detail, that the function of the cranial bones is to cover this brain, and that their organic correlations are produced in growing to its soft parts—much as the correlations of measurements on crab-shells are produced by those shells having grown to the "plan" of the crabs' soft bodies. Notwithstanding its plastic character the brain moulds the head shape and size, through fetal days and early youth, and if that plastic body be distorted artificially, or pathologically through the presence of fluid, the bones will grow to the distorted

form. A man inherits the head-shape of his ancestry only indirectly; what he directly inherits on this view is the brain, which lays on its bone-case according to its plan. It does not do to dogmatise in the absence of a far more elaborate study of cranial correlations than we have provided, but some such theory as the present one would be certainly helpful in interpreting what we have learnt of cranial correlations, namely: that the correlations of non-homologous parts, especially of their indices (which roughly exclude the growth factor), are small in the absence of one character covering a second; that the so-called organic correlations are really due to growth, roughly equal growth to the larger or smaller size of the brain; and that the racial and familial characteristics of the skull are thus transferred to brain modelling. The staff corps of control-determinants working to plan may not be wholly abolished, but it is removed from the field of craniology. Craniometric correlations are only a feeble reflex of cerebral correlations, and anthropometric measurements, however valuable for racial differentiation, may be of small importance in distinguishing those cerebral characteristics on which the psychical differences of races depend. There is not known at present a single correlation between an anthropometric and a psychical character which is of the slightest prognostic value. Yet on the present hypothesis it is the skull which is determined by the organ of psychical activities; it is not an inherited or racial brain-case, which fixes the development of the several parts of the skull. Some day a study of the variability and correlation of measurements on the brain—which can probably only be done by the use of endocranial casts—may show us more than craniometric correlations can do. Phrenology is only an absurd science because it rushed down a blind alley, before the brain itself had been adequately studied; long racial series of endocranial casts with a wider knowledge of the local distribution of cerebral activities might lead a rejuvenated phrenology to modest but none the less useful conclusions, if the new science must lack the ambitious dogmatism of the old. But all such science, new or old, stands or falls according as it discovers, or fails to discover, *numerically* determined correlations between physical and psychical characters exceeding let us say 0·4; this is the least value which is of the slightest importance for prognosis.

We have regretted the absence of a full study of anatomical correlations for these and these only would throw strong light on our suggestions as to the origin of the organic cranial correlations. We can, however, make a fairly detailed study in this direction in the case of the frontal, parietal and occipital arcs in relation to (i) the total sagittal arc and (ii) the maximum cranial length. This will suffice to indicate something of the relative importance of component and covering factors, and of the organic relationship of independent bones.

The following table as far as it extends confirms strongly the remarks we have previously made. Where one character (as S_1, S_2 or S_3) is a component of another (S) we obtain high or substantial correlations; where one character (L) covers another we obtain smaller, but still considerable correlations. When the lengths measured are the lengths of independent bones, we obtain significant but relatively much smaller correlations—the true organic correlations. These may even be negative

Table of Sagittal Arcs and Maximum Length Correlations.

	Maximum Length L	Total Sagittal Arc S	Frontal Arc S_1	Parietal Arc S_2	Occipital Arc S_3
L ♂	—	+·7405±·0103	+·4822±·0173	+·3766±·0194	+·5094±·0169
L ♀	—	+·6968±·0145	+·4225±·0230	+·2972±·0253	+·4424±·0227
S ♂	+·7405±·0103	—	+·6917±·0118	+·5991±·0145	+·5273±·0164
S ♀	+·6968±·0145	—	+·6790±·0151	+·3799±·0239	+·4925±·0212
S_1 ♂	+·4822±·0173	+·6917±·0118	—	+·1861±·0217	+·1545±·0222
S_1 ♀	+·4225±·0230	+·6790±·0151	—	+·1141±·0272	+·1074±·0280
S_2 ♂	+·3766±·0194	+·5991±·0145	+·1861±·0217	—	−·1587±·0221
S_2 ♀	+·2972±·0253	+·3799±·0239	+·1141±·0272	—	−·3422±·0247
S_3 ♂	+·5094±·0169	+·5273±·0164	+·1545±·0222	−·1587±·0221	—
S_3 ♀	+·4424±·0227	+·4925±·0212	+·1074±·0280	−·3422±·0247	—

as in the case of the correlation of the parietal and occipital arcs*. This negative correlation indicates a compensatory process, e.g. when the occipital bone is large then the parietals are smaller; actually, considering the cranial bones to develop as a covering for the brain, we might anticipate that such negative correlations would be fairly frequent, and *a priori* we might have supposed the positional variability of the bregma might, quite as well as that of the lambda, lead to a negative correlation of frontal and parietal arcs. That it does not is probably to be attributable to the dominance of the size factor, i.e. that skulls are large and skulls are small in all their parts. We cannot but believe that when the "anatomical correlations" have been determined for a long series of crania we shall have a series of correlations which will throw light on and may be interpreted by what is known, or may yet be discovered of the manner and periods of growth and development of the several cranial bones. It is a point not without interest or practical importance for the reconstruction of the skull from fragments that the entire sagittal arc can be best determined from the frontal arc, while the maximum length of the skull can be most accurately ascertained from the occipital arc. It is sad to think how much labour has been devoted to the reconstruction of palaeolithic crania, and to the living head from these reconstructed crania. Yet as far as we are aware these reconstructions have not been made with any due regard to the numerical correlations between parts of the skull, and particularly in the occipital region some of these reconstructions can only be described as exceedingly improbable. It would be of the highest help to such work to have a very full system of anatomical correlations and the corresponding regressions available. Still the present correlations may, even as they stand, be of service for medico-criminal purposes, provided we deal with modern man and with a race not too divergent from the ancient Egyptian.

* The female correlation is here more than double the male. We believe this to be due to the frequent development of large muscular attachment in the inionic region in the male upsetting the true relationship of the two arcs.

In the case of palaeolithic man great caution must be exercised; all we can say is that it would seem to us more useful to complete the Neanderthal or Pithecanthropus skull-caps from what we know of the association of the parts of the human skull in general, than to try and adapt a portion of the La Chapelle cranium to their defects, or even proceed with a pure work of the imagination. Whether it would be possible to reduce all results for variation and correlation to those of the variation and correlation of anatomical characters, and these again to a relatively few factors of which the two most important may be "planned" size X, and general growth Y, it is far too early yet to say; it might even be well at starting to assume that a correlation between X and Y may exist. Thus if κ_t be any character, we might assume if p_t and q_t be constants for that individual character:

$$\kappa_t = p_t X + q_t Y,$$

and if σ_t be the standard deviation of the tth character we have:

$$\sigma_t^2 = p_t^2 \sigma_X^2 + q_t^2 \sigma_Y^2 + 2 p_t q_t \sigma_X \sigma_Y r_{XY}.$$

We may write further for the correlation of the tth and t'th characters:

$$\sigma_t \sigma_{t'} r_{tt'} = p_t p_{t'} \sigma_X^2 + q_t q_{t'} \sigma_Y^2 + (p_t q_{t'} + q_t p_{t'}) \sigma_X \sigma_Y r_{XY}.$$

Let us put:

$$\lambda_{tt'} = \sigma_t \sigma_{t'} r_{tt'}, \quad \lambda_{tt} = \sigma_t^2, \quad u = \sigma_X^2, \quad v = \sigma_Y^2, \quad w = \sigma_X \sigma_Y r_{XY}.$$

Then our equations will be of the form:

$$\lambda_{tt} = p_t^2 u \quad + q_t^2 v \quad + (p_t q_t + p_t q_t) w,$$
$$\lambda_{tt'} = p_t p_{t'} u + q_t q_{t'} v + (p_t q_{t'} + p_{t'} q_t) w,$$

where the λ's are known. There will be $2n + 3$ unknowns, i.e.,

$$p_1, \ q_1; \ p_2, \ q_2; \ \ldots p_n, \ q_n, \text{ and } u, \ v, \ w$$

if there be n characters, while there are $n + \frac{1}{2}n(n-1)$, or $\frac{1}{2}n(n+1)$, equations. We should require n to be at least 5 before we should have enough equations to find the unknowns—there would in this case be two equations to spare. If we took 6 characters, we should require 15 equations out of 21 with 6 controls, and if 10 characters 23 equations out of a possible 55 with 32 controls, and so on. It would probably be wise to test our hypothesis of solving for the "best" values of 15 unknowns from 21 equations by least squares. Then every additional character would introduce two unknowns, so that its standard deviation and one correlation would be all that would be needful in order to determine its correlation with *all* the remaining characters.

It may be that a reduction to two factors (excluding common components and "covering" factors) may not suffice to account for the elaborate system of cranial anatomic correlations, but some such system seems more reasonable than to suppose that individual bones of the skull are predestined by an elaborate system of "determinants" to grow to a definite size and shape, so that they may aptly fit their neighbours.

Unfortunately it does not seem to me that the elaborate algebraic and numerical work necessary for solving even with two factors only such a system of equations

as are given above would be well spent on the few single bone measurements included in our anthropometric data. We have not enough simple measurements to test the efficiency of the hypothesis. Our analysis of the anthropometric correlations can only be said to suggest such an investigation, but it is well to postpone it, until a worker can be found, who, discarding anthropometric characters, will devote his energies to measuring characters on the individual bones of the skull. If any such worker will come forward, he will find in the Biometric Laboratory a homogeneous series of some 1700 crania at his disposal and ready help from its staff on the algebraical side. It is not a problem which can be completed in a few months' work, but it is, we venture to think, a very important problem. Does after all a most complicated organ like the cranium depend upon a vast system of hereditary determinants or can its adult form be described by aid of two or three constants associated with each centre of ossification and two or three factors peculiar to the skull as a whole? With the aid of such a relatively simple system, it might be possible to throw back heredity, whether racial or familial, on brain heredity, and thus dimly grasp how selection of psychical and motor activities has moulded human brains, and so indirectly influenced racial cephalography, and to some extent physiognomy.

There is, however, one point which is I hold well illustrated in the present memoir. The highest correlations—if we place on one side the measurements of homologous bones or organs—arise from the presence of common factors, covering factors, or the covering of a common factor. Much insight is thus afforded into the nature of the "common cause groups" on which correlation is supposed to depend. Where we find a correlation between, say, Palatal and Upper Face Indices, we are not in the first place to regard this as a mysterious organic relationship; we must remember that palatal breadth is a portion of facial breadth, and that this common factor is a "common cause group." It is precisely similar to the process of finding the correlation between successive draws of n cards from a pack, where to form the second draw of n we leave s cards of the draw on the table, supplementing them by $n - s$ fresh cards. In this way we create an arbitrary correlation (depending on the size of s) between our draws. The small correlation of indices of the skull which have no common or covering factors serves to demonstrate how much the high correlation of certain common factor cranial indices is really non-organic, or what I have formerly termed spurious.

CANCER AND GOITRE.

By PERCY STOCKS, M.D.

CONTENTS.

		PAGE
I.	Introduction	364
II.	Goitre and cancer distributions in Switzerland	366
	(*a*) Sources of data	366
	(*b*) Correlation by cantons—cancer of all organs with goitre	367
	(*c*) Correlation by districts—cancer of all organs with goitre	369
	(*d*) Correlation by cantons—cancer of digestive tract with goitre	373
	(*e*) Goitre rate in school children and cancer mortality	377
III.	Goitre and cancer frequency in other countries	378
IV.	Goitre and cancer distributions in Norway	380
V.	Goitre and cancer distributions in U.S.A.	381
VI.	Post-mortem evidence of relation between thyroid enlargements and cancer	383
	(*a*) Source of data	383
	(*b*) Correlation of thyroid anomalies with cancer	384
	(*c*) Nature of thyroid anomalies met with	387
	(*d*) Site of primary cancer	387
	(*e*) Type of primary growth	387
VII.	Cancer of thyroid gland	391
VIII.	Cancer and Graves' Disease in England and Wales	392
	(*a*) Source of data	392
	(*b*) Distribution of Graves' Disease as indicated by standardized mortality rates	393
	(*c*) Correlation with cancer mortality	395
IX.	Cretinism and other thyroid diseases in England and Wales	398
X.	Conclusions	399
XI.	References	400

I. Introduction.

At the present time, when every week brings forth a new theory of the causation of cancer, one is naturally loth to augment the confusion by seeming to add another. It will be advisable therefore at the outset to disclaim any such intention and to state that the incentive for undertaking and publishing this research has been the possibility that from it might conceivably arise a means of diminishing in some degree the incidence of this terrible disease by a simple and practicable prophylactic measure, whilst leaving its cause as yet undecided. The discovery of the cause of a disease has not always been the principal factor contributing to its prevention or cure, as evidenced by tuberculosis, which has still to be fought mainly by prophylactic measures which were not inspired by the discovery of the tubercle bacillus.

The considerations which prompted this investigation were briefly as follows:

(i) The most goitrous country in Europe shows also the highest cancer mortality.

(ii) Endemic goitre is one of the more obvious manifestations of inadequacy of the thyroid gland, and its causation has been shown by M^{c}Carrison(1) and others to be either deficiency in iodine supplied to the body or inability of the body to assimilate iodine adequately.

(iii) The liability to cancer increases steadily after middle age, whilst thyroid function diminishes.

(iv) The goitrous thyroid gland is itself commonly affected by cancer, the normal thyroid uncommonly, and the hyper-functioning thyroid of Graves' Disease rarely if ever.

(v) Prophylactic administration of iodine in small doses to school children on a large scale in some of the most goitrous areas of Switzerland has in the last few years produced an astonishing decrease in goitre incidence.

(vi) It has been the clinical experience of some physicians in this country(2) that administration of iodine has seemed to retard the rate of growth of malignant tumours in some inoperable cases.

The question at once arises, Is there a common factor involved in the incidence of goitre and cancer, or can we go even farther and say that subnormal functioning of the thyroid gland renders the body more liable to be attacked by cancer? If either of these questions could be answered affirmatively it would very probably follow that the simple methods of prophylaxis which are proving so successful in combating the development of goitre in young people might also be effective in some measure, if applied from middle life onwards, in maintaining a resistance against cancer.

It may at once be objected that cancer is also common in countries where goitre is infrequent (e.g. England) and *vice versâ* (e.g. the Himalayas), but since hypo-function of the thyroid gland may exist without obvious goitre, and on the other hand a predisposition to cancer alone would not necessarily result in cancer unless the causative agent were present, these facts do not *à priori* rule out either of the above hypotheses.

In the attempt to throw some light upon this question the following principal lines of attack were commenced:

(*a*) To select one or more countries in which goitre is sufficiently common to give a reliable index of the degree of *hypo*-function of the thyroid in the populations of small sub-divisions, and determine what correlation, if any, exists between goitre prevalence and cancer mortality in these different divisions.

(*b*) To select a country in which Graves' Disease is sufficiently common to give some indication of the degree of *hyper*-function of the thyroid in the populations of small areas, and determine whether or not cancer mortality tends to be less in those areas.

(*c*) To determine from a parallel series of cancer and non-cancer post-mortem records whether enlargements or anomalies of the thyroid gland are more common in the one group than in the other, and also to what extent the thyroid gland itself is attacked by cancer.

Some of the results of these three main investigations are detailed in the following sections, together with some other data which are of interest in this connection. The investigation of (*a*) has as yet only been completed for Switzerland, but preliminary studies of the same problem in Norway and the U.S.A. are also included in the present paper.

II. Goitre and Cancer Distributions in Switzerland.

(*a*) *Sources of Data.* This study has only been made possible by the invaluable assistance I have received from Professor F. de Quervain of Bern, whose experience of goitre is unequalled. To him I express most heartily my gratitude for his cordial help during my visit to Bern, and I also acknowledge my indebtedness to the Chief of the Swiss Public Health Service and to Drs Carriére and Otto Stiner for kindly furnishing me with the statistical material.

In 1919 Dr Bayard of Zermatt published a paper(3) in which he propounded a theory that in advancing age the body cells gradually lose their affinity for thyroiodin and consequently their vitality, and that deficiency in the supply of thyroiodin to the cells, which occurs in goitre, produces in the same way a premature ageing of the tissues; he further suggested that in such conditions the tissues would become more liable to cancerous degeneration. The only evidence which he produced in support of the last suggestion was that the average weights of the thyroid gland of children aged 6—10 in Bern, Munich and Kiel were respectively 18·6 gms, 11·9 gms, 7·4 gms, and that the cancer mortalities in these localities in 1900 were 132, 98 and 57 per 100,000. In January 1923 Dr Renaud of Lausanne, in a discussion of cancer mortality in Switzerland(4), pointed out that 73 °/₀ of cancers in males and 50 °/₀ in females were of the digestive tract, but this and the apparent increase in cancer generally he was inclined to attribute to better diagnosis of internal cancer. After the commencement of my research in July 1923, Dr Hunziker of Adliswil, in a short paper read to the Swiss Goitre Commission on Sept. 22, 1923, and published in 1924(5), compared the percentage of recruits rejected for goitre in 1908–12 with the total cancer mortality rates for 1918–20 in 146 districts (excluding the three Southern cantons), and showed evidence of a slight positive relation, whereas with phthisis mortality there was a slight negative relation. The cancer rates were not, however, corrected for age or sex distribution, nor for uncertified deaths, nor was any account taken of the great differences in populations of the areas dealt with, so that this evidence was inconclusive. In the same Bulletin, Dr Stiner(6) reported upon the percentage of goitre in recruits called up for examination in the years 1921–22 in every district of Switzerland, and the data and maps given in that report form the basis of the goitre statistics used in this research. This more recent material has the advantage that it includes the *total* goitre rate in addition to the rejection rate, many cases of slight goitre being

accepted for military service, and I have used the total rates throughout as probably giving a better index of goitre prevalence.

The statistics of cancer mortality available were (i) deaths from cancer of each organ by sexes in each of the 25 cantons for the year 1920, (ii) deaths from cancer of each organ by sex and age for Switzerland as a whole in the years 1901–20, (iii) deaths from cancer in general for each sex in the 186 sub-districts for 1906–15. The age and sex distributions of the cantonal populations were available from the census returns of 1910 and 1920, and also the total populations of the sub-districts and the number of these deaths uncertified as to cause. It was therefore possible to standardize the cancer deaths for differences in sex and age distribution and also to correct for uncertified deaths. The last factor is not of great importance except in Uri, Graubünden and Valais. In the quinquennium 1916–20, out of approximately 290,000 deaths in Switzerland as a whole, 11,942 (about 4·0 °/₀) were of doubtful or unknown cause, but one half of these occurred in the three cantons referred to. Thus in the sparsely populated districts of the Valais where the long valleys leading up into the main chain of the Alps are almost impassable throughout a great part of the year, 4581 deaths out of a total of 12,427 were medically uncertified and of doubtful cause. The correction factors for each district in Valais and Graubünden were therefore computed separately and found to be very large for the Conthey, Hérens, Visp, Martigny, St Maurice, Goms, Sierre and Raron areas of the Valais, and for the Vorderrhein and Glenner districts of the Grisons. For the remainder of Switzerland the correction is small but has been applied throughout. The goitre rate, being obtained from males of a uniform age, does not require correction, but the assumption is made that this rate for any canton or district is proportional to the goitre rate in the population of that area as a whole*.

(*b*) *Correlation by Cantons—Cancer of all organs with Goitre.* In Table I is given an extract from the statistics for the cantons. The populations of the 25 cantons do not form a distribution which can be satisfactorily dealt with by correlation, owing to the very large populations of the cantons of Bern and Zürich. In order to obtain a more continuous distribution and to avoid overweighting the very populous cantons, the following re-arrangement was made:

Bern was divided into five sub-areas: N., consisting of the northern districts as far south as Biel, Büren and Neuveville; N.W.C., consisting of Aarberg, Bern, Erlach, Fraubrunnen, Laupen and Nidau; N.E., Aarwangen, Burgdorf, Trachselwald, and Wangen; S.E.C., Konolfingen, Schwarzenburg, Seftigen, Signau and Thun; S., the remaining southern districts.

Zürich was divided into four areas: N., Andelfingen and Winterthur; City of Zürich district; C., Bulach, Dielsdorf, Hinwil and Pfäffikon; S.W., the remaining four districts near the Lake of Zürich.

Vaud was divided into S.E., consisting of Aigle, Lausanne, Lavaux, Pays d'Enhaut and Vevey, and W., the remainder of the canton.

St Gallen was divided into N.E., consisting of Gossau, Rorschach and St Gallen proper, and S.W., the remainder.

* It is of course possible that differences in sex distribution might make this assumption not strictly accurate, but the variations in goitre rate dealt with are so great as compared with the slight variations in sex distribution that such a correction would not sensibly affect the calculations. Thus coefficient of variation of goitre rate is 55 °/₀, of sex ratio 2 °/₀, and of cancer rate 24 °/₀.

TABLE I. *Cancer and Goitre in Switzerland, by Cantons (for explanations see text).*

Canton (for sub-divisions see text)	Census population 1920	Total cancer deaths 1901—20		Total cancer deaths 1920		Deaths from cancer of digestive* tract 1920		Correction factor for uncertified deaths	Standardizing factor for cancer deaths	Standardized cancer mortality rate, all organs 1901—20†	Corrected No. of deaths from cancer		Ratio cancer of stomach to all cancers	Deaths from all causes 1920	Estimated goitre rate‡ 1921—22 per cent.
		Male	Female	Male	Female	Male	Female				All organs 1920	Digestive tract* 1920			
Appenzell Int.	14,614	290	228	7	8	6	6	1·089	·997	195·1	82	55	·42	255	8·5
„ Ext.	55,354	858	892	25	49	17	30	1·019	·875	135·4				766	
Aargau E.	110,220	3221	2842	171	131	135	79	1·036	·949	131·7	147	104	·45	3607	22·6
Aargau W.	130,556										150	106			10·6
Baselstadt	140,708	1364	1697	85	108	57	40	1·010	1·176	131·8	229	115	·23	1618	7·5
Baselland	82,390	812	790	49	42	37	19	1·016	1·061	112·1	98	60	·37	1033	8·1
Bern N.	165,160	6957	6217	378	325	286	154	1·021	1·049	105·1	188	118	·38	8886	2·7
Bern N.E.	104,281										124	78			15·4
Bern N.W.C.	201,497										212	133			16·7
Bern S.E.C.	130,291										136	85			23·5
Bern S.	73,165										92	57			11·6
Fribourg	143,055	1734	1605	102	100	78	54	1·055	1·004	127·1	219	140	·37	2442	14·6
Geneva	171,000	1713	2334	118	134	66	57	1·036	·934	125·0	244	119	·27	2487	9·1
Glarus	33,834	632	563	33	27	25	13	1·009	·783	143·0	84	51	·42	543	7·5
Uri	23,973	280	197	14	13	9	7	1·189	1·082	137·6			·38	389	12·5
Graubünden	119,854	1249	1278	66	53	45	25	1·103	·901	112·7	123	72	·41	1912	11·8
Luzern	177,073	2947	2272	158	118	119	71	1·010	·932	149·6	258	180	·50	2691	25·1
Neuchatel	131,349	1360	1558	63	76	45	40	1·026	1·043	116·6	149	91	·34	1668	1·3
Nidwalden	13,956	266	199	8	13	7	10	1·009	·967	165·9	45	28	·45	292	17·5
Obwalden	17,567	275	214	18	9	8	4	1·028	·860	126·4				277	25·0
St Gallen N.E.	113,638	4092	4023	191	209	145	126	1·015	1·044	149·5	137	94	·51	4195	12·5
St Gallen S.W.	181,905										283	193			20·2
Schaffhausen	50,428	575	606	28	36	22	20	1·008	·923	118·1	60	39	·47	749	17·2
Schwyz	59,731	1012	869	52	44	32	28	1·058	·981	166·1	145	92	·41	980	14·7
Zug	31,567	476	461	28	13	21	6	1·005	1·057	177·3			·51	460	17·5
Solothurn	130,617	1529	1337	94	71	69	31	1·015	1·112	135·2	186	113	·36	1719	8·2
Tessin	152,256	1156	1625	67	85	47	42	1·021	·847	77·8	131	77	·53	2940	4·3
Thurgau	135,933	1808	1847	106	108	85	63	1·014	·966	136·2	210	145	·51	2004	18·6
Valais	128,246	626	632	38	50	25	29	1·584	·942	79·1	142	87	·43	2287	17·4
Vaud W.	156,705	3032	3741	161	188	102	66	1·040	·991	111·3	193	93	·27	4540	7·7
Vaud S.E.	160,793										167	80			8·9
Zürich N.	86,787	6455	6997	388	413	276	220	1·014	1 016	137·0	129	80	·42	7069	30·0
Zürich City	248,566										317	196			12·5
Zürich C.	97,278										187	116			9·6
Zürich S.W.	105,971										192	119			13·1

* I.e. oesophagus, stomach, intestines and rectum.

† Based upon 1910 census populations; mean annual corrected rate per 100,000 living.

‡ From recruiting statistics.

Aargau was divided into E., Baden, Bremgarten, Lenzburg, Muri and Zurzach, and W., the remaining districts.

The two small cantons of *Innerrhoden* and *Ausserrhoden* were combined (Appenzell), as were also the adjoining pairs of cantons *Obwalden-Nidwalden*, *Glarus-Uri*, *Schwyz-Zug*.

This re-arrangement into 31 areas produced a more satisfactory distribution as regards population, and also as regards cancer deaths and estimated goitre cases.

In order to avoid the spurious correlation introduced by correlating ratios, and at the same time give greater weight to the more populous areas, the method used has been that suggested by Professor Pearson(7), of correlating actual numbers of cases and obtaining the partial coefficient when population is constant.

In this case the estimated numbers of cases of goitre in each area 1921–22 were correlated with the corrected and standardized numbers of deaths from cancer occurring in the same areas in 1920, and these numbers were also correlated with the respective census populations of 1920. The result was as follows:

	Correlation coefficients
Deaths from cancer of all organs (corrected) with estimated goitre cases	$r = \cdot6038 \pm \cdot0770$
Deaths from cancer with population	$r = \cdot8831 \pm \cdot0267$
Goitre cases with population	$r = \cdot5649 \pm \cdot0825$
Deaths from cancer of all organs with goitre cases, population constant	$r = \cdot2711 \pm \cdot1122$

It is conceivable, of course, that a correlation of this order might arise from differences in general unhealthiness of the areas considered, which might be supposed to influence mortality from all diseases and at the same time affect the incidence of goitre. To test this point I have correlated deaths from *all causes other than cancer* with goitre frequency for the same areas, with the following result:

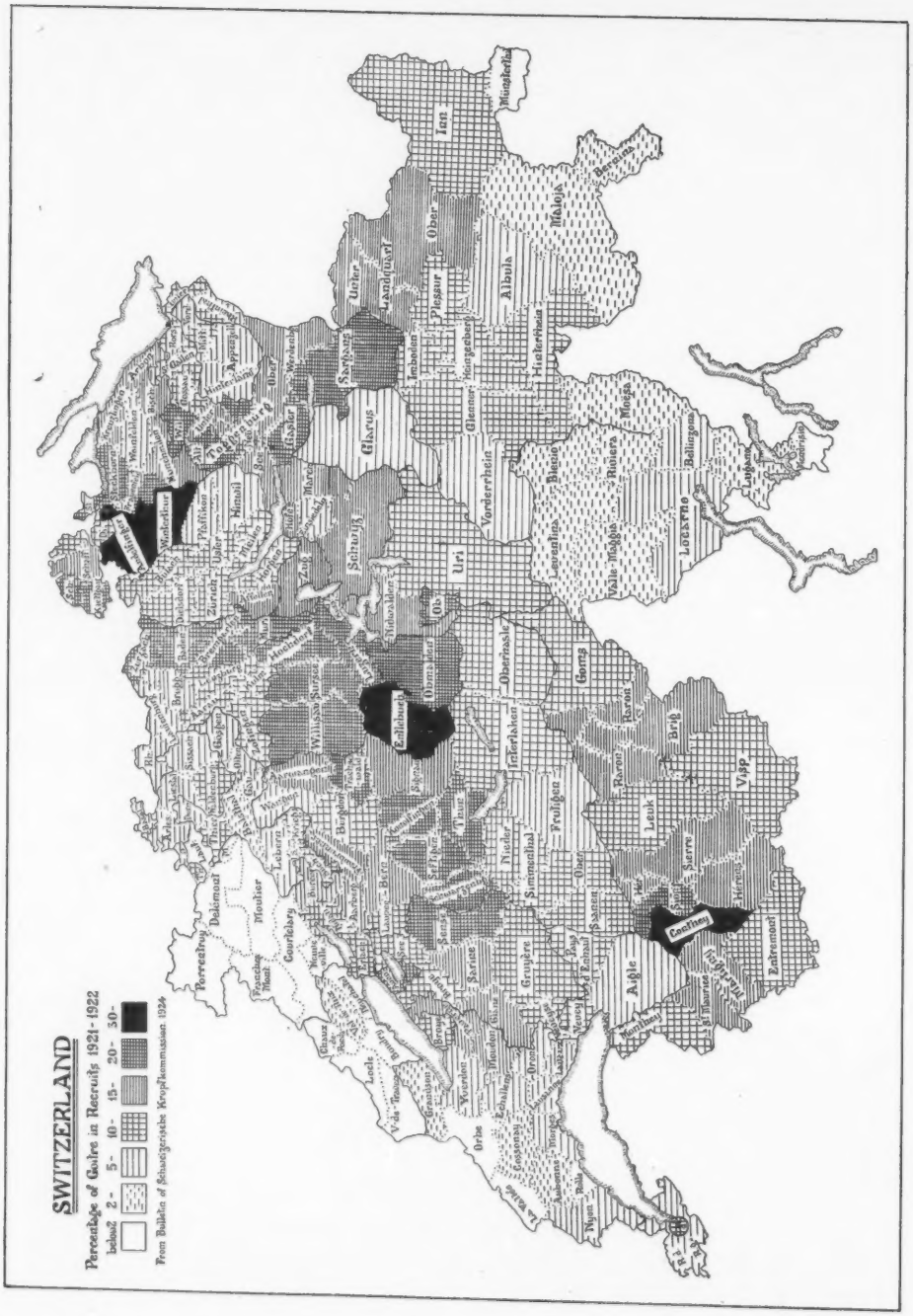
Deaths from all causes other than cancer (corrected) with goitre cases	$r = \cdot5625 \pm \cdot0821$
Deaths from all causes other than cancer with goitre cases, population constant	$r = \cdot1039 \pm \cdot1198$

This figure, in itself insignificant, measures the relation between general unhealthiness and goitre frequency. If we attempt to eliminate this factor by making non-cancer deaths constant, the result is

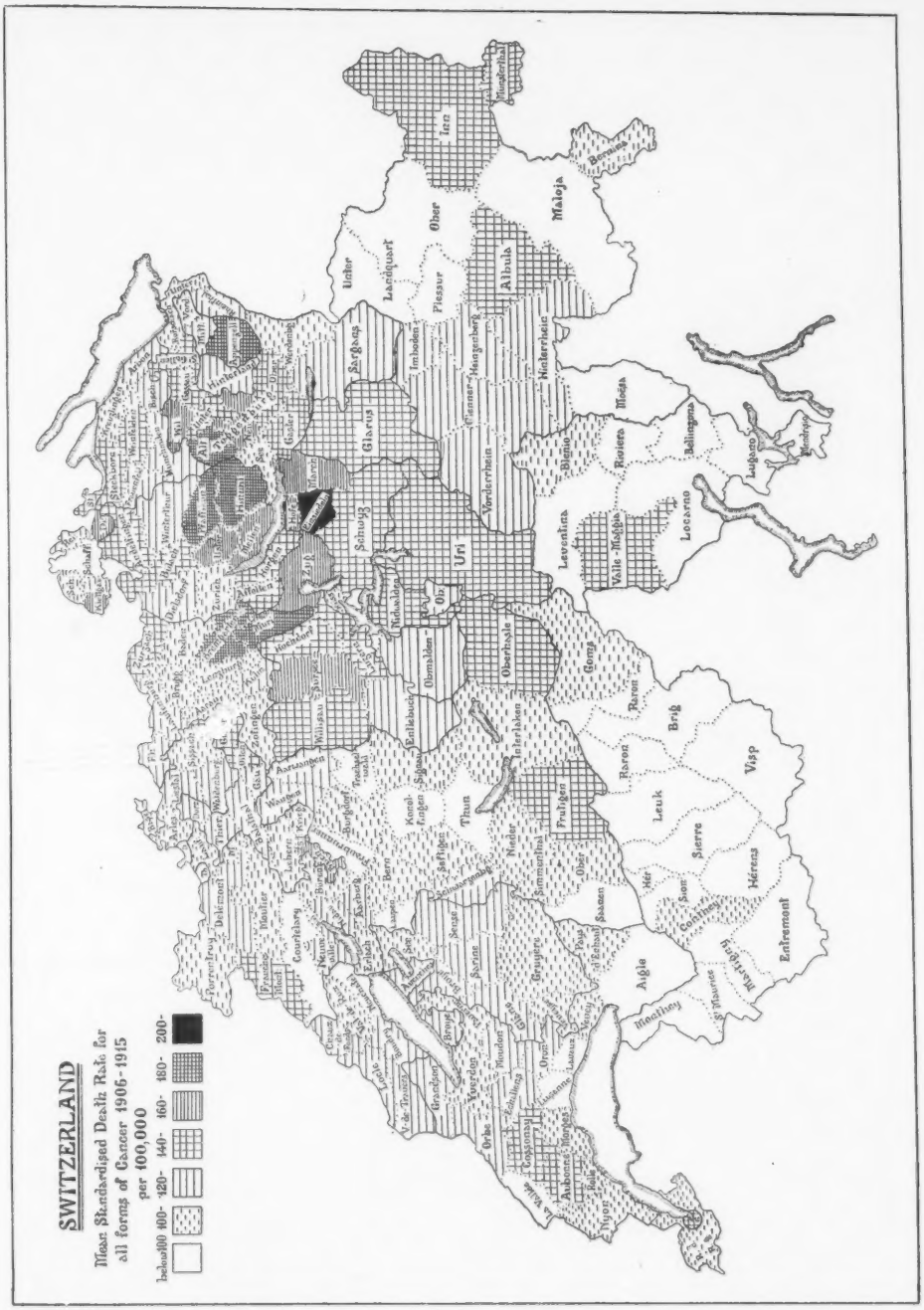
Deaths from cancer of all organs with goitre cases, population and non-cancer deaths being constant	$r = \cdot3600 \pm \cdot1054$

so that we can say at any rate that the general unhealthiness factor is not responsible for the correlation obtained between cancer and goitre.

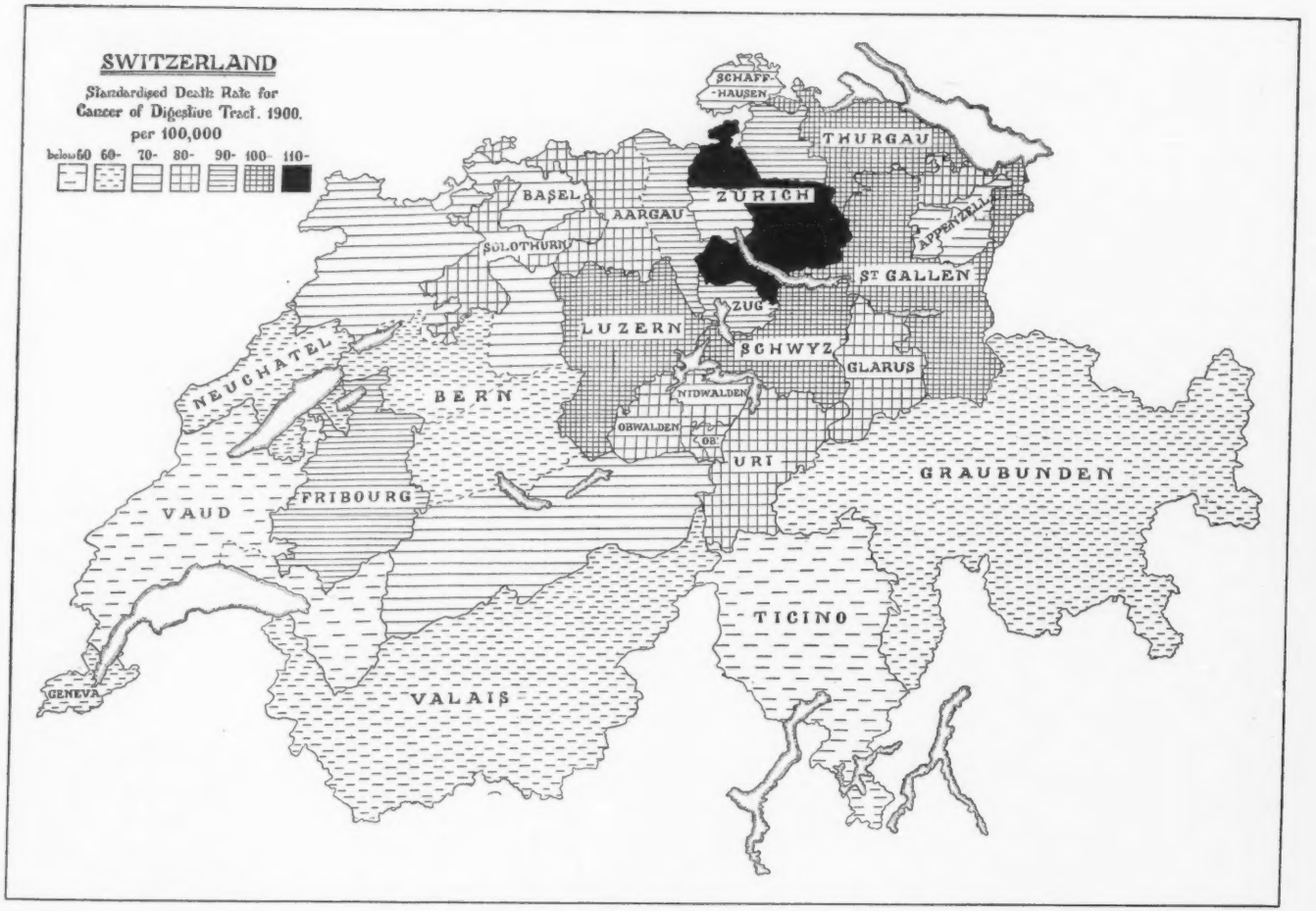
(*c*) *Correlation by Districts—Cancer of all organs with Goitre.* The 1921–22 goitre rates for the 186 districts of Switzerland are shown on the map (Map I) and the estimated numbers of cases of goitre calculated from these rates are given in Table II. The total cancer deaths in the period 1906–15 standardized for sex and age, and corrected for uncertified deaths, are also given in the Table, together with the populations and mean standardized cancer mortality rates. The


SWITZERLAND
Percentage of Goitre in Recruits 1921-1922
below 2 - 5 - 10 - 15 - 20 - 30 -
From Bulletin of Schweizerische Kropfkommission 1924
Ticino
Uri
Glarus
Graubünden
Valais
Vaud
Geneva
Neuchâtel
Jura
Bern
Lucerne
Zug
Schwyz
Unterwalden
Fribourg
Solothurn
Basel
Aargau
Zürich
Schaffhausen
Thurgau
St. Gallen
Appenzell
Engelberg
Andermatt
Airolo
Bellinzona
Lugano
Locarno
Chiasso
Martigny
Montreux
Lausanne
Nyon
Basel
Zürich
Winterthur
Schaffhausen
Rheinfelden
Aarau
Baden
Brugg
Olten
Solothurn
Biel
Bern
Thun
Interlaken
Luzern
Sursee
Sarnen
Stans
Altdorf
Schwyz
Einsiedeln
Glarus
Chur
Davos
Arosa
Landquart
Ilanz
Disentis
Bellinzona
Locarno
Lugano
Chiasso
Fribourg
Bulle
Romont
Payerne
Yverdon
Neuchâtel
La Chaux-de-Fonds
Le Locle
Delémont
Porrentruy
Saignelégier
Moutier
Courtelary
Sion
Martigny
Montreux
Vevey
Lausanne
Nyon
Aigle
Monthey
Saxon
Leuk
Visp
Brig
Zermatt
Goms
Oberwallis
Unterwallis
Entlebuch
Emmental
Oberaargau
Seeland
Simmental
Saanen
Gruyère
Broye
Glâne
Veveyse
Greyerz
Sense
Murten
Seebezirk
Laupen
Schwarzenburg
Konolfingen
Signau
Trachselwald
Burgdorf
Fraubrunnen
Wangen
Aarwangen
Oberhasle
Interlaken
Frutigen
Niedersimmental
Obersimmental
Saanen
Gruyère
Broye
Glâne
Veveyse
Greyerz
Sense
Murten
Seebezirk
Laupen
Schwarzenburg
Konolfingen
Signau
Trachselwald
Burgdorf
Fraubrunnen
Wangen
Aarwangen
Oberhasle
Interlaken
Frutigen
Niedersimmental
Obersimmental

MAP I.


SWITZERLAND
Mean Standardised Death Rate for all forms of Cancer 1906-1915 per 100,000
below 100
100-
120-
140-
160-
180-
200-

MAP II.


SWITZERLAND
Standardised Death Rate for Cancer of Digestive Tract. 1900. per 100,000
below 60
60-
70-
80-
90-
100-
110-
SCHAFF-HAUSEN
THURGAU
BASEL
AARGAU
ZURICH
APPENZELL
SOLOTHURN
ST GALLEN
ZUG
LUZERN
SCHWYZ
GLARUS
NEUCHATEL
BERN
NIDWALDEN
OBWALDEN
URI
GRAUBUNDEN
VAUD
FRIBOURG
TICINO
GENEVA
VALAIS

Map III.

last are also shown graphically on Map II. The populations given are those of the 1920 census, which have been used in the correlation tables, but the mortality rates are based on the populations of 1910 at the centre of the period covered. It was found impossible to obtain a satisfactory statistical distribution of populations without omitting districts of over 60,000 population* and combining several adjoining districts. The districts omitted were Bern, Basel, Zürich, Luzern, Lausanne, Winterthur, St Gallen and Genève Rive-Gauche; those combined were Balsthal-gäu with Balsthal, St Maurice-Martigny, Sierre-Hérens, Brig-Raron, Leuk-Visp, Oron-Lavaux, Imboden-Heinzenburg, and Saanen-Niedr. Simmenthal-Ober Simmenthal. This left 160 areas and the result of correlating by the same method as in (*b*) was as follows:

Deaths from cancer of all organs (corrected) with estimated goitre cases	$r = \cdot6459 \pm \cdot0302$
Deaths from cancer with population	$r = \cdot8613 \pm \cdot0134$
Goitre cases with population	$r = \cdot6241 \pm \cdot0317$
Deaths from cancer of all organs with goitre cases, for population constant	$r = \cdot2729 \pm \cdot0493$

This value agrees closely with that obtained in the last section, but we have much reduced the probable error.

(*d*) *Correlation by Cantons—Cancer of Digestive tract with Goitre.* The coefficients obtained above were sufficiently large to suggest the existence of some common factor at work in the incidence of the two disease in various districts. An examination of the statistics of cancer of various organs of the body shows that cancers of the oesophagus and stomach are entirely responsible for the excessive cancer mortality in Switzerland. This is shown in Table III, where the mean death rates for 1901–20 in Switzerland are compared with those for England and Wales for the period 1901–10. Thus the mortality per 100,000 from cancer of oesophagus and stomach in males amounted to 75·4 as compared to 21·6, whilst for all other organs combined the rates were respectively 45·7 and 55·7. In females the rate for oesophagus and stomach cancer was 46·2 for Switzerland, 16·2 for England and Wales, whilst for all other organs combined the rates were respectively 72·0 and 86·5. This remarkable tendency of cancer to attack the upper part of the digestive tract in Switzerland suggested a possibly more intense relation to goitre frequency, and I have therefore correlated the number of deaths from cancer of the digestive tract (oesophagus, stomach, intestine and rectum) in 1920, with estimated goitre cases in 1921–22 for the 31 areas for which data are to be found in Table I. The results were as follows:

Deaths from cancer of digestive tract (corrected) with estimated goitre cases	$r = \cdot7130 \pm \cdot0596$
Deaths from cancer of digestive tract with population	$r = \cdot8566 \pm \cdot0322$
Goitre cases with population	$r = \cdot5649 \pm \cdot0825$
Deaths from cancer of digestive tract with goitre cases, population constant	$r = \cdot5380 \pm \cdot0861$

* Such a procedure is of course open to criticism but on the other hand this simply amounts to excluding the large towns and confining attention to the country areas.

TABLE II.

Cancer and Goitre in Switzerland by Districts—Mortality from Cancer 1906–15 *and Goitre rates in recruits* 1921–22.

Canton and district	Population 1920	Mean annual corrected cancer death rate 1906—15 *	Corrected number of cancer deaths 1906—15	Estimated goitre cases 1921—22 †
AARGAU				
Aarau	29,362	114·2	321	3,670
Baden	38,922	105·7	394	9,730
Bremgarten ...	21,280	191·9	391	5,320
Brugg	20,524	118·7	234	1,540
Kulm	21,497	124·7	257	2,690
Laufenburg ...	14,201	132·9	181	1,060
Lenzburg	21,206	117·6	239	2,650
Muri	14,519	181·6	253	3,630
Rheinfelden ...	13,767	122·2	161	1,030
Zofingen	31,205	117·7	352	3,900
Zurzach	14,293	142·8	195	3,570
AUSSERRHODEN (Appenzell Ext.)				
Hinterland	25,380	128·5	362	1,900
Mittelland ...	14,350	129·9	195	1,790
Vorderland ...	15,624	114·8	188	1,170
BASELSTADT				
Stadtbezirk ...	135,976	134·7	1,771	10,200
Landbezirk ...	4,732	119·1	55	350
BASEL-LAND				
Arlesheim	36,558	98·1	333	2,740
Liestal	18,817	123·4	216	1,410
Sissach	17,340	107·1	172	1,300
Waldenburg ...	9,675	134·6	121	1,210
BERN				
Aarberg	19,175	121·3	229	3,350
Aarwangen ...	28,782	128·4	364	5,040
Bern	135,152	110·5	1,278	23,650
Biel	35,145	96·4	336	2,660
Büren	13,053	113·3	146	980
Burgdorf	32,467	102·1	327	4,060
Courtelary... ...	26,093	99·3	255	0
Delémont	18,564	122·2	224	180
Erlach	8,017	100·3	79	1,000
Franches-Montaignes	9,933	157·6	154	100
Fraubrunnen ...	14,613	92·6	149	2,560
Frutigen	12,553	145·4	181	940
Interlaken... ...	28,039	115·1	317	3,500
Konolfingen ...	31,345	90·0	277	7,840
Laufen	8,487	109·0	91	80
Laupen	9,547	80·2	85	1,190
Moutier	23,745	101·7	238	240
Neuveville	4,546	122·1	55	0
Nidau	14,993	132·8	196	1,870
Oberhasle	6,507	159·8	103	2,060
Porrentruy ...	25,324	116·4	290	250
Saanen	6,063	96·9	58	760
Schwarzenburg ...	11,138	112·3	135	2,780
Seftigen	21,790	117·3	252	5,450
Signau	25,035	101·2	250	4,380
Simmenthal Niedr.	12,454	103·9	127	1,560
,, Ober	7,549	113·7	85	940
Thun	40,983	92·2	372	10,250
Trachselwald ...	24,418	98·5	237	6,100
Wangen	18,614	122·8	252	1,400
FRIBOURG				
Broye	15,670	131·6	206	1,960
Glâne	14,911	132·6	198	2,240
Gruyère	27,372	117·1	321	3,280
Sarine	39,049	133·3	521	5,850
See	15,491	117·5	182	1,700
Sense	22,058	128·1	283	4,410
Veveyse	8,504	110·7	94	1,060
GENÈVE				
Ville de G. ...	56,292	147·5	752	7,040
Rivedroite	27,092	111·7	274	2,030
Rivegauche ...	87,616	113·6	901	6,570
GLARUS	33,834	148·2	494	2,540
GRAUBÜNDEN (Grisons)				
Albula	6,532	154·7	99	490
Bernina	4,968	139·9	68	620
Glenner	11,565	133·2	150	1,450
Heinzenburg ...	6,791	139·0	92	850
Hinterrhein ...	2,561	127·6	32	320
Imboden	6,057	122·9	73	760
Inn	6,840	144·4	108	860
Landquart Ober ...	16,213	52·1	82	2,840
,, Unter...	13,200	87·7	113	2,310
Maloja	9,660	81·8	77	340
Moësa	6,191	87·4	77	220
Münsterthal ...	1,575	157·2	24	0
Plessur	20,608	96·5	194	2,580
Vorderrhein ...	7,093	125·6	87	530
INNERRHODEN (Appenzell Int.)				
Appenzell-i-Rh. ...	14,614	193·5	289	1,100
LUZERN				
Entlebuch	16,883	121·4	208	5,490
Hochdorf	22,516	151·0	343	4,500
Luzern	73,614	140·6	847	18,200
Sursee	32,022	178·5	576	8,000
Willisau	32,038	155·3	502	8,000
NEUCHATEL				
Boudry	16,107	127·8	210	160
Chaux-de-Fonds ...	39,516	120·2	481	400
Locle	18,083	136·6	250	180
Neuchatel	30,803	83·6	292	310
Val de Ruz ...	9,719	104·3	103	490
Val Travers ...	17,121	127·7	221	170
NIDWALDEN	13,956	158·5	219	2,440
OBWALDEN	17,657	122·4	209	4,830
ST GALLEN				
Gaster	8,458	156·8	142	1,690
Gossau	14,651	146·3	369	1,830
Rheinthal Ober ...	19,226	123·1	253	2,400
,, Unter ...	21,126	104·0	235	2,640
Rorschach... ...	24,154	111·9	289	3,020
St Gallen	74,833	142·0	715	9,350
Sargans... ...	21,189	138·3	314	5,240
See	16,554	113·0	306	2,900

TABLE II—(*continued*).

Canton and district	Population 1920	Mean annual corrected cancer death rate 1906—15*	Corrected number of cancer deaths 1906—15	Estimated goitre cases 1921—22†
ST GALLEN (*continued*)				
Toggenburg Alt ...	12,697	199·6	271	3,170
,, Neu ...	12,129	166·5	216	3,030
,, Ober ...	11,583	140·1	174	2,030
,, Unter	24,846	159·0	423	6,210
Werdenburg ...	19,742	111·8	236	3,450
Wil	14,355	165·5	254	3,840
SCHAFFHAUSEN				
Klettgau Ober ...	3,259	127·9	38	570
,, Unter ...	4,123	167·7	63	720
Reiath	5,026	128·5	60	630
Schaffhausen ...	31,159	97·3	277	5,450
Schleitheim ...	3,102	111·5	32	500
Stein	3,759	148·8	51	750
SCHWYZ				
Einsiedeln	8,228	228·2	184	620
Gersau	2,003	84·8	17	70
Höfe	5,715	170·8	95	1,000
Küssnacht	4,099	112·5	55	510
March	13,424	162·8	214	2,010
Schwyz	26,262	158·0	406	4,590
SOLOTHURN				
Balsthal-gäu ...	7,186	189·1	122	250
Balsthal	10,700	128·1	123	370
Bucheggberg ...	5,839	170·1	89	730
Dorneck	7,768	132·8	92	580
Gösgen	11,462	153·7	158	860
Kriegstetten ...	20,523	114·0	210	2,570
Lebern	21,137	101·1	192	1,590
Olten	25,643	143·7	330	1,540
Solothurn	13,065	150·2	176	980
Thierstein ...	7,294	131·1	86	910
TESSIN (TICINO) ...				
Bellinzona ...	21,174	60·1	131	1,590
Blenio	6,298	100·4	65	220
Leventina	9,960	91·4	93	350
Locarno	25,488	82·3	215	1,910
Lugano	51,946	78·2	417	1,820
Mendrisio	26,877	72·5	199	270
Riviera	6,160	65·7	42	220
Valle Maggia ...	4,353	125·9	56	150
THURGAU				
Arbon	26,676	131·1	302	4,670
Bischofszeil ...	17,772	120·1	213	3,110
Diessenhofen ...	4,632	160·0	73	810
Frauenfeld ...	18,857	122·1	228	4,710
Kreuzlingen ...	19,664	104·2	262	3,440
Münchwilen ...	19,894	136·6	269	3,480
Steckborn ...	12,659	144·4	181	3,160
Weinfelden ...	15,779	149·0	233	1,970
URI	23,973	141·2	312	3,000
VALAIS				
Brig	9,997	64·0	64	1,750
Conthey	9,855	101·9	100	2,950
Entremont ...	9,117	86·0	78	1,140
Goms	4,037	117·9	48	500
Hérens	7,709	73·8	57	1,160
Leuk	7,227	85·1	61	900
Martigny ...	14,719	84·7	125	2,580
Monthey ...	12,964	67·6	88	1,620
Raron	6,861	76·4	52	1,200
St Maurice ...	8,087	79·0	64	1,410
Sierre	15,740	45·8	72	2,750
Sion	12,207	63·1	77	3,050
Visp	9,726	69·6	65	1,220
ZUG	31,569	166·6	469	5,520
VAUD				
Aigle	23,576	92·7	219	1,770
Aubonne ...	7,925	145·8	115	200
Avenches ...	5,053	136·7	69	1,260
Cossonay ...	11,663	142·3	156	410
Echallens ...	9,060	137·7	125	680
Grandson ...	12,027	118·3	142	420
Lausanne ...	83,376	94·6	789	6,240
La Vallée ...	6,189	96·6	60	620
Lavaux	9,817	99·8	98	740
Morges	17,393	104·3	181	130
Moudon	11,103	138·4	154	1,940
Nyon	14,738	107·1	158	1,110
Orbe	18,657	97·8	182	190
Oron	6,330	128·7	81	470
Payerne	12,317	108·8	134	2,160
Pays d'Enhaut ...	5,313	112·6	60	660
Rolle	6,386	116·2	74	790
Vevey	38,711	108·6	421	4,840
Yverdon	17,864	118·3	211	1,340
ZÜRICH				
Affoltern ...	13,522	146·6	192	2,370
Andelfingen ...	18,420	141·8	252	5,520
Bulach	25,551	146·0	358	3,190
Dielsdorf ...	15,938	122·5	188	1,990
Hinwil	36,942	183·0	654	2,770
Horgen	46,303	141·0	630	5,790
Meilen	25,055	171·8	417	3,130
Pfäffikon ...	18,847	186·0	339	1,410
Uster	21,091	168·4	344	2,640
Winterthur ...	68,367	123·0	813	20,510
Zürich	248,566	116·8	2621	31,070

* Based on 1910 populations, corrected for uncertified deaths, and age-sex distribution of canton as a whole.

† I.e. on assumption that both sexes and all ages are equally affected; the actual cases would be larger, since females are more liable to goitre than males.

TABLE III.

Mortality from Cancer of Different Organs in Switzerland as compared with England and Wales.

Location of cancer	Switzerland, 1901—1920				England and Wales, 1901—1910	
	Total Deaths		Rate per 100,000		Rate per 100,000	
	Males	Females	Males	Females	Males	Females
Skin	608	879	1·65	2·30	1·78	1·16
Lips	263	61	0·71	0·16	1·21	0·10
Tongue	988	80	2·68	0·21	4·29	0·48
Mouth, pharynx, larynx	2,324	332	6·30	0·87	4·67	1·15
Oesophagus	7,257	1,245	19·66	3·26	5·09	1·59
Stomach	20,586	16,398	55·77	42·98	16·53	14·62
Intestines	2,311	3,117	6·26	8·17	6·40	8·13
Rectum	2,254	1,650	6·11	4·32	7·84	6·13
Peritoneum	184	459	0·50	1·20	0·65	1·49
Pancreas	652	592	1·77	1·55	1·45	1·28
Liver and gall bladder	2,536	3,989	6·87	10·45	9·57	13·61
Uterus	—	6,673	—	17·44	—	22·41
Ovaries	—	1,104	—	2·89	—	2·05
External genitalia ...	191	418	0·52	1·10	2·31	—
Breast	28	5,492	0·07	14·39	0·14	17·27
Lung	420	246	1·14	0·64	1·02	0·75
Kidney	271	242	0·73	0·63	0·84	0·80
Bladder, prostate, &c.	1,860	358	5·04	0·94	2·40	0·96
Thyroid gland	580	563	1·57	1·48	0·20*	0·55*
Other organs	1,388	1,218	3·76	3·19	10·92	8·20
Totals	44,701	45,116	121·10	118·24	77·31	102·73

* Rates for 1913—1922, earlier figures not being available.

Making non-cancer deaths constant as well as population with the object of eliminating the effect of general unhealthiness of some areas, the partial correlation becomes

Deaths from cancer of digestive tract with goitre cases, population and non-cancer deaths being constant ... ... $r = \cdot 6360 \pm \cdot 0721$

There can be little question of the significance of these coefficients, and a comparison of Map No. I with Map No. III shows at once a remarkable similarity in the distributions of goitre and cancer of the digestive organs. The goitre centre in the Valais has apparently no corresponding centre for cancer, but as I have pointed out, the cancer rates for certain areas of the Valais cannot be regarded as reliable since it is probable that the uncertified deaths included a larger percentage of cancer than those certified. It seems reasonable to conclude therefore that cancer of the stomach and oesophagus is more common in goitrous areas, and that there is some common factor at work in the production of these conditions.

I have tested this finding in another way by correlating for 23 cantons (combining the Appenzell provinces and Obwalden with Nidwalden owing to the small numbers) the percentage of stomach cancers to all cancers directly with the percentage goitre rate in recruits. In this correlation the general healthiness of districts is not involved, nor is it evident why there should be appreciable spurious correlation between the ratios. The result is:

Relative frequency of stomach cancers to all cancers with goitre rate ... ... $r = \cdot 5936 \pm \cdot 0911$

a value of the same order as before*. In a later section the same method has been applied to different countries.

(*e*) *Goitre rate in School children and Cancer mortality.* In some 19 districts of Switzerland data are available of the percentages of school children presenting goitre, the sources of this data being as follows:

(i) Professor de Quervain, H. Kerzmann and others(8) examined 7565 children in Bern, 641 in Neuchatel, 641 at Le Locle and 701 at Le Chaux-de-Fonds, and made a detailed classification of their findings as regards the condition of the thyroid gland at different ages for boys and girls. The classification used was briefly: O = not palpable, I = normal, II = diffuse slight enlargement, III = definite diffuse goitre, IV = definite nodular goitre, V = large diffuse goitre, VI = large nodular goitre. Regarding, for the purpose of comparison with the other data, categories O—I—II as normal, and III—VI as goitrous, the total percentages of school children found to be goitrous in the four localities were:

	Boys	Girls	Children
Bern	53·1	56·3	54·8
Le Chaux-de-Fonds ...	20·9	35·9	28·5
Le Locle	26·9	37·4	32·0
Neuchatel	32·6	35·2	34·3

(ii) Dr Jaeger of Rhäzuns(9) examined 1334 school children in the district of Heinzenburg and classified them in the same groups, with actual measurements of the gland. Of these 830 were goitrous (categories III—VI), a percentage of 62·2.

(iii) Dr Steinlin of St Gallen(10) reported upon the percentage of goitrous school children in the 14 districts of the canton of St Gallen, using the same classification. In all 44,500 were examined, from 197 primary and 40 secondary schools. The percentages with goitre (categories III—VI) were as follows: Neutoggenburg 48·5, Wil 51·6, Obertoggenburg 54·9, Unterrheinthal 55·9, Rorschach 57·9, Werdenburg 60·0, Sargans 62·6, Gossau 63·9, Untertoggenburg 64·2, Gaster 65·0, See 67·0, St Gallen 68·6, Oberrheinthal 70·4, Alttoggenburg 77·2, or for the whole canton 61·9.

From the above data I have correlated the goitre percentages of the 19 districts with the corrected cancer mortality rates for 1906–15 and find $r = \cdot 3670 \pm \cdot 1336$.

* If the canton of Ticino, south of the Alps, is omitted, the coefficient is $r = \cdot 7841 \pm \cdot 0554$.

III. Goitre and Cancer frequency in other Countries.

It is well known that goitre is endemic in other countries than Switzerland, as far apart as Norway and Eastern France, the Himalayas and the Andes. It is difficult or impossible to obtain, however, any estimate of the actual frequency of goitre in most countries, and such figures as I have been able to collect are briefly summarized below. In the country districts of Sweden, goitre is prevalent to the extent of 7 %*; it is hoped to give the figures in a later paper. Norway is dealt with in Section IV and I am also able, by the courtesy of the Director of Public Health Services at Kristiania, to give statistics for a most important endemic area in central Norway. At Naes in Hedmark the communal medical officer, Dr Schiötz, made an accurate survey of the whole district in 1914 and found 177 goitrous in a population of 3550, 158 females in a total of 1843, and 19 males in 1707, giving percentages of 8·6 and 1·1, or 5 % of all persons. It is further estimated that in this endemic area there are about 2400 cases in a population of 48,000. In other parts of Norway goitre is by no means so frequent. Regarding Great Britain we are told that goitre is moderately frequent in Derbyshire, Nottingham, Westmorland, Cumberland, parts of Yorkshire and Wales, and also occurs in Buckinghamshire, Hampshire, Norfolk, Sussex, Surrey, Monmouth, Northumberland, and in Roxburgh, Selkirk, Peebles, Ayrshire, Lanark and some other Scottish counties to a slight degree(11), but the only source from which I have been able to obtain an estimate of goitre frequency is a Report(12) on the examination of recruits by the Ministry of National Service during the period 1917–18. In the Liverpool area nine men were rejected for goitre and 11 for Graves' Disease out of 20,704 examined. In the West Midland area 174 were rejected for "goitre" out of 217,510 examined, but this figure includes Graves' Disease. These results give percentages of ·043 for Liverpool and ·080 (including exophthalmic goitres) for the Midlands, as compared with 7·2 % of rejections of Swiss recruits. The same report shows that in a series of 10,000 Scottish recruits 50 had enlarged thyroids of whom seven had symptoms of Graves' Disease, giving a goitre percentage of ·43, somewhat higher than in England.

In Belgium 1910–14 an average of 58 recruits were rejected annually for goitre(13), which probably represents a rate of about ·1 %. In Italy it was estimated in 1883 that 129,000 people were goitrous, or about ·5 % of the population. In France, 1873, 370,000 were estimated as goitrous, or over 1 % of the population, and more recent figures show 1200 rejections per 300,000 recruits annually (·4 %). In S. Bavaria, Schittenburg and Weichardt's figures show 77·89 % of school children to present goitre in some degree, which on comparison with analogous Swiss data probably represents a goitre percentage of 15—20 in the male population. In Thibet and S.E. Mongolia it is stated that one-third of the population are goitrous. In the U.S.A. Kerr(14) found at Camp Lewis, Washington, 1276 recruits with large or moderately large goitres out of 21,182 examined, or 6 %, the percentages for 11 States being—Washington 11·0, Oregon 8·6,

* According to an estimate kindly made for me by Mr R. Moosberg of Stockholm.

Montana 7·3, Idaho 6·6, Wyoming 6·4, Minnesota 5·5, Utah 5·1, N. Dakota 3·7, S. Dakota 2·0, Nevada 1·1, California 0·5. The Draft Board however classed only 4 °/₀ of recruits as goitrous in Washington State. At Jefferson Barracks, Missouri, 1·63 °/₀ of 65,507 men had goitre (Smith(15)). In 1913 Clark(15) found 1234 cases of goitre in 13,836 school children in West Virginia, or 9 °/₀. The distribution of goitre in U.S.A. as shown by the report of the War Department on defects found in drafted men (1920, Table 23) has been summarized by McClendon(16) and correlated with analyses of iodine content of water supplies; the percentage increases from 0—0·1 in the Southern States to 1·5—10 in the N.W. and Lake regions. At Chicago Olson found 7 °/₀ of males and 18 °/₀ females to present enlarged thyroids. At New York Mayo found ·3 °/₀ goitrous and at Washington University Hall found 18 °/₀ of men students and 31 °/₀ of women students with enlarged thyroids. These and other figures are utilised in Section V.

TABLE IV.

Estimated Goitre prevalence and Ratio of Cancer of Stomach to Cancer of all organs in various countries.

Area	Estimates of goitre frequency in males per 1000	Period of cancer data	Mortality from cancer of stomach, per 100,000			Mortality from cancer of all organs, per 100,000			Percentage of all cancer deaths	
			Males	Females	All	Males	Females	All	Stomach cancer	Stomach and liver cancer
									All	All
England and Wales	·4—·8	1908–10	16·5	14·6	15·6	77·3	102·7	90·4	17·2	30·0
Scotland	4	1906–10	21·8	23·0	22·5	81·3	116·9	99·7	22·5	33·8
U.S.A. (Southern)(c)	0—1	1900	—	—	4·1(b)	11·3	22·7	16·9	24·4	30·0
U.S.A. (E. Coast)(c)	about 1	1900	—	—	18·0(b)	42·4	79·2	61·0	29·5	43·6
Italy	5	1906–10	—	—	19·6	—	—	63·6	30·8(a)	—
U.S.A. (Central S.)(c)	1—5	1900	—	—	7·6(b)	17·7	31·3	24·4	31·2	40·0
Sweden (cities) ...	1—40	1905, 1911	—	—	—	90·9	113·5	103·1	34·4(d)	35·6
U.S.A. (Central N.)(c)	5—20	1900	—	—	17·2(b)	38·2	59·5	48·6	35·5	47·7
Switzerland (all) ...	130	1901–20	55·8	43·0	49·0	121·1	118·2	119·6	41·0	48·4
U.S.A. (N.W. States)(c) ...	20—110	1900	—	—	17·6(b)	34·0	44·6	38·9	45·1	57·7
Sweden (rural areas) ...	70	1905, 1911	—	—	—	98·1	95·4	96·7	47·9(d)	48·8
Bavaria	150? in south	1905–10	62·9(a)	55·0(a)	55·8(a)	92·5	114·1	103·0	—	56·6(a)
Norway (endemic areas)(e)	10—50	1920–21	75·0(f)	60·3(f)	69·7	112·7(f)	108·0(f)	113·7	61·3	67·1

The above have been arranged as nearly as possible in order of increasing ratio cancer of stomach to cancer of all organs. Some of the cancer statistics have been extracted from Hoffman's tables (29).

(*a*) Includes cancer of oesophagus also. (*b*) Corrected for cases where site of cancer was unspecified. (*c*) For explanation see Section V. (*d*) Proportion obtained from cancer census of 1905; mortality rates for 1911. (*e*) For explanation see Section IV. (*f*) From 1920 figures only.

A comparison of cancer mortality rates in various countries is difficult owing to differences in race, methods of certification and reliability of data. For the present purpose however these objections can be largely eliminated by considering the selection rate of cancer for the stomach—i.e. proportion of stomach cancers to all cancers of known site—in each country where such data are available. In some instances it is impossible to separate stomach and liver cancers and in many cases these are necessarily liable to confusion, so that the site-rates for stomach and liver combined have been given as well as for stomach alone wherever possible. In Table IV (p. 379) I have arranged a number of countries or states as nearly as possible in order of increasing tendency for cancer to attack the stomach, as shown by the last two columns. This table is of course very fragmentary and incomplete, but as far as it goes it confirms the results already obtained for the sub-divisions of Switzerland, since it can be seen at a glance that, generally speaking, the countries with the highest selective rate for cancer of the stomach are also the most goitrous, and *vice versâ*.

IV. Goitre and Cancer Distributions in Norway.

Through the courtesy of Dr H. M. Gram, Director of the Norwegian Public Health Services, I have been able to briefly investigate the goitre and cancer distributions in Norway in time for publication with this paper, and to Dr Gram and also to Dr F. G. Gade, Secretary of the Cancer Research Fund in Norway, I express my hearty appreciation of their kindness in providing the necessary data, which entailed considerable trouble. Regarding goitre prevalence a questionnaire was sent to all the County Medical Officers and the replies were as follows :

Goitre is of uncommon occurrence throughout the counties of *Vest-Agder*, *Hordaland*, *Møre*, *Sør-Trøndelag*, *Nordland*, *Troms* and *Finnmark*. In *Vestfold Fylke* (county) it is sporadic in the districts of Andebu, Hoff, Lardal, Sandeherad, Holmestrand, Svelvig and Strömm, and rare elsewhere. In *Akerhus Fylke* goitre is endemic in Eidsvoll and Hurdal districts, sporadic in Aker, Enebak, Fet and Asker, but rare in the remaining districts. In *Buskerud Fylke* there are endemic centres in Nore-Opdal and Modum, and sporadic cases in all other districts. In *Hedmark Fylke* goitre is frequent round Lake Mjösen but uncommon elsewhere. In *Opland Fylke* it is again common near Lake Mjösen, and sporadic in Torpa, Etnedal, Snertingdal, Gausdal, Öyer, and Ringebu, but rarely found in the mountainous regions of Valdres or Gudbrandsdalen. In *Sogn og Fjordane Fylke* goitre is frequent round Lake Opstryn, but rare in the rest of the county. In the cities of *Kristiania* and *Bergen* goitre is usually said to be rare, and no data are at present available relating to the remaining counties of *Østfold*, *Telemark*, *Aust Agder*, *Nord Trøndelag*.

I have therefore divided the counties for which reports are available into two groups, placing the cities in a third group, thus—(1) Counties in which there are endemic centres where goitre is frequent* or in which sporadic cases occur not uncommonly, (2) Counties where goitre is infrequent, (3) Cities where goitre is said to be rare. The counties comprising group (1) have a total census population of some 800,000 but the aggregate population of the districts where goitre is prevalent to the extent of 5 % or more does not exceed 80,000, so that probably the

* Actual statistics for one such area have been given in Section III, showing a frequency about 5 %.

average frequency in this county group lies between the limits 10—50 per 1000. In counties of group (2), and probably also in the cities, the average frequency does not exceed about 1 per 1000.

In Table V (p. 382) the aggregate mortality figures for cancer of the various digestive organs have been worked out for the three groups, the total cancer mortality being also given. The populations were compiled from the recent report of the Census of 1920 (30). The age and sex distributions of each county were obtained from the same source, and the standardizing factors for cancer deaths were calculated for each group of counties on the basis of the distribution for Norway as a whole. Cancer deaths by age and sex were obtained from the *Statistical Year Book* for 1920 (31). The census being taken on December 1, 1920, I have used for calculating death rates the combined deaths during 1920 and 1921 which Dr Gade kindly furnished me with. Only deaths from carcinoma are given, sarcoma being excluded. The deaths from all causes are also included at the end of the table together with the percentage of these whose cause was unknown. Since this percentage is by no means the same in the three groups, all the cancer death rates have been corrected for these uncertified deaths as well as for age and sex distributions.

A study of the resulting death rates reveals the following:—(1) For cancer of all organs the goitrous counties have a mortality rate 113·7 as compared with 94·4 for non-goitrous counties. (2) This *excess of cancer mortality is almost entirely due to cancer of the stomach*, the rates being 69·7 and 52·2 respectively, other parts of the digestive tract showing also a slight excess in every case, whilst the rate for cancer of other organs is slightly lower in the goitrous counties, as is also the general death rate. (3) As in other countries, the cities have a much larger general cancer death rate than county areas (134·2) but it is noteworthy that the rate for cancer of the stomach in the cities is very little different from that in the non-goitrous counties (55·2, 52·2) whereas for cancer of all other organs the cities show considerably higher rates than the counties. This is well shown by the lower ratio of stomach cancer to all cancers in the cities.

These figures appear to afford confirmation of a direct correlation between mortality from cancer of the stomach and prevalence of goitre.

V. GOITRE AND CANCER DISTRIBUTIONS IN UNITED STATES OF AMERICA.

The sources from which goitre frequency in the various States of America can be estimated have been referred to in Section III, the chief of these being the survey carried out by Kerr at Camp Lewis and the report of the Draft Board. For the purposes of a preliminary investigation, I have divided the U.S.A. into five geographical areas corresponding as nearly as possible with the percentage of goitre in males as indicated by this material.

The States comprising these divisions are as follows:

I. *North-Western States* (goitre rates from 2°/₀ to 11°/₀), Washington, Oregon, District of Columbia, Idaho, Montana, Wyoming, Utah, N. Dakota, Minnesota, Wisconsin.

II. *Central States, Northern belt* (goitre rates from 0·5 to 2°/₀), California, Nevada, Colorado, S. Dakota, Iowa, Illinois, Indiana, Michigan, Ohio, Pennsylvania, New York State.

TABLE V. *Cancer Mortality in Goitrous and Non-goitrous areas of Norway*, 1920–1921.

Norwegian Counties classified according to prevalence of goitre	Census population Dec. 1, 1920	Age and sex Corrective factor for cancer mortality‡	Total number of deaths 1920—1921							Mean corrected annual mortality rates per 100,000 living§							Percentage ratio cancer of stomach to all cancer	Deaths from all causes 1920–1	Mean corrected death rates per 100,000	Percentage of deaths with cause unknown
			Cancer of all organs	Cancer of stomach	Cancer of oesophagus	Cancer of intestines	Cancer of rectum	Cancer of liver	Cancer of other organs	Cancer of all organs	Cancer of stomach	Cancer of oesophagus	Cancer of intestines	Cancer of rectum	Cancer of liver	Cancer of other organs				
Counties where goitre is frequent or sporadic in certain districts*	810,153	1·0376	1744	1069	42	50	51	102	430	113·7	69·7	2·7	3·3	3·3	6·6	28·0	61·3	18231	1167	1·81
Counties where goitre is uncommon†	1,040,402	·9905	1845	1021	45	55	35	99	590	94·4	52·2	2·3	2·8	1·8	5·1	30·2	55·3	26385	1263	6·41
Cities of Kristiania and Bergen (goitre said to be rare)	349,926	1·0984	839	345	61	34	35	52	312	134·2	55·2	9·8	5·4	5·6	8·3	49·9	41·1	8603	1350	1·92

* Akerhus, Buskerud, Vestfold, Hedmark, Opland, Sogn og Fjordane (see text). † Vest-Agder, Rogaland, Hordaland, Møre, Sør-Trøndelag, Nordland, Troms, Finnmark (see text).
‡ Calculated from aggregate of census distributions by age and sex of the counties in the group, using census population of Norway as standard.
§ Corrected for uncertified deaths and for age and sex distributions.

TABLE VI. *Cancer Mortality in Goitrous and Non-goitrous areas of U.S.A.*, 1900.

American States classified according to prevalence of goitre	Census population 1900	Estimated goitre frequency in males†	Deaths from all causes 1900	Number of deaths from all forms of cancer 1900			Cancer deaths by organs					Mortality Rates per 100,000					Percentage Ratio to all cancers with specified site	
				Males	Females	Totals	Total deaths where site was specified	Cancer of stomach	Cancer of liver	Cancer of abdomen	Cancer of rectum	All causes	Cancer, all forms			Cancer of stomach‡	Cancer of stomach	Cancer of stomach and liver
													Males	Females	All			
North-Western States*	6,124,320	Over 2%	66,166	1120	1264	2384	1718	775	216	86	43	1080	34·0	44·6	38·9	17·6	45·1	57·7
Central States, Northern Belt*	32,188,059	0·5 to 2%	395,210	6281	9361	15642	11914	4228	1451	891	401	1228	38·2	59·5	48·6	17·2	35·4	47·7
Central States, Southern Belt*	16,256,291	0·1 to 0·5%	196,527	1464	2500	3964	2784	870	244	132	78	1209	17·7	31·3	24·4	7·6	31·2	40·0
East Coast States*	8,848,465	about 0·1 %	154,963	1862	3533	5395	4060	1196	574	395	168	1751	42·4	79·2	61·0	18·0	29·5	43·6
Southern States*	12,184,980	0 to 0·1%	160,903	700	1362	2062	1425	348	79	63	33	1320	11·3	22·7	16·9	4·1	24·4	30·0

* For explanation of these divisions see text. † Estimates based on recruiting records and other sources mentioned in Section III.
‡ Corrected by increasing in proportion of total cancer deaths to all cancer deaths with specified site.

III. *Central States, Southern belt* (goitre rates from 0·1 to 0·5 °/₀), Nebraska, Kansas, Oklahoma, Missouri, Kentucky, Tennessee, Virginia, W. Virginia, N. Carolina, S. Carolina.

IV. *East Coast States* (goitre rates about 0·1 °/₀), Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, Connecticut, New Jersey, Delaware, Maryland.

V. *Southern States* (goitre rates from 0 to 0·1 °/₀), Arizona, Arkansas, Mississippi, Alabama, Louisiana, Georgia, Florida.

The only complete cancer statistics available to me at the time of writing being those of the 12th Census relating to the year 1900 (32), I have used these figures in compiling Table VI, in which the aggregate deaths in each group of States from all causes, cancer generally and cancer of the stomach, liver, intestines and rectum are given, and the crude mortality rates deduced from these. Since a certain proportion of cancer deaths were certified merely as "cancer," the total deaths for which the site of the growth was specified have also been given, and the percentage ratios in the last two columns are based on these totals. An examination of the table shows that: (1) Mortality from all causes tends in these groups to be inversely related to goitre prevalence (as in Norway and Switzerland). (2) Mortality from cancer in general increases from south to north and is to this extent correlated with goitre prevalence, the East Coast States being however exceptional; but the many important factors involved, such as density of population and proportion of negroes to whites, which vary greatly in passing from Southern to Northern States, render such a comparison of little value. (3) Mortality rates for cancer of the stomach are more nearly correlated with goitre prevalence, but the densely populated East Coast States are again exceptional. (4) The *selection rate of cancer for the stomach shows a remarkable correspondence with the prevalence of goitre*, increasing steadily through the five areas from 24·4 °/₀ where goitre is rare to 45·1 °/₀ where goitre is very prevalent. This last index of the tendency to gastric cancer is more reliable than the absolute mortality rate since most of the important sources of variation are eliminated, and its evident correlation with goitre confirms the results obtained in Switzerland and Norway.

VI. Post-mortem Evidence of Relation between thyroid enlargements and Cancer.

(*a*) *Source of Data.* I have to acknowledge my great indebtedness to Professor Lazarus Barlow and the Cancer and General Research Committee of the Middlesex Hospital for granting me permission to investigate the excellent post-mortem records of that Hospital, and to the former for his very cordial assistance in carrying this out. The main object in view was to examine the records of 500 post-mortems on patients who died from cancer and 500 post-mortems on patients who died from diseases other than cancer, and compare the frequency of thyroid anomalies noted in each series. It must be confessed that difficulties arose owing to the personal factor of the observer, and the percentages of thyroid anomalies noted by different observers were not the same. For the most part, however, the two parallel series of post-mortems were done by the same observers, though not in quite the same

* For explanation of these divisions see text.
‡ Corrected by increasing in proportion of total cancer deaths to all cancer deaths with specified site.
† Estimates based on recruiting records and other sources mentioned in Section III.

proportions, and the effect of the personal factor has been tested in the case of one pathologist, *X*, whose excellent reports inspired perhaps the most confidence; thus

	Cancer Cases			Non-Cancer Cases		
	Male	Female	All	Male	Female	All
Number of post-mortems by pathologist *X* ...	96	213	309	73	28	101
Number of these showing thyroid anomalies ...	16	49	65	1	3	4
Percentage showing thyroid anomalies ...	16·7	23·0	21·0	1·4	10·7	4·0

These percentages are not much different from those found by combining all observers (see next section) and would produce in the sex distribution of 1000 cases a total of 104 thyroid anomalies in the cancer group and 25 in the non-cancer group as compared with 93 and 19 respectively actually found by combining all observers. It is therefore possible to state with some confidence that the personal factor is not of sufficient importance to invalidate the conclusions arrived at. It is of course possible that the cancer post-mortems were on the whole more carefully carried out than those on non-cancer cases, but this is not proved by the records and it seems certain that this factor could not account for the correlation obtained, though conceivably it has to some extent increased it. Thus, taking into account only those enlargements which were evidently too obvious to have escaped even superficial examination, the percentages are as follows:

	Male	Female	All
Percentages of *gross* enlargements of thyroid gland in cancer cases	7·9	14·3	12·2

which figures are still greatly in excess of the *total* anomalies recorded in the non-cancer series.

(*b*) *Correlation of Thyroid Anomalies with Cancer.* It is now possible to consider the results as a whole.

			Male	Female	All
500 Cancer Post-mortems (165 males, 335 females)	Thyroid abnormal apart from secondary cancer	Goitrous or fibrotic	20	62	82
		Goitrous with added secondary cancer	2	9	11
	Normal thyroid invaded by cancer	Primary	—	1	1
		Secondary	9	13	22
	Thyroid normal		134	250	384
	Percentage showing thyroid anomalies (apart from secondary cancer) ...		13·3	21·2	18·6
500 Non-Cancer Post-mortems (301 males, 199 females)	Thyroid abnormal		6	13	19
	Thyroid normal		295	186	481
	Percentage showing thyroid anomalies ...		2·0	6·5	3·8

Thus, of males dying of cancer 13·3 °/₀ showed anomalies of the thyroid gland as compared with 2·0 °/₀ of males dying of other diseases; of females dying of cancer 21·2 °/₀ showed thyroid anomalies as compared with 6·5 °/₀ of females dying of other diseases.

In London in 1922 the proportion of cancer deaths to total deaths was 9·714 °/₀, hence 500 cancer deaths would occur in a total of 5147 deaths, and we obtain for the general population at death

	Thyroid anomalous apart from secondary cancer	Thyroid not obviously abnormal	Totals
Cancer present ...	93	407	500
Cancer absent ...	$19+x$	$481+y$	4647
Totals ...	$112+x$	$888+y$	5147

Assuming for the moment that the general non-cancerous population of London presents at death the same percentage of thyroid anomalies as a Middlesex Hospital post-mortem non-cancerous population, then $\frac{19+x}{481+y}=\frac{19}{481}$ (approximately 4 °/₀) and the table becomes

	Thyroid +	Thyroid −	Totals
Cancer + ...	93	407	500
Cancer − ...	177	4470	4647
Totals ...	270	4877	5147

which gives a tetrachoric correlation $r = \cdot4587 \pm \cdot0286$.

In the above no account has been taken of sex distribution, which is not the same in the three populations considered. Treating each sex separately and proceeding as before:

Males. In London, 1922, cancer deaths formed 9·1098 °/₀ of total deaths, hence 165 cancer deaths would occur in a total of 1811 deaths and we have

	Thyroid +	Thyroid −	Totals
Cancer + ...	22	143	165
Cancer − ...	$6+x'$	$295+y'$	1646
Totals ...	$28+x'$	$438+y'$	1811

which becomes on the same assumption as before

22	143	165
33	1613	1646
55	1756	1811

giving $r = \cdot 4905 \pm \cdot 0572$.

Females. In London, 1922, cancer deaths formed 10·671 °/₀ of total deaths, hence 335 cancer deaths would occur in a total of 3139 and the table becomes:

71	264	335
183	2621	2804
254	2885	3139

giving $r = \cdot 3758 \pm \cdot 0330$.

In all the above calculations it has been assumed that the frequency of thyroid anomalies in the general non-cancerous population of London at death is the same as in the corresponding hospital population, i.e. 4 °/₀. The only data of which I am aware which throw any light upon this are to be found in an enquiry into the physical condition of school children in London carried out by Dr C. J. Thomas for the County Council in 1917(17), which showed that enlargement of the thyroid was present in 1·4 °/₀ of boys and 3·9 °/₀ of girls, or 2·6 °/₀ of all children examined. We are of course dealing with a very different age period and thyroid anomalies may develop after school age, but on the other hand many adolescent enlargements disappear. On this question of age incidence the Bern data of Professor de Quervain(8) throws some light; thus in Bern the percentage of boys showing any thyroid enlargement whatever (i.e. categories II—VI, see Section II(*e*)) increased from 71·8 °/₀ at age 7 to a maximum of 93·5 °/₀ at age 15, and in girls from 74·6 °/₀ at age 7 to 94·4 °/₀ at 15; or considering only the grosser enlargements the percentages for boys were 33·6 at 7 and 80·5 at 16. In contrast to the last figure, recruits at age 19 showed only 15—20 °/₀ of goitres, so that a great decrease in thyroid enlargements must occur immediately after adolescence. This is also shown by the other Swiss data for schools already quoted, and if we may conclude that a similar decrease occurs in this country, it follows that 2·6 °/₀ is too high an estimate for the frequency of thyroid enlargements in London adults. On *a priori* grounds also it seems probable that a population of hospital patients would present a larger percentage of thyroid anomalies than the general population, since some would go to hospital on this account, but there seems no reason to suppose that this would be the case with the cancer cases. It is therefore tolerably certain that 4 °/₀ is a maximum figure. Supposing the real ratio to be 3 °/₀, then $r = \cdot 5126 \pm \cdot 0287$, or if the ratio is 2 °/₀, $r = \cdot 5926 \pm \cdot 0282$, i.e. the correlation becomes larger. There is evidently therefore a considerable correlation between anomalies of the thyroid gland and cancer generally at the time of death, and the suggestion that the thyroid enlargements might be secondary to cancer cannot be supported by any evidence of which I am aware.

(*c*) *Nature of Thyroid Anomalies met with.* A summary of the various thyroid anomalies found in the 93 cancer post-mortems is given in Table VII, the additional 23 cases where thyroid was affected by cancer only not being included.

The 93 cases have been roughly grouped in order in the following table, which includes also a parallel classification of the non-cancer cases with thyroid anomalies.

Thyroid anomaly	Cancer cases						Non-Cancer cases		
	Without secondary growth in thyroid			With added secondary growth in thyroid					
	Male	Female	All	Male	Female	All	Male	Female	All
Simple parenchymatous enlargement	9	23	32	1	—	1	3	3	6
Adenomatous enlargement	1	13	14	1	7	8	—	5	5
Calcified nodules	2	9	11	—	—	—	—	—	—
Cystic changes	4	14	18	—	1	1	—	4	4
Atrophy or fibrosis ...	4	3	7	—	1	1	2	—	2
Other abnormalities ...	—	—	—	—	—	—	1*	1†	2
Totals ...	20	62	82	2	9	11	6	13	19

* Congestion only. † Exophthalmic goitre.

The causes of death in the non-cancer cases were nephritis (4), phthisis (2), diabetes, enlarged pituitary, aneurysm, gastric ulcer, gall stones, cirrhosis of liver, gangrene of intestine, ulceration colitis, ruptured liver, mastoiditis, thyroid tumour causing bronchopneumonia, exophthalmic goitre, and cardiospasm probably arising from exophthalmic goitre. Thus one, and probably two, of these 19 cases had exophthalmic goitre. Evidence of exophthalmic goitre was not found in the cancer cases. It would appear that the adenomatous types of enlargement are more liable to become attacked by secondary cancer than other types of anomaly. The mean age at death of the cancer cases showing thyroid anomalies other than secondary cancer was 55·6 years, of those with normal thyroids 55·2, and of those with primary or secondary cancer in otherwise normal thyroids 49·5 years.

(*d*) *Site of Primary Cancer.* The 500 cancer cases are analysed in Table VIII (p. 390) as regards the part of the body primarily attacked, and the percentage site-rates are given for each sex for the normal and abnormal thyroid groups. Taking both sexes together, primary cancer of the stomach formed 7·5 °/₀ of all cancers in those with abnormal thyroids, and 4·7 °/₀ in those with normal thyroids, a difference which goes to support the results obtained in Sections II (*d*) and III. The buccal cavity on the other hand was the site of primary cancer in 4·3 °/₀ of thyroid cases as compared with 11 °/₀ in other cases. Here again it is noteworthy that the lips, tongue and mouth show much lower site-rates in Switzerland where goitre is common than in England and Wales (see Table III).

(*e*) *Type of Primary Growth.* The last columns of Table VIII show that in 411 out of 500 cases the primary growth was carcinoma, in 33 sarcoma, and in 56 endo-

TABLE VII. *Summary of Thyroid Anomalies found in 93 Cancer Post-Mortems (taken from a series of 500 post-mortems on Cancer cases at the Middlesex Hospital).*

Sex	Age	Type of primary cancer	Site of primary	Condition of thyroid gland as described by pathologist	
				Naked eye examination	Additional information from microscopic examination
Male	19	Sarcoma	Rib	Large, nearly 2 oz.	—
,,	20	Endothelioma	? Pituitary	Rather large	—
,,	65	Carcinoma	Bladder	Larger than usual	—
,,	36	,,	Rectum	Enlarged; L. lobe $1\frac{3}{4}$ oz.	—
,,	70	,,	Face	Somewhat enlarged	—
,,	53	Sarcoma	Penis	Enlarged to about twice usual size	—
,,	59	Carcinoma	Tongue	Enlarged	No malignancy
,,	69	,,	,,	R. lobe larger than normal; L. very small	—
,,	55	,,	Oesophagus	Much enlarged	No malignancy
,,	45	Sarcoma	Neck	Both lobes considerably enlarged	Simple adenoma
,,	63	Carcinoma	Larynx	Cystic; large cavity $\frac{1}{2}$″ diam. in L. lobe filled with liquefied colloid; cyst $\frac{1}{5}$″ diam. in R. lobe	Simple cystic change with colloid distension and increased fibrous stroma
,,	67	,,	Stomach	Large cystic goitre	—
,,	45	Sarcoma	Mesentery	L. lobe contains cyst as large as small marble	—
,,	46	Carcinoma	Stomach	L. lobe contains cyst $\frac{1}{5}$″ diameter	—
,,	52	,,	Rectum	Contains a small focus of calcification	—
,,	51	Malignant lymphoma	Tonsil	Colour variation in one lobe	—
,,	65	Carcinoma	Stomach	Decidedly smaller than normal	—
,,	67	,,	Rectum	Rather fibrotic	—
,,	59	,,	Stomach	Fibrotic	—
,,	52	,,	Oesophagus	Slightly fibrotic	—
,,	54	,,	,,	R. lobe enlarged and mass of pale growth	Some degenerated thyroid tissue present; mostly secondary growth
,,	41	,,	Tongue	Upper part invaded by direct extension	Adenomatous change and invasion by growth
Female	58	Carcinoma	Rectum	Slightly enlarged	—
,,	21	Lympho-sarcoma	Mediastinum	Both lobes enlarged 2″ × 1″	Normal thyroid tissue
,,	65	Sarcoma	Neck	Greatly enlarged, 6 or 7 times normal	Adenomatous with degeneration in small areas separated by fibrous trabeculae partly calcified
,,	60	Carcinoma	Face	Slight bilateral simple enlargement	—
,,	59	,,	Bladder	Large	—
,,	47	Endothelioma	Cervix uteri	Slight enlargement on L. side	Alveolar spaces distended with colloid
,,	39	,,	,,	R. lobe enlarged $1\frac{1}{2}$″ × $1\frac{1}{4}$″ × $\frac{3}{4}$″	Spaces in fibrous stroma—no malignancy
,,	37	Carcinoma	,,	About twice usual size	—
,,	42	,,	,,	Slight enlargement, gelatinous appearance	Normal structure
,,	42	,,	,,	Each lobe about twice usual size	—
,,	47	,,	,,	About twice usual size	—
,,	63	,,	,,	General enlargement	—
,,	70	,,	,,	L. lobe increased to size of small orange	—
,,	76	,,	,,	L. lobe enlarged about 2—3 times	—
,,	17	Sarcoma	Uterus	Enlarged, about 2″ across; parenchymatous goitre	Alveoli distended with colloid, no malignancy
,,	62	Carcinoma	,,	Rather large	—
,,	44	Endothelioma	Breast	Enlarged to 2—3 times usual size—normal appearance on section	—
,,	45	Carcinoma	,,	Twice usual size; slight goitre	—
,,	51	,,	,,	Somewhat enlarged	—
,,	56	,,	,,	Parenchymatous goitre	—
,,	52	,,	Gall bladder	Slightly enlarged; no adenoma	—
,,	21	,,	Rectum	Enlarged to about $2\frac{1}{2}$ to 3 times usual—simple hypertrophy	—
,,	54	,,	Stomach	Enlarged to about twice usual size	—
,,	48	,,	Cervix uteri	Greatly enlarged, cystic on section with calcified deposit in one lobe	Simple adenomatous changes
,,	57	,,	Rectum	Small adenoma L. lobe	—
,,	81	,,	Anus	Enlarged; fibrotic appearance	Simple adenoma
,,	62	,,	Breast	Somewhat enlarged; contains circumscribed mass size of a small pea	Non-malignant adenoma
,,	53	,,	,,	Nodule in L. lobe	Adenoma with increase of fibrous tissue and irregular proliferation of alveolar cells
,,	51	,,	,,	Considerably enlarged by adenoma; L. lobe about 3 times usual size	—

TABLE VII.—(*continued*).

Sex	Age	Type of primary cancer	Site of primary	Condition of thyroid gland as described by pathologist	
				Naked eye examination	Additional information from microscopic examination
Female	68	Carcinoma	Cervix uteri	Isthmus enlarged by adenomatous mass $\frac{5}{8}$" diameter	—
,,	33	,,	,,	R. lobe large; L. small adenoma	—
,,	35	,,	,,	L. lobe contains small growth $\frac{5}{8}$" diameter	Alveolar spaces greatly distended with colloid; areas of pigmentation
,,	81	,,	Breast	Small translucent adenoma	—
,,	55	,,	Cervix uteri	R. lobe contains colourless mass size of large pea	Alveolar spaces greatly distended with colloid
,,	72	,,	Bladder	Small adenoma $\frac{1}{4}$" diameter	Adenoma
,,	62	,,	Breast	Thought to be secondary growth	Non-malignant
,,	67	Endothelioma	Cervix uteri	L. lobe contains calcified mass size of pea	Large vesicles full of colloid
,,	48	Carcinoma	,,	R. lobe contains small calcified adenoma	—
,,	64	,,	,,	Both lobes considerably enlarged, $2\frac{1}{2} \times 1\frac{1}{2}$"; L. lobe has small calcified nodule	—
,,	62	,,	Uterus	Small calcified nodule	—
,,	55	Endothelioma	Breast	Several small white nodules	No malignancy
,,	37	Carcinoma	,,	Small yellow-white mass in one lobe $\frac{1}{5}$" diameter	No malignancy
,,	50	,,	Rectum	Small calcified nodule	—
,,	50	,,	Peritoneum	L. lobe has small calcified mass; R. fibrotic nodule	—
,,	70	,,	Breast	Tumour in L. lobe size of hen's egg, calcified	—
,,	66	,,	Face	Several small cysts in both lobes	—
,,	73	,,	Breast	Cystic, one $\frac{1}{4}$", several $\frac{1}{8}$"	Enlarged alveoli and small cysts
,,	64	,,	Rectum	Rather small; cystic adenoma in L. lobe	—
,,	52	Endothelioma	Intestine	Slightly enlarged, cystic	Cyst-adenoma
,,	69	Carcinoma	Stomach	Small cysts	—
,,	53	,,	Kidney	Very much enlarged on R. with large cyst; L. pale	—
,,	60	? Carcinoma	Uterus	L. lobe size of hen's egg, cystic	Increased fibrous tissue; degeneration and irregularity of acini
,,	87	Carcinoma	,,	Slight adenomatous change	Cystic
,,	64	Endothelioma	Breast	Enlarged by adenomatous masses, largest $2" \times 1" \times \frac{3}{4}"$; cystic degeneration	—
,,	62	,,	,,	Enlarged, especially L. lobe, containing cysts, one calcified	Increased fibrous tissue and degeneration of acini. No malignancy
,,	58	Carcinoma	,,	Cystic tumour, chiefly of R. lobe	Degenerated thyroid tissue
,,	63	,,	,,	Enlarged; R. lobe cystic	Several cysts and dilated acini
,,	56	,,	Pancreas	Cyst containing clear fluid lined by pigment layer, size $1\frac{1}{2}" \times 1" \times \frac{3}{4}"$	Adenoma
,,	50	,,	Stomach	Both lobes enlarged; R. cystic	Chronic inflammatory changes only
,,	63	,,	Breast	Mottled appearance in parts	—
,,	72	Endothelioma	,,	Rather small	—
,,	67	Carcinoma	Rectum	Fibrotic with calcified area in L. lobe	—
,,	41	,,	Breast	Slight adenomatous change and infiltration by new growth	—
,,	54	,,	,,	L. lobe invaded by new growth	Growth mixed with thyroid gland tissue which has undergone secondary change especially loss of colloid
,,	65	,,	,,	Pale, granular and hard, with minute cysts	New growth and shrunken connective tissue containing a few colloid cysts
,,	73	,,	,,	Several small dark masses, largest about $\frac{1}{2}$" diameter	Many distended alveoli in both lobes; in L. several centres of new growth differing from breast carcinoma and probably of independent origin
,,	62	Endothelioma	,,	Somewhat enlarged and cystic and invaded by growth	Adenomatous change and also invasion by growth
,,	47	,,	,,	Slight enlargement	ditto
,,	49	Carcinoma	,,	Slight enlargement with several colloid cysts, largest $\frac{1}{2}$"	Simple adenoma invaded by malignant growth
,,	57	,,	,,	Enlarged; with growth	Cells arranged in masses in gland acini
,,	58	,,	,,	Small nodules of new growth in addition to simple adenoma	New growth present

TABLE VIII. *Site and Type of Primary Growth in 500 Cancer Post-Mortems.*

Site of Primary	Males						Females						Type of Cancer					
	Thyroid abnormal (with or without secondary cancer)		Secondary cancer in normal thyroid	Normal thyroid not invaded	Normal thyroid all cases	All cases	Thyroid abnormal (with or without secondary cancer)		Secondary cancer in normal thyroid	Normal thyroid not invaded	Normal thyroid all cases	All cases	Thyroid abnormal			Thyroid normal		
													(with or without secondary cancer in thyroid)					
	No. of cases	Site rate	No. of cases	No. of cases	Site rate		No. of cases	Site rate	No. of cases	No. of cases	Site rate		Carcinoma	Sarcoma	Other	Carcinoma	Sarcoma	Other
Buccal cavity ...	4	18·2	1	39	28·0	44	—	—	—	6	2·3	6	3	1	—	45	—	1
Pharynx	—	—	2	3	3·5	5	—	—	—	—	—	—	—	—	—	4	1	—
Oesophagus	3	13·6	2	16	12·6	21	—	—	1	1	·8	2	3	—	—	20	—	—
Stomach	4	18·2	—	9	6·2	13	3	4·2	—	10	3·8	13	7	—	—	18	1	—
Liver and gall bladder	—	—	—	3	2·1	3	1	1·4	—	3	1·1	4	1	—	—	5	—	1
Pancreas	—	—	—	1	·7	1	1	1·4	—	4	1·4	5	1	—	—	4	—	1
Peritoneum	1	4·5	—	—	—	1	1	1·4	—	3	1·1	4	1	1	—	1	1	1
Intestines	—	—	—	4	2·8	4	1	1·4	—	8	3·0	9	—	—	1	10	2	—
Rectum and anus ...	3	13·6	1	16	12·0	20	7	9·9	—	23	8·7	30	10	—	—	39	1	—
Female genital organs	—	—	—	—	—	—	22	31·0	—	83	31·5	105	18	1	3	66	3	14
Breast	—	—	—	—	—	—	28	39·5	9	87	36·4	124	21	—	7	80	1	15
Skin	2	9·1	—	8	5·5	10	3	4·2	2	9	4·3	14	4	1	—	19	—	—
Kidneys	—	—	—	1	·7	1	1	1·4	—	3	1·1	4	1	—	—	—	—	4
Larynx	1	4·5	2	10	8·4	13	—	—	1	1	·8	2	1	—	—	12	1	1
Bone	1	4·5	—	8	5·5	9	—	—	—	3	1·1	1	—	1	—	4	5	2
Thyroid	—	—	—	—	—	—	—	—	1	—	·4	3	—	—	—	—	1	—
Other	3	13·6	1	16	12·0	20	3	4·2	—	6	2·3	9	3	2	1	10	9	4
Total	22	—	9	134	—	165	71	—	14	250	—	335	74	7	12	337	26	44

thelioma or other variety of malignant growth. Sarcoma forms 7·5 °/₀ and carcinoma 79·5 °/₀ of the cases with abnormal thyroid, as compared with 6·3 °/₀ and 82·8 °/₀ respectively of the other cases.

VII. CANCER OF THYROID GLAND.

An excellent and extensive monograph on the subject of thyroid cancers has recently been published by Bérard and Dunet(18). Amongst many interesting observations is the statement (Chap. V) that cancerous thyroid tissues are very poor in iodine, a fact which has a bearing upon the present investigation since our results seem to point to an antagonism between the iodine-rich thyroid secretion and cancer. Primary cancer in a normal sized thyroid is of rare occurrence. Feilchenfeld(19) found it in one case out of 507 cancers, Reichelmann(20) in five out of 711, Redlich(21) in three out of 496, Ménétrier in three out of 330, Chiari in one out of 452, Jeanseline and Barbe in three out of 1374, and in our own series of 500 from Middlesex Hospital records it occurred once. Totalling all these cases it appears that in regions where goitre is uncommon primary thyroid cancer forms about 4 per 1000 of all cancers. The mortality figures for England and Wales 1913–22 (see Table IX) show 1419 deaths from thyroid cancer out of a total of 418,346 deaths from all cancers, or 3·4 per 1000.

In contrast to this we have the following figures from goitrous areas:

Bern	Limacher(23).....	1 in 93	=11	per 1000 cancers
St Gallen	Mortality in 1920	6 in 396	=15	„ „ „
Switzerland	„ „ 1901–20.....1143 in 89,817		=12·2	„ „ „
Lyon	Bérard (18) 1908–22.....	38 in 837	=44	„ „ „

showing that thyroid cancer is relatively very much more common in goitrous areas than elsewhere.

From Wilson's(22) statistics of the Mayo Clinic covering 25 years, pre-existent goitre was proved to have been present in 266 out of 297 cases (88 °/₀) of thyroid cancer; Slesinger(24) proved it to have existed in 90 °/₀, Bérard and Dunet in 81 °/₀, Speese and Brown(25) in 98°/₀. Operative statistics show that in endemic areas about 8 °/₀ of goitres become malignant and in other areas about 3 or 4 °/₀. According to Bérard the most common form of precancerous lesion is the adenomatous goitre. The single case of primary cancer from the Middlesex Hospital series was a mixed cell sarcoma in a woman aged 75.

As regards *secondary cancer* of the thyroid the literature contains little reference to the question. It seems probable that the normal thyroid is only attacked by metastasis in a very advanced stage of the disease, but it may be involved earlier by direct extension from neighbouring parts. In 500 cases, 22 showed secondary cancer in an otherwise normal thyroid and 11 in an abnormal thyroid. An analysis of these 33 cases reveals some interesting facts.

		By direct extension			By metastasis		
		No.	Site of Primary	Type	No.	Site of Primary	Type
Secondary cancer of thyroid	Males	10	2 larynx 3 oesophagus 2 pharynx 2 tongue 1 neck	Carcinoma „ „ „ Branchioma	1	Rectum	Carcinoma
	Females	2	1 larynx 1 oesophagus	Carcinoma „	20	18 breast	13 Carcinoma 1 Sarcoma 4 Endothelioma
						1 groin	Sarcoma
						1 skin	Melanotic Sarcoma

Of 11 males, 10 showed involvement by direct extension and one by metastasis from the rectum (the adrenals being also involved). Of 22 females extension from neighbouring organs only occurred in two, and of the 20 cases of metastasis no less than 18 were breast cancers, the other two being sarcomata. All these showed very widespread metastases throughout the body. It is also of interest to note that in 11 out of 93 cancer cases with anomalous thyroids (11·8 °/₀) the gland became secondarily involved, whereas in only 22 out of 407 cancer cases with previously normal thyroids (5·4 °/₀) did this occur; and that in eight of the 11 cases with a precancerous lesion this was of the adenomatous type. Both these findings agree with statistics for primary cancer already quoted and go to prove that the goitrous thyroid is more liable than the normal gland to be attacked, not only by primary, but also by secondary cancer, a fact which I have not found to have been proved hitherto. Whilst this all goes to prove that depressed thyroid activity is favourable to cancer incidence, symptoms of hyper-thyroidism have occasionally been met with in late stages of malignant thyroid tumours(26), but I have been unable to find records of cancer, either of the thyroid or other organs, developing in a patient with pre-existent Graves' Disease, and such cases must be extremely rare.

VIII. Cancer and Graves' Disease in England and Wales.

(*a*) *Source of Data.* The absence of information as to the frequency or geographical distribution of Graves' Disease is astonishing. The only statistical data readily available as to the case-rate in Britain was the Report of the Ministry of National Service(12) in which we are told that 11 cases were found in 20,704 recruits examined in the Liverpool area, or ·053 °/₀, and 6—7 cases in 10,000 recruits examined in Scotland, or ·06—·07 °/₀. Hospital statistics are of little value for this purpose and we are left with mortality rates as the only means at present of estimating the distribution of the disease in this country. It must be admitted that, since the case mortality is comparatively low and must be influenced by facilities for treatment to a considerable extent, mortality rates may not give a correct index of the relative

morbidity rates in different areas, but they probably give over a period of years a fair indication of this. In the decade 1913–1922, 4874 persons died primarily of the disease in England and Wales, giving a mean annual mortality rate of 1·35 per 100,000 living. Of these 4414 were females and 460 males, giving rates of 2·26 per 100,000 in females and 0·28 in males, calculated on the mean civilian population*. In Liverpool the standardized death rate over the same period was 1·04, and the case-rate in males as shown by the recruiting data 53 per 100,000. Assuming that the case mortality is the same in Liverpool as in the country as a whole we arrive at the estimate of 69 cases per 100,000 civilian males in England and Wales in 1918, or a total of 9500 males with the disease. Sattler(26) found that of 3800 collected cases in Germany, 3120 were females and 680 males, a ratio of 4·6 : 1, whereas 127 cases treated at Guy's Hospital in 1908–1917 gave a ratio of 17 : 1. Probably therefore we may assume that the sex-distribution of cases in England is similar to that of deaths from the disease (9·4 : 1), and applying these ratios to the population as a whole, a rough estimate of 100,000 persons affected by Graves' Disease in England and Wales is arrived at.

In 1921 Campbell(27), using crude death rates for 1913–19, concluded that the disease was more prevalent in western than eastern counties and near the coast than inland, but no account was taken of the greater proportion of females in coast resorts, which would greatly influence the rates (e.g. using 1913–22 figures of Table IX, the crude mortality rate for the large watering places is 2·25 per 100,000, which becomes 1·80 when standardized, whereas for midland industrial towns the crude rate is raised). It therefore seemed advisable to work out the distribution in greater detail, to standardize the mortality rates and then correlate the distribution so obtained with cancer mortality, also standardized. If depressed thyroid activity is really favourable to cancer incidence we might expect to find a small negative relationship between the distributions of a disease characterized by hyper-thyroidism and cancer, and such a finding would give further support to our conclusions. The mortality data from various types of thyroid disease for the decade 1913–22, extracted from the records of the Registrar General, were kindly lent to me by Colonel Smallman of the Ministry of Health, to whom I express my indebtedness for his assistance. The working-out of this data has entailed very heavy labour, and only the preliminary results can as yet be given.

(*b*) *Distribution of Graves' Disease as indicated by Standardized Mortality Rates.* The period dealt with was of course unsuitable for work of this kind owing to great disturbances in population occasioned by the War, but it was essential to take at least a complete decade, and data prior to 1913 were not available. It was therefore necessary to compute the age distributions of each of the 233 registration areas for each year separately, and in doing so to use a large age group of 15—45 for males and assume that all the deficiency in the total civilian populations of districts† for the war years 1914–18 from the expected figures arose by subtraction

* From figures supplied to me by the Registration Authority the mean mortality rates in Holland during the same period were respectively 1·09 and ·19, and in Amsterdam 1·22 and ·38.

† I.e. as given by the Registrar General as having been determined from National Registration and rationing statistics.

of population from this age group alone, the other age groups being obtained by interpolating from the reliable figures of 1913 and the census of 1921. Such a method provided the nearest approximation that could be arrived at for these years and probably introduced no very serious error. The age and sex distributions of civilian deaths from Graves' Disease and cancer for the whole country having been found, the respective corrective factors for each area were calculated by reducing to the mean civilian population of England and Wales during the decade 1913–22. Mortality rates reduced to such a standard are not of course comparable with the more usual standardized rates, but are uniformly higher since the removal of five or six million of the healthiest males for several years in the middle of this decade necessarily raised all the mortality rates of the residual population. For the purpose of working out a relative geographical distribution within England and Wales and correlating the distributions of diseases, this standard population was the most useful, and it is difficult to see how it would be possible or profitable to reduce the rates to any standard population of pre- or post-war years.

In Table IX (p. 396) will be found the total deaths from Graves' Disease in 1913–22 in various divisions of the country, and the standardized mortality rates. These rates indicate that (i) mortality has an inverse relation with density of population, thus—rural districts 1·73, urban districts 1·36, county boroughs 1·22, London ·84 per 100,000; (ii) mortality is higher generally in counties fringing the coast than inland, thus—rural coast areas 1·90, rural inland 1·50, urban coast areas 1·43, urban inland 1·27—but the large towns as a whole do not show any such difference; (iii) mortality in the east and south coast counties and boroughs is much lower than in those fringing the west coast and the S.W. corner of England, with the exception of several large south coast watering places which show high rates; (iv) of the inland areas the Welsh border counties show much higher rates than any other and these are also closer to the west coast than any other inland counties; (v) the coal-mining counties of Glamorgan, Durham and Monmouth have low rates. The westerly distribution of the disease can be easily shown by writing down from Table IX all the areas with mortality rates above two per 100,000—they are Cumberland, Westmorland, Lancashire, Cheshire, S.W. counties, N. Wales, S.W. Wales, and Welsh border counties. This suggests some relation to climatic conditions, since a similar distribution is found for rainfall. M^{c}Clendon(16) has roughly demonstrated a relation between iodine content of water supplies in American States and the frequency of goitre. Hunziker(28) in a recent monograph of great interest, compares analyses of drinking waters from several areas in Switzerland and France, and points to their influence on goitre frequency. In the absence of knowledge as to iodine contents of water supplies in England and Wales I suggest the possibility that those in the West are richer in iodine than those in the East, and for this reason—the greatest source of iodine is the sea and the atmosphere in the vicinity of the sea has been found to contain appreciable quantities derived from spray and consequently the rainfall near the coast is also richer in iodine. Since the prevailing winds are westerly the areas bordering the west coast would be affected by this factor to a greater extent than those on the east, and the

midland areas least of all. The suggestion that the distribution of Graves' Disease might be explained in this manner is merely a tentative one and needs demonstration. M^{c}Clendon's conclusion that this distribution is on the contrary inversely related to iodine supply cannot be accepted without further proof.

(*c*) *Correlation with Cancer mortality.* An examination of the standardized mortality rates shown in Table IX indicates that (i) cancer mortality shows on the whole a direct relation with density of population, thus—rural districts 100·8 per 100,000, urban districts 111·5, county boroughs 119·3, London 115—116; (ii) in rural areas it is usually higher inland than near the coast, and (iii) higher near the east coast than near the west; (iv) when urban and rural areas are combined and county boroughs excluded the highest rates are for N. Wales 119·5, Cambridgeshire (with Ely and Peterborough) 118·6, Middlesex 117·9 and Cheshire 116·5; (v) the coal-mining areas have exceptionally low rates; (vi) of the large towns those of Lancashire, Cheshire and West Riding have the highest rates. It is noticeable on comparing (i) (ii) (iii) above with (i) (ii) (iii) in the last section that there is an inverse relation with mortality from Graves' Disease. I have therefore correlated the deaths from the two diseases in different areas, using various methods and distributions to determine whether there is a real negative relationship.

A. Correlating the mortality rates directly and obtaining the partial coefficient when population, age-sex corrective factor for cancer, and age-sex corrective factor for Graves' Disease are made constant. It is doubtful whether spurious correlation is eliminated by such a procedure but the results are given for comparison with those obtained by the other method.

(α) Dividing the country into 100 urban and rural areas, excluding the county boroughs and combining urban with rural in a few smaller counties to obtain a better distribution:

Corrected mortality rate for cancer with corrected mortality rate for Graves' Disease ...	$r = -\cdot2161 \pm \cdot0643$
Partial coefficient for population constant ...	$r = -\cdot2191 \pm \cdot0627$
Partial coefficient for population and both corrective factors constant	$r = -\cdot2483 \pm \cdot0618$

(β) Dividing the country into 200 areas including county boroughs, combining some small areas as before:

Correlation between mortality rates	$r = -\cdot1949 \pm \cdot0459$
Partial coefficient for population constant ...	$r = -\cdot1944 \pm \cdot0448$

B. Correlating the numbers of deaths by the method suggested by Professor Pearson(7), obtaining a partial coefficient when population and corrective factors for each disease are constant. In using this method I endeavoured to divide the country into areas whose populations were more nearly equal in density, so that the differing effect of urbanisation on these diseases might have less weight. This was done as follows: excluding county boroughs, the urban and rural districts for

TABLE IX.

Mortality from Cancer and Diseases of Thyroid Gland in England and Wales, 1913–22.

	Area	Mean civilian population 1913–22	Cancer of all Organs		Graves' Disease		Cancer of Thyroid		Other Diseases of Thyroid	
			Total Deaths	Mean Standardized death-rate	Total Deaths	Mean Standardized death-rate	Total Deaths	Mean Standardized death-rate	Cretinism Total Deaths	Other Total Deaths
	Rural Districts									
Coast Areas	East Coast Counties[a] ...	1,251,484	16,603	102·2	215	1·73	57	—	32	132
	Durham	408,735	3,063	92·5	41	1·20	13	—	5	23
	Cumberland	92,040	1,160	93·5	23	2·49	3	—	1	15
	Westmorland	34,369	506	112·9	9	2·42	4	—	2	9
	Lancashire	232,099	2,439	96·4	64	2·64	11	—	2	30
	Cheshire	182,153	2,071	105·6	50	2·69	12	—	4	15
	South-West Counties[b] ...	597,963	7,813	96·5	135	2·09	24	—	17	72
	South Coast Counties[c] ...	857,562	11,184	103·1	154	1·74	42	—	26	104
	North Wales[d]	269,488	3,857	119·5	58	2·19	15	—	5	40
	South-West Wales[e] ...	188,442	2,269	100·3	44	2·33	11	—	4	25
	Glamorgan	216,644	1,645	93·8	34	1·86	6	—	2	9
	Total Coast Areas ...	4,330,979	52,610	97·5	827	1·90	198	·38	100	474
Inland Areas	Monmouth	42,936	423	84·1	5	1·20	—	—	2	3
	Welsh Border Counties[f]	89,139	1042	98·9	23	2·80	6	—	3	8
	West Midland Counties[g]	323,668	3996	95·4	43	1·30	26	—	6	44
	South Midlands (Thames Valley)[h]	946,574	12,218	104·5	130	1·32	42	—	21	83
	East Midland Counties[i]	449,655	6,410	106·7	53	1·16	21	—	11	33
	Central Midlands[k] ...	529,116	5,755	102·2	86	1·70	24	—	8	50
	North Midlands[l] ...	773,442	7,464	99·4	123	1·75	31	—	13	63
	Total Inland Areas ...	3,154,530	37,308	101·8	463	1·50	150	·41	64	284
	Total Rural Districts	7,485,509	89,918	100·8	1290	1·73	348	·39	164	758
	County Boroughs									
Coast Towns	West Coast Watering Places[o]	221,502	3,233	129·4	47	1·81	8	—	6	27
	West Coast Ports ...	1,053,238	10,985	122·8	110	1·12	31	—	16	55
	East Coast Ports ...	1,310,849	13,555	119·1	129	1·07	36	—	21	93
	South Coast Ports ...	548,550	6,481	111·9	61	1·09	17	—	8	35
	South Coast Watering Places	311,683	5,082	119·1	73	1·80	16	—	1	13
	Total Coast Towns ...	3,445,822	39,336	119·7	420	1·20	108	·33	52	223
London.	N. of Thames	2,511,983	31,980	115·3	212	·74	77	—	30	188
	S. of Thames	1,834,784	22,975	116·6	192	·99	64	—	21	127
	Total Metropolitan Boro's	4,346,767	54,955	115·9	404	·84	141	·30	51	315

(*a*) Northumberland, Yorkshire N. and E. Ridings, Lincs., Norfolk, Suffolk, Essex (including East Ham and Southend in U.D.). (*b*) Somerset, Devon, Cornwall. (*c*) Kent, Sussex, Hants., I. of Wight, Dorset. (*d*) Flint, Denbigh, Carnarvon, Anglesey, Meirioneth. (*e*) Carmarthen, Cardigan, Pembroke. (*f*) Montgomery, Brecknock, Radnor. (*g*) Hereford, Salop, Worcester. (*h*) Gloucester, Oxford, Bucks., Berks., Wilts., Middlesex, Surrey. (*i*) Bedford, Herts., Hunts., Northants, Cambs., Rutland, Ely, Peterboro. (*k*) Warwick, Staffs., Leicester. (*l*) Notts., Derby, Yorks. W. Riding. (*o*) Blackpool, Southport, Wallasey.

TABLE IX.—(*continued*).

Mortality from Cancer and Diseases of Thyroid Gland in England and Wales, 1913–22.

	Area	Mean civilian population 1913–22	Cancer of all Organs		Graves' Disease		Cancer of Thyroid		Other Diseases of Thyroid	
			Total Deaths	Mean Standardized death-rate	Total Deaths	Mean Standardized death-rate	Total Deaths	Mean Standardized death-rate	Cretinism Total Deaths	Other Total Deaths
	URBAN DISTRICTS									
Coast Areas	East Coast Counties[a] ...	1,698,885	19,204	109·0	183	1·08	62	—	24	141
	Durham[m]	549,219	4,830	109·4	56	1·18	15	—	4	36
	Cumberland[n]	168,994	2,027	120·8	30	1·83	8	—	3	13
	Westmorland	25,371	352	108·6	8	2·66	6	—	1	1
	Lancashire	1,450,524	15,524	112·0	277	1·84	64	—	23	108
	Cheshire	416,637	5,121	121·3	74	1·76	34	—	6	39
	South-West Counties[b] ...	489,423	6,964	105·2	113	1·92	17	—	16	40
	South Coast Counties[c] ...	1,255,353	17,015	118·0	165	1·21	50	—	19	105
	North Wales[d]	177,865	2,676	124·8	35	1·74	14	—	2	18
	South-West Wales[e] ...	124,658	1,312	102·4	23	1·85	10	—	—	8
	Glamorgan	572,680	3,993	98·0	50	1·07	12	—	5	38
	Total Coast Areas ...	6,929,609	79,018	111·5	1014	1·43	292	·41	103	547
Inland Areas	Monmouth	296,872	1,927	89·4	16	·69	9	—	2	14
	Welsh Border Counties[f]	40,485	584	118·8	13	2·96	3	—	1	5
	West Midland Counties[g]	304,598	3,847	110·6	47	1·48	21	—	5	21
	South Midlands (Thames Valley)[h]	2,077,891	25,029	114·8	253	1·14	69	—	35	120
	East Midland Counties[i]	581,388	7,481	115·3	80	1·31	27	—	12	55
	Central Midlands[k] ...	716,369	7,150	107·5	75	1·12	37	—	17	45
	North Midlands[l] ...	1,643,927	17,283	111·7	246	1·53	51	—	24	129
	Total Inland Areas ...	5,661,530	63,301	111·5	730	1·27	217	·38	96	389
	TOTAL URBAN DISTRICTS	12,591,139	142,319	111·5	1744	1·36	509	·40	199	936
	COUNTY BOROUGHS (*cont.*)									
Inland Towns	Industrial Towns: Lancashire & Cheshire	2,126,767	23,883	124·5	342	1·56	71	—	30	136
	Industrial Towns: West Riding	1,590,424	18,766	123·3	185	1·11	57	—	16	112
	Industrial Towns: Midlands	2,282,890	24,161	116·4	243	1·09	92	—	36	119
	Industrial Towns: Southern[p]	831,444	9,403	115·8	72	·85	23	—	11	52
	Mining centres[q] ...	487,159	4,511	106·9	62	1·40	26	—	3	20
	Agricultural centres[r] ...	847,153	11,094	114·6	112	1·22	44	—	17	78
	Total Inland Towns ...	8,165,837	91,818	119·0	1016	1·22	313	·41	113	517
	TOTAL COUNTY BOROUGHS ...	11,611,659	131,154	119·3	1436	1·22	421	·38	165	740
	ENGLAND AND WALES ...	36,035,074	418,346	112·4	4874	1·33	1419	·38	579	2749

(*a*) Northumberland, Yorkshire N. and E. Ridings, Lincs., Norfolk, Suffolk, Essex (including East Ham and Southend in U.D.). (*b*) Somerset, Devon, Cornwall. (*c*) Kent, Sussex, Hants., I. of Wight, Dorset. (*d*) Flint, Denbigh, Carnarvon, Anglesey, Meirioneth. (*e*) Carmarthen, Cardigan, Pembroke. (*f*) Montgomery, Brecknock, Radnor. (*g*) Hereford, Salop, Worcester. (*h*) Gloucester, Oxford, Bucks., Berks., Wilts., Middlesex, Surrey. (*i*) Bedford, Herts., Hunts., Northants, Cambs., Rutland, Ely, Peterboro. (*k*) Warwick, Staffs., Leicester. (*l*) Notts., Derby, Yorks. W. Riding (including Wakefield). (*m*) Includes Darlington. (*n*) Includes Carlisle. (*p*) Bristol, Croydon, West Ham. (*q*) Swansea, Cardiff, Merthyr, Newport. (*r*) Lincoln, Gloucester, Northampton, Ipswich, Norwich, Exeter, Oxford, Bath, York, Reading, Canterbury, Worcester, Chester.

each county (except Lancashire* and West Riding) were combined and some of the small adjoining counties were also compounded with one another. This gave 42 areas and the results of correlation were:

Cancer deaths, corrected, with population ...	$r = \cdot 9944 \pm \cdot 0011$
Deaths from Graves' Disease, corrected, with population	$r = \cdot 8800 \pm \cdot 0235$
Cancer deaths with deaths from Graves' Disease	$r = \cdot 8678 \pm \cdot 0257$
Corrective factor for cancer with population ...	$r = \cdot 4470 \pm \cdot 0833$
„ „ „ Graves' Disease with population	$r = \cdot 2701 \pm \cdot 0965$
„ „ „ cancer with cancer deaths	$r = \cdot 4284 \pm \cdot 0850$
„ „ „ Graves' Disease with cancer deaths	$r = \cdot 2370 \pm \cdot 0982$
„ „ „ cancer with deaths from Graves' Disease ...	$r = \cdot 4054 \pm \cdot 0870$
„ „ „ Graves' Disease with deaths from Graves' Disease	$r = \cdot 2161 \pm \cdot 0992$
„ „ „ Graves' Disease with corrective factor for cancer	$r = \cdot 9011 \pm \cdot 0196$
Partial correlation of cancer deaths with deaths from Graves' Disease, population and both corrective factors constant	$r = -\cdot 2208 \pm \cdot 0990$

In this last proceeding, urban and rural districts having been combined before correlating, county boroughs omitted, populations of counties made constant and age-sex distributions also constant, the effect of differing densities of population has been thereby practically eliminated, and there still remains a small negative correlation between cancer and Graves' Disease mortalities which seems to be significant. Taken by itself a correlation of this order would not, in view of the great complexity of the factors involved in correlating death rates, be sufficient ground for building any hypothesis upon, but as forming an additional link in the chain of evidence for an antagonism between the thyroid secretion and cancer it is perhaps worthy of consideration.

IX. Cretinism and other thyroid diseases in England and Wales.

In the decade 1913–1922 579 deaths from cretinism were registered. The crude mortality rate per 100,000 living was ·22 in rural districts, ·16 in urban districts, ·14 in county boroughs, and ·12 in London. Taking urban and rural districts together, the rates for coast and inland areas as a whole did not differ significantly (·180, ·181). The south-west counties showed a high rate (·303);

* The group of Lancashire Urban districts was omitted along with the county boroughs to which it is more akin.

south coast counties ·213; east coast ·181; all west coast counties (excluding S.W.) ·157; Lancashire alone ·148; Welsh coast counties alone ·145. The relation of the last five groups is directly opposed to that found for Graves' Disease, where the order is inverted. London however gave a low rate (·117) and the coal-mining centres still lower (·097), as in the case of Graves' Disease.

The group of other thyroid conditions is a heterogeneous group containing myxedema, some thyroid tumours and no doubt some exophthalmic goitres; it would be futile therefore to attempt to draw any conclusions from the distribution of this group. When correlated with cancer mortality there is no significant relation—for 100 areas excluding large towns $r = -·0775 \pm ·0674$, and for 200 areas including large towns $r = -·0654 \pm ·0476$.

X. CONCLUSIONS.

(1) The distributions of goitre in recruits and school children and of cancer mortality in Switzerland show a small positive correlation which is apparently not accounted for by differences in healthiness as measured by the general death rate.

(2) This correlation of general cancer mortality with goitre is found to arise from a very definite regional correlation between goitre and cancer of the stomach and oesophagus.

(3) The selection rate of cancer for the stomach is generally high in other goitrous regions of Europe and America, and low in areas where goitre is infrequent and this relation also holds good for sub-divisions of Norway and United States of America.

(4) The examination of 1000 London post-mortem records leads to the conclusion that enlargements and other anomalies of the thyroid gland are considerably more frequent in cancer cases than in non-cancer cases at death, and these anomalies are of the types usually associated with depressed thyroid function.

(5) The enlarged or adenomatous thyroid gland is attacked more frequently than the normal gland both by primary and secondary cancer; the great majority of the cases in which a normal thyroid was secondarily involved were recurrent breast cancers with widespread metastasis.

(6) Graves' Disease probably affects about 100,000 persons in England and Wales; its prevalence as indicated by corrected mortality rates is higher in rural than town areas, in coast than in inland areas, and on the West Coast than on the East.

(7) After correcting as far as possible for all known factors, the mortality from Graves' Disease in the counties shows a small negative correlation with cancer mortality; this, as might be expected, is not so evident in the large towns.

(8) The above findings seem to indicate that defective functioning of the thyroid gland is favourable to the incidence of cancer of the stomach, and possibly of other organs also.

(9) This leads to the suggestion that iodine prophylaxis applied after middle age, on the same lines as it is now being successfully applied to young persons in Switzerland for prevention of goitre, or some other form of thyroid administration, might result in diminishing cancer incidence. That it is at all likely to seriously influence the growth of cancer once started is not suggested, though retardation might result.

(10) It is admitted that some of the above conclusions are open to criticism, and that some of the facts might be explained in other ways, but before rejecting the hypothesis in the face of such evidence as there is, I contend that further research is called for, including if possible a direct experimental test of the results of prolonged iodine or thyroid administration to a cancerous strain of animals.

In conclusion I have to thank Professor Karl Pearson for many helpful suggestions, Miss Ida M^{c}Learn for drawing the maps, and Miss Noel Karn for her assistance in statistical work.

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MISCELLANEA.

I. Historical Note on the Origin of the Normal Curve of Errors.

By KARL PEARSON, F.R.S.

It is usual to attribute the discovery of the Normal Curve of Errors to Gauss. This is solely due to the fact that Laplace's *Théorie analytique des Probabilités* was published in 1812, and to this most writers have referred. But Laplace's *Mémoire sur les Probabilités* was published in the *Histoire de l'Académie de Sciences* in 1778, and this memoir contains the normal curve function, and emphasises the importance of tabulating the probability integral. Nay, we may go further and say that Laplace in 1774 (*Mémoires...présentés à l'Académie*, T. VI. p. 6) when discussing Bayes' Theorem had also reached the exponential curve of errors as an approximation to the hypergeometrical series. All Gauss' work falls into the 19th century. His *Theoria motus corporum coelestium* was published in 1809, his theory of least squares and his theory of combination of observations being of a still later date. There is, I think, not a doubt that Laplace's name ought to be associated with the normal curve and the probability integral before Gauss'.

But in studying De Moivre I have come across a work which long antedates both Laplace and Gauss.

The matter is a very singular one historically. De Moivre published in 1730 his *Miscellanea Analytica*, still a mine of hardly fully explored wealth. Many copies of this work have attached to them a *Supplementum* with separate pagination, ending in a table of 14 figure logarithms of factorials from 10! to 900! by differences of 10. But only a *very few* copies have a second supplement, also with separate pagination (pp. 1—7) and dated Nov. 12, 1733. This second supplement could only be added to copies sold three years after the issue of the original book, and this accounts for its rarity. Dr Todhunter in writing his *History of the Theory of Probability* appears to have used the 1730 issue of the *Miscellanea Analytica*, and so never come across this supplement*.

It is entitled: *Approximatio ad Summam Terminorum Binomii* $(a+b)^n$ *in Seriem expansi.* This supplement contains the first treatment known to me of the probability integral, and essentially of the normal curve. It antedates Laplace's discussion by half-a-century. De Moivre finds the ratio of the maximum term of a binomial to the term at distance x (in his symbols l) from the maximum by means of what we should now term *Stirling's Theorem*. He supposes his power so large, that practically we may use

$$m! = \text{const.} \times \sqrt{m}\, e^{-m}\, m^m.$$

He determines the constant which he calls B by the theorem that the hyperbolic logarithm of B is given by:

$$\log_e B = 1 - \tfrac{1}{12} + \tfrac{1}{360} - \tfrac{1}{1260} + \tfrac{1}{1680},$$

which gives $\log_{10} B = \cdot 399{,}2235$ to the terms written down, or $B = 2\cdot 5074$.

Thus
$$m! = 2\cdot 5074 \times \sqrt{m}\, e^{-m}\, m^m.$$

* All three copies of this work in the University College Library have the first, only one the second supplement. The sole copy in the British Museum Library has the first but not the second. Of the three copies at Cambridge (University Library) one has the first supplement only, the other two neither supplement (information through the kindness of Mr W. H. Macaulay). The Royal Society has two copies, one has the first, the other neither supplement. The Bodleian Library has a single copy with neither supplement. The University Library, Edinburgh, has two copies, one has the first, the other neither supplement.

Then as De Moivre puts it: "Vir Doctissimus mihique amicissimus *Jacobus Stirling*," who after me brought himself to the subject by a very different process, showed that the value of B was the square root of the ratio of the circumference of a circle to its radius, i.e. what we now call $\sqrt{2\pi}$, the value of which is 2·506,628. I consider that the fact that Stirling showed that De Moivre's arithmetical constant was $\sqrt{2\pi}$ does not entitle him to claim the theorem, and that it is erroneous to term it Stirling's Theorem. De Moivre calls the constant $\sqrt{c}$, and he now proceeds to show by aid of this theorem:

(i) That the ratio of the term at distance x from the maximum term to the maximum term for a large n has for its hyperbolic logarithm $-x^2/2\sigma^2$ in our notation, or

$$y = y_0\, e^{-x^2/(2\sigma^2)}.$$

He takes two cases $(1+1)^n$ and $(a+b)^n$. In the first case he shows that

$$\sigma = \sqrt{\tfrac{1}{2} \times \tfrac{1}{2} \times n},$$

and in the second case that

$$\sigma = \sqrt{\frac{a}{a+b} \times \frac{b}{a+b} \times n}.$$

He does not name σ (our standard deviation of the binomial $= \sqrt{pqn}$) but he fully realises its importance and that all deviations must be measured in terms of it.

(ii) He indicates (Lemma II. p. 6) that the position of the maximum term is given by nq, and he shows that James Bernoulli's Problem depends for its proper solution on evaluating the ratio of the sum of the terms between $nq-x$ and $nq+x$ to the sum of the total series, and thus introduces for the first time the probability integral.

(iii) He finds the value of y_0, for he says the ratio of the maximum term of $(a+b)^n$ to the sum of all the terms (i.e. y_0 to the total population N of our notation) is

$$\frac{a+b}{\sqrt{abnc}} \text{ (i.e. } y_0/N = \frac{1}{\sqrt{2\pi}\,\sigma} \text{ of our notation).}$$

(iv) To obtain the probability integral

$$\int y_0\, e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}\, dx$$

he expands the exponential by the exponential theorem, puts $x = ls\sigma$ and integrates. He finds what we should now term α, for $x/\sigma = 1$, and gives its value as 0·341,344; its true value to six figures is 0·341,345, so that he is correct to a unit in the sixth significant figure.

De Moivre is less successful when (the expansion of his series being too laborious) he evaluates the probability integral by quadratures, thus he gives for our α when

$$x/\sigma = 2 : \cdot 95428 \text{ instead of } \cdot 95450,$$
$$x/\sigma = 3 : \cdot 99874 \text{ instead of } \cdot 99730.$$

His theory is correct, but his quadrature formula not accurate enough when he includes the point of inflexion of the normal curve. Bernoulli had stated that by taking n sufficiently large it was possible to make the sum of t terms round the mode of the binomial bear a ratio to the whole series differing by less than any assignable quantity from unity. Bernoulli's proof of this is laborious, unsatisfactory and without insight into the standard deviation of the binomial varying as root of n. This insight was first gained by De Moivre, and what is now termed Bernoulli's Theorem, namely, that accuracy increases inversely as the square root of the number of experiences, was first due to De Moivre. Modern proofs of that theorem follow De Moivre's lines in this *Supplementum*, but never, as far as I am aware, appeal to the proof in the *Ars conjectandi*—which I am fairly confident that modern text-book writers have never examined; they would find grave difficulty in recognising it as the Bernoulli's Theorem, with which they are familiar.

De Moivre's treatment is essentially our modern one. He takes for example an event equally likely to succeed or fail and asks what is the probability in 3600 trials that success will lie between 1750 and 1850 occurrences. The mean is 1800, and the range ± 50. The standard deviation is $\sqrt{\frac{1}{2}\frac{1}{2}n}=30$. The ratio is $50/30=5/3$, and he says in effect we must calculate the value of the integral (pp. 5—6)

$$\int_{-5/3}^{+5/3}\frac{1}{\sqrt{2\pi}}-e^{-\frac{1}{2}t^2}dt.$$

This we now do from our tables of the probability integral, but De Moivre was compelled to expand and integrate or to use quadratures. "Si Contingentia & Non-contingentia Eventum se habuerint inter se ut a ad b, Problemata de summatione Terminorum aeque facile solventur ac illa in quibus Contingentia & Non-contingentia sunt inter se in ratione aequalitatis" (p. 7).

The same matter is dealt with twenty-three years later in the edition of 1756 of *The Doctrine of Chances*, pp. 243—250. Todhunter in his *History of the Theory of Probability*, Arts. 324 and 335, passed over the topic most superficially. He did not grasp that this supplement gives priority to De Moivre not only as the discoverer of the normal curve, but also in the theorem we now term Stirling's. Nor did he realise that what we now term Bernoulli's Theorem—or the measure of accuracy depending on the inverse square root of the size of the sample—was entirely due to De Moivre and not to Bernoulli. Nor did he in the least appreciate the immense range of application which De Moivre himself foresaw for his problem; for him it was a theological problem, he was determining the frequency of irregularities from the Original Design of the Deity. Without grasping this side of the matter, it is impossible to understand the history of statistics from De Moivre through Derham and Süssmilch to Quetelet, culminating in the modern principle of the stability of statistical ratios. No one had any true inkling of the idea of the probable deviation of a statistical ratio before De Moivre.

When we remember how common a process it is even to-day to deduce the normal curve from the limit to a binomial, I think it is only fitting to drop the association of that curve with the name of either Laplace or *à fortiori* Gauss and to credit De Moivre with its discovery. In the same way we ought at least to speak of the De Moivre-Stirling Theorem, if we retain in part Stirling's name as contributor of the $\sqrt{2\pi}$.

II. A Preliminary Inquiry into the Density of the Living Male Human Body.

By FRANK SANDON, M.A.

In 1912 Mr R. Sandon published in the *Field* of 1st June a description of a method that he devised for testing the capacity of any particular individual to float (i.e. to remain motionless, upright, or horizontal, with the mouth above water, in ordinary fresh water, without the exertion of any force). The writer recently had an opportunity of developing this work among some 80 boys of Highgate School: he wishes to thank all those who submitted to the experiments and the members of the school scientific society who helped him in the observations.

The object of the experiments was to determine the frequency distribution of the density of the human body, text-books of various kinds giving all values from ·9 to 1·1, and to find relations between these anthropometrical values and others easily determined. For this purpose the following series of measurements and calculations were proposed in each case.

(*a*) Date, name of experimenter and hydrometer value of the density of the water in the school bath at the time of experiment.

(*b*) Name of subject.

(*c*) Age in completed years and months at the date of experiment.

(*d*) Weight, stripped, in stones and pounds.

(*e*) Height, barefoot, in feet and inches.

(*f*) Apparent weight, in pounds, of subject when immersed in the water with lungs fully and comfortably inflated.

(*g*) Apparent weight (in lbs.) of subject when immersed in the water with lungs as deflated as possible.

(*h*) Calculated lung capacity from *f* and *g* in pounds of water,

(*k*) and hence in cubic feet.

(*l*) Spirometer reading of lung capacity ("vital capacity").

(*m*) Sturdiness ratio, calculated from *d* and *e*.

(*n*) Density, inflated, calculated by Archimedes' Principle from the formula $d/(d-f)$, i.e. from $1+f/d$, on the assumption that the density of the water is 1·000.

(*o*) Density, deflated, calculated as $1+g/d$.

The following remarks should be made in this connection.

(1) The hydrometer should be a sensitive direct reading one, capable of recording densities from ·995, say (the density of fresh water at 30° C.) to 1·030 (for dense sea-water) as the third decimal place proves to be desirable. In the work below it had to be omitted as no hydrometer reading to more than two decimal places was available; the reading was, throughout, by the instrument used, 1·00.

(2) It was found in the paper above referred to that the adult male living human body as a rule required a weight of one or two lbs. to sink it. Two weights were therefore used, weighing in water $3\frac{3}{4}$ lbs. and $7\frac{1}{2}$ lbs. respectively. A spring balance reading by ounces to 4 lbs. was also used, and was read as correctly as possible to the nearest $\frac{1}{4}$ lb., vertical oscillations preventing more precise values. The sinker was not needed in the deflated experiments and a balance reading to more than 4 lbs. is sometimes required in these cases. With boys, however, it was found difficult in the limited time available to get them to show sufficient confidence to immerse themselves, lungs deflated, for the 10—12 seconds necessary to get a reading. Non-swimmers were found usually to be lacking in confidence to release themselves from the side of the bath even with lungs inflated. The slightest effort to swim or move alters the readings by perhaps several pounds. It was found that the best way was to go down in a crouching position, with both hands holding the weight beneath the knees, and with the loop of the string that passes under the arm-pits arranged to be at the back of the body. The water must be kept quite still.

(3) In the short time available the readings and calculations *g* to *l*, inclusive, and of *o* had unfortunately to be omitted in all but a few cases. The results obtained in this connection, however, indicate an apparent change in the weight immersed of some 6 lbs., equivalent to $\frac{1}{10}$th cu. ft., or 173 cu. ins., this representing the volume of reserve air, tidal air, and some complementary air, and giving a specific gravity, deflated, of 1·025. This last point needs further investigation; it seems a striking fact which biologists may find of evolutionary importance that the density should approximate so closely to that of sea-water.

(4) The sturdiness ratio* is the value of (weight in lbs.)/(height in ins.)^3. If two individuals have the same relative proportions and are the same average density then their weights will be proportional to their volumes, i.e. to the cube of their heights, and the sturdiness ratio will be the same in each case. It will alter for either one of two reasons:

(*a*) The lankiness or solidity of build, i.e. a variation in average girth regarded purely from the geometrical point of view.

(*b*) A difference in average density, due to the relative amounts of fat, muscle and air, etc. contained in the body, the bones, etc.

* In the Report of the Chief Medical Officer of the Board of Education for 1910 (pp. 31—32) a nutritional index is used, on the same principle as the above, and calculated as $\frac{100\times\sqrt[3]{(\text{weight in kilos})}}{\text{height in cms.}}$.

It was hoped by use of this ratio to get confirmation of a suspicion that had been gaining ground in the writer's mind that fat people do not necessarily float better than those of spare build. The values of the ratio were read off from a nomogram, so that the third significant figure is unreliable.

(5) The density was calculated by first determining f/d. Usually f was negative, i.e. a boy holding the $3\frac{3}{4}$ lb. weight seemed to weigh, say, $2\frac{1}{2}$ lbs., giving f as $-1\frac{1}{4}$. The positive value indicates a reading where the boy is intrinsically heavier than fresh water: only two such were noted. Since the value of the apparent weight was correct only to the nearest $\frac{1}{4}$ lb. the error in the density is about ·001 or ·002. A further error of not more than another ·001 or ·002 will arise in taking the density of the bath water used as exactly 1·000.

The frequency curve of the ages of the boys was as follows:

Age	12—	13—	14—	15—	16—	17—	18—	Total	Median at 15 yrs. 5 mos.
Cases	5	7	18	22	12	10	6	80	

This gave rise to the following sturdiness ratio distribution:

S.R.	·000	375—	400—	425—	450—	475—	500—	525—	550—	575—	Total	Median at
Cases		8	22	23	17	6	2	1	0	1	80	·000432

This is a skew frequency curve limited at one end and unlimited at the other. This is what might be anticipated: the boys at the smaller values "have not an ounce of fat on them," they are as spare as well-nourished healthy boys could be. There is no corresponding upper limit to this ratio and thus we have one case of a boy under 5 feet who weighs nearly 9 stone. The distribution has been obtained by lopping off males above 18 years but it seems most unlikely that any Gaussian normal distribution would result if observations were taken among a homogeneous population.

The specific gravity frequency distribution for the same 80 boys is as follows:

Sp. Gr.	·915—	·925—	·935—	·945—	·955—	·965—	·975—	·985—	·995—	Total	Median at
Cases	1	0	0	4	4	16	17	26	12	80	·984

This is another skew frequency curve, limited at the upper end and unlimited at the other (apparently: the population dealt with is too small to make it worth while calculating the curve). This implies that the normal healthy young male does not exceed a certain value, say 1·005, and thus all such if placed in sea-water (density, say, 1·025) would remain on the surface. The great majority of them would remain on the surface of fresh water and probably all of those below the median value (·984) could float motionlessly horizontally with the mouth above water. At the lower end of the curve we should presumably approach asymptotically the density of human fat (or of air in the case of deep-chested people!) but the non-pathological cases will probably tail down pretty quickly and values below ·90 be extremely rare. The one case ·915— was actually 923 and was that of a boy of 13 yrs. 9 mos., weight 6 st. $0\frac{1}{2}$ lbs., height 4 ft. $6\frac{3}{4}$ ins., apparent weight, inflated, $6\frac{1}{2}$ lbs.

Although the population considered was so small, it seems worth while to get some indication of the order of the correlations concerned. We may first take those between age, height, and weight: the values obtained are:

Between		r	S.D.
Age and Weight	...	·5852	·0588
Height and Weight	...	·8458	·0254
Age and Height	...	·7100	·0443

These are the sorts of values anticipated: the regression is practically linear.

For density we have the following values:

Correlation with		r	S.D.	η
Age ...	...	·1726	·1085	$\eta_{ad} = ·295$, $\eta_{da} = ·2736$
Weight	...	·0199	·1113	
Height	...	·0009	·1108	
Sturdiness	...	−·2600	·1040	$\eta_{sd} = ·3075$, $\eta_{ds} = ·3894$

The P.E. is thus large in all cases and we cannot safely generalize, but it appears as if the specific gravity of a boy is less the sturdier he is and the greater the older he is, and that it does not depend (directly and as far as simple correlations help us) on such factors as weight and height. The density of boys, in other words, changes (in the physical sense!) as they get older rather on account of a readjustment of their build than directly on account of such obvious features as height and weight. The three more striking results can be represented in fourfold form.

For age and specific gravity we have:

Age / Sp. Gr.	Under 15	15 and over	Totals
Under ·982 ...	19	18	37
·982 and over ...	11	32	43
Totals	30	50	80

indicating that, of the boys examined, more than half the younger ones were light and nearly two-thirds of the older ones were heavy: the lighter boys were equally young and old while of the heavier boys three-quarters were above 15. In Sheppard's form, dividing the distributions at the medians, the table obtained is:

	Younger	Older	Totals
Buoyant	24	16	40
Not so Buoyant ...	16	24	40
Totals	40	40	80

The variation of the sturdiness ratio with age is as follows: the correlation table gives $r = -\cdot0934$, indicating that the older a boy the sparer he is, but a Sheppard's table gives the result:

	Younger	Older	Totals
Spare ...	22	18	40
Sturdy ...	18	22	40
Totals ...	40	40	80

giving the contrary result. This arises partly, no doubt, because the ages include the periods of "shooting-up" and of "filling-out," so that the regression is far from linear.

For sturdiness and sp. gr. we have:

S.R. / Sp. Gr.	Under 432	432 and over	Totals
Under ·982 ...	16	21	37
·982 and over ...	24	19	43
Totals	40	40	80

or as a Sheppard's table:

	Spare	Sturdy	Totals
Buoyant	18	22	40
Not so Buoyant ...	22	18	40
Totals	40	40	80

These tables give some idea of the correlations found, but it is necessary that far more observations should be obtained to draw correct conclusions and to calculate the multiple and partial correlations. It may be noted that a spurious correlation will arise in correlating the sturdiness ratio with either weight or height. The regressions of this variable, particularly with height and with specific gravity, seem to be far from linear and it appears doubtful whether it is of any value for the discussion: perhaps if trunk girth were taken as well results of value would be noted.

A table is attached giving all the measurements employed.

TABLE I.

Measurements.

Age		Weight		Height		S. R. ·000	Sp. Gr. (1000ths)
yr.	mo.	st.	lbs.	ft.	in.		
12	4	7	1	5	1	446	972
12	7	6	7	4	9⅝	474	989
12	8	6	2¾	4	10⅝	426	968
12	9	5	6¼	4	6½	470	971
12	10	8	11¾	4	11¼	588	970
13	2	7	6¾	5	4⅛	402	998
13	4	5	8	4	8⅜	430	958
13	4	6	4	4	7⅜	510	969
13	4	6	4	4	11	428	986
13	9	6	0½	4	6¾	526	923
13	9	6	2¼	5	0¼	394	957
13	11	6	8½	5	0	424	973
14	1	6	8½	5	1⅛	396	981
14	1	8	0	5	0⅜	498	995
14	1	8	2½	5	1¼	503	948
14	1	8	3	5	6	402	1000
14	2	9	1	5	6¾	432	971
14	3	8	5	5	6⅛	406	989
14	4	6	11	5	2	412	950
14	5	11	2	5	8¾	438	997
14	6	6	3	4	10⅞	422	980
14	6	6	13	5	2½	398	992
14	7	6	8	4	11	448	995
14	7	7	3½	5	0½	456	971
14	7	7	5	4	11½	422	969
14	8	7	11½	5	3⅛	432—	975
14	9	9	0¼	5	7⅜	410	978
14	10	6	12	5	2⅛	400	992
14	10	7	6½	5	0¼	472	981
14	11	5	9	4	8⅛	439	984
15	0	6	13	5	0½	440	972
15	0	8	3½	5	2⅞	480	972
15	0	9	9¾	5	10½	388	991
15	1	6	5¼	5	1⅛	388	994
15	1	7	1	5	1¼	428	970
15	2	7	9	5	2	452	970
15	3	7	1	5	2	428	1000
15	3	8	7	5	8¾	376	981
15	4	7	13¼	5	4⅛	420	1002
15	4	10	1	5	9	435	993
15	5	8	1¾	5	5¼	412	985
15	6	9	5	5	9	398	998
15	6	10	1	5	7½	462	991
15	7	6	4¾	4	11½	420	972
15	7	9	4	5	5¾	466	956
15	8	8	8	5	4⅛	450	985
15	9	7	4	5	3	410	993
15	9	8	5	5	4¾	441	994
15	9	8	9½	5	5¼	445	990
15	9	8	12	5	4½	461	1002
15	9	9	1	5	5¼	462	994
15	11	9	1	5	9¼	387	986
16	0	8	8	5	4½	454	981
16	2	9	3½	5	7¼	428	983
16	3	7	4	5	1⅝	450	980
16	4	9	13½	5	6½	482	964
16	5	9	0	5	6⅛	448	992
16	5	9	6½	5	8¼	418	980
16	7	9	9	5	7½	440	946
16	7	10	13	5	11½	418	976
16	8	7	8½	5	3⅛	418	965
16	8	9	5¼	5	4⅝	484	949
16	10	8	5½	5	5¼	426	988
16	11	9	1	5	6⅛	464	990
17	1	8	3	5	5½	412	996
17	1	8	9½	5	8	400	988
17	2	8	3	5	5⅝	412	1000
17	2	9	7	5	7⅝	430	987
17	4	9	3¾	5	8⅜	400	983
17	6	10	6	5	6¾	490	990
17	6	10	13¼	5	11½	420	985
17	8	9	10	5	7½	450	974
17	9	10	2	5	6⅛	482	986
17	10	11	2½	5	10⅝	440	1000
18	0	10	12½	5	9⅛	452	990
18	1	9	5	5	6⅛	450	975
18	2	11	3	6	0¾	410	984
18	3	11	10	5	10½	462	983
18	5	11	2¼	5	11	433	981
18	7	9	9¼	5	8¼	430	991

SUMMARY OF RESULTS.

(1) By use of Archimedes' Principle the specific gravity of the living human body, with lungs deflated or inflated, can be found.

(2) In the case of 80 boys whose ages followed a skew distribution between 12 yrs. 4 mos., and 18 yrs. 7 mos. with median at 15.5, skew distributions of sturdiness and of specific gravity were obtained.

(3) The sturdiness ratio distribution is limited at the lower (sparer) end and unlimited at the other (sturdier). The mode is near the spare end.

(4) The sp. gr. distribution is limited at 1·002, has a median at ·984, and tails down to ·923. All these boys could therefore remain motionless in sea-water, all but a very few in fresh water and a fair proportion could learn to float in fresh water with their mouths above the surface.

(5) With increasing age, boys get heavier and taller, but their sturdiness does not change to any marked extent.

(6) Although there is little alteration in their sturdiness there is considerable change in their sp. gr.

(7) The specific gravity of a boy increases as he gets older.

(8) Many more values are required to enable sound conclusions to be drawn.

TABLE II.

Frequency Distributions.

Age. Median between 15.4 and 15.5.

12.4—13.0—	13.8—14.4—	15.0—15.8—	16.4—17.0—	17.8—18.4—	Totals
5 4	9 12	15 10	9 7	7 2	80
9	21	25	16	9	80

Weight. Median at 8.5 (two cases here).

5.6—6.1—	6.10—7.5—	8.0—8.9—	9.4—9.13—	10.8—11.3—	Totals
4 11	10 7	13 12	10 5	6 2	80
15	17	25	15	8	80

Height. Median between $5.4\frac{5}{8}$ and $5.4\frac{3}{4}$.

4.6—4.8—	4.10—5.0—	5.2—5.4—	5.6—5.8—	5.10—6.0—	Totals
3 3	7 12	9 14	15 10	6 1	80
6	19	23	25	7	80

Sturdiness Ratio. Median immediately below 432. (Work in millionths.)

368—384—	400—416—	432—448—	464—480—	496—512—	528—544—	560—576—	Totals
1 7	13 19	11 14	5 5	3 1	0 0	0 1	80
8	32	25	10	4	0	1	80

Specific Gravity. Median between ·983 and ·984. (Work in thousandths.)

922—928—	934—940—	946—952—	958—964—	970—976—	982—988—	994—1000—	Totals
1 0	0 0	4 2	1 5	14 10	12 16	9 6	80
1	0	6	6	24	28	15	80

TABLE III. Correlation Tables.

Specific Gravity.

Sp. Gr.	Age.						Weight.						Height.						Sturdiness Ratio.						
	12.4	*13.8*	*15.0*	*16.4*	*17.8*	Totals	*5.6*	*6.10*	*8.0*	*9.4*	*10.8*	Totals	*4.6*	*4.10*	*5.2*	*5.6*	*5.10*	Totals	*368*	*400*	*432*	*464*	*496*	and *588*	Totals
923—	0	1	0	0	0	1	1	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	1	0	1
946—	0	3	1	2	0	6	1	1	1	3	0	6	0	2	3	1	0	6	1	1	1	2	1	0	6
958—	3	1	0	2	0	6	3	2	0	1	0	6	2	2	1	1	0	6	0	4	0	1	1	0	6
970—	3	8	8	2	3	24	5	8	6	3	2	24	1	11	4	6	2	24	2	8	10	3	0	1	24
982—	2	4	9	8	5	28	3	3	11	7	4	28	2	1	7	14	4	28	3	13	8	4	0	0	28
994—	1	4	7	2	1	15	2	3	7	1	2	15	0	3	8	3	1	15	2	6	6	0	1	0	15
Totals	9	21	25	16	9	80	15	17	25	15	8	80	6	19	23	25	7	80	8	32	25	10	4	1	80

Sturdiness Ratio.

S. R.	Age.						Weight.						Height.					
	12.4	*13.8*	*15.0*	*16.4*	*17.8*	Totals	*5.6*	*6.10*	*8.0*	*9.4*	*10.8*	Totals	*4.6*	*4.10*	*5.2*	*5.6*	*5.10*	Totals
368	0	3	5	0	0	8	3	1	2	2	0	8	0	3	1	3	1	8
400	4	9	7	10	2	32	6	10	10	3	3	32	1	7	12	9	3	32
432	1	5	11	2	6	25	2	5	8	5	5	25	1	5	7	9	3	25
464	2	1	2	4	1	10	2	1	2	5	0	10	2	1	3	4	0	10
496	1	3	0	0	0	4	2	0	2	0	0	4	2	2	0	0	0	4
and *588*	1	0	0	0	0	1	0	0	1	0	0	1	0	1	0	0	0	1
Totals	9	21	25	16	9	80	15	17	25	15	8	80	6	19	23	25	7	80

Weight.

Weight.	Age.						Height.					
	12.4	*13.8*	*15.0*	*16.4*	*17.8*	Totals	*4.6*	*4.10*	*5.2*	*5.6*	*5.10*	Totals
5.6	6	7	2	0	0	15	6	9	0	0	0	15
6.10	2	7	7	1	0	17	0	7	10	0	0	17
8.0	1	6	11	7	0	25	0	3	11	11	0	25
9.4	0	0	5	6	4	15	0	0	2	12	1	15
10.8	0	1	0	2	5	8	0	0	0	2	6	8
Totals	9	21	25	16	9	80	6	19	23	25	7	80

Height.

Height.	Age.					
	12.4	*13.8*	*15.0*	*16.4*	*17.8*	Totals
4.6	4	2	0	0	0	6
4.10	4	10	5	0	0	19
5.2	1	4	13	5	0	23
5.6	0	5	6	9	5	25
5.10	0	0	1	2	4	7
Totals	9	21	25	16	9	80

III. Note by "Student" with regard to his paper "On Testing Varieties of Cereals." *Biometrika*, Vol. XV. p. 271 *et seq.*

p. 279. I wish to apologise to the readers of *Biometrika* for having allowed it to appear that I was the author of the term 'Variance' defined as the square of the Standard Deviation. It was first used by Mr R. A. Fisher in 1918 in a paper entitled: "The Correlation between relatives on the Supposition of the Mendelian Inheritance." *Trans. Roy. Soc. Edin.* 52^2: 399—433; and he has published many papers since in which the word has been used.

p. 283. Line thirteen from top *for* $n\sum_1^m(\bar{X}_p-X)^2$ *read* $n\sum_1^m(\bar{X}_p-\bar{\bar{X}})^2$.

Line four from bottom *for* $m\sum_1^n(X_q-X)^2$ [in some copies] *read* $m\sum_1^n(\bar{X}_q-\bar{\bar{X}})^2$.

Line three from bottom

for $\sum_1^m\sum_1^n(X_{pq}-\bar{X}_p-\bar{X}_q+X)^2$ *read* $\sum_1^m\sum_1^n(X_{pq}-\bar{X}_p-\bar{X}_q+\bar{\bar{X}})^2$.

IV. Extract from a Letter of Dr E. I. Gumbel to the Editor of *Biometrika* concerning his paper in the present issue, pp. 283—296.

Ich muss leider einen Fehler berichtigen, der in meiner Arbeit "Eine Darstellung der Sterbetafel" vorhanden ist. Es heisst auf S. 291, da wo ich für die 3 Spezialfälle das mittlere Alter der Lebenden berechne, beim Fall $n=0$: "Das mittlere Alter der über x jährigen ist also hier stets grösser als die Lebenserwartung eines x jährigen. Dies gilt, wie mir Herr Professor Pearson mitgeteilt hat, ganz allgemein für die Gompertz-Makehamsche Formel, für die ja der Fall $n=0$ einen Spezialfall darstellt." An Stelle dessen muss es heissen: "Wie mir Professor Pearson mitgeteilt hat, gilt für die Gompertz-Makehamsche Formel im allgemeinen der Satz

$$x+E(x)>\mathring{e}_x$$

während hier in dem $n=0$ entsprechenden Spezialfall das Gleichheitszeichen eintritt."

Mein Versehen ist dadurch entstanden, da ich die Grösse $E(x)=\frac{1}{l(x)}\int_x^\omega l(x)\,dx$ als Lebenserwartung bezeichne, während bei Ihnen die Grösse $x+E(x)$ als Lebenserwartung bezeichnet wird.

Einige Zeilen später heisst es: "Die Lebenserwartung eines Neugeborenen grösser ist als das mittlere Alter der Lebenden, lässt sich also nicht auf alle Alter erweitern." Hier fehlt die Bemerkung, "da $\mathring{e}_x$ sehr bald viel grösser als $E(x)$ wird."

Ich wäre Ihnen ausserordentlich verbunden, falls es Ihnen noch möglich wäre, diese Aenderung und diesen Zusatz in meinem Artikel anzubringen. Sollte es nicht mehr möglich sein, so würde ich bitten, dass diese Bemerkung als Berichtigung erscheint.

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CONTENTS

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CONTENTS

PAGE

I. On an Interspecific Hybrid of *Digitalis*. By ERNEST WARREN, D.Sc. (London). (With six plates and four figures in the text) 205

II. On the Problem whether two given Samples can be supposed to have been drawn from the same Population. By E. C. RHODES, M.Sc. (London) . 239

III. On the Difference and the Doublet Tests for Ascertaining whether two Samples have been drawn from the same Population. By KARL PEARSON, F.R.S. 249

IV. The Skull of Robert the Bruce, King of Scotland, 1274—1329. By KARL PEARSON, F.R.S. (With sixteen plates and three figures in the text—one folding) 253

V. A Pedigree of Congenital Dislocation of Lenses. By C. H. USHER, M.B., Ch.B. (With three diagrams in the text) 273

VI. Eine Darstellung der Sterbetafel. Von E. I GUMBEL, Heidelberg. (With three figures in the text) 283

Note. To prove that the Expectation of Life at any Age is greater than the Mean Age of all Persons living greater than this Age. By KARL PEARSON, F.R.S. 297

VII. On the Relationship between Strength of Grip and Certain Mental and Sensory Characters. By WILLIAM ARTHUR, M.A. (With nine diagrams in the text) 299

VIII. On the Biometric Constants of the Human Skull. By KARL PEARSON, F.R.S. and ADELAIDE G. DAVIN, B.Sc. (With one folding table) 328

IX. Cancer and Goitre. By PERCY STOCKS. (With three maps in the text) . 364

Miscellanea:

(i). Historical Note on the Origin of the Normal Curve of Errors. By KARL PEARSON 402

(ii). A Preliminary Inquiry into the Density of the Living Male Human Body. By FRANK SANDON, M.A. 404

(iii). Note by "Student" with regard to his paper "On Testing Varieties of Cereals." 411

(iv). Note by Dr GUMBEL correcting certain passages in his memoir "Eine Darstellung der Sterbetafel" 411

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